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PREFACE.

THIS work is intended partly, but not entirely, for the use of students preparing for musical examinations. The Universities of Cambridge and London set papers in Acoustics at their examinations for musical degrees, and a paper in Acoustics also forms part of the Cambridge Middle Class Examinations. There is, it is believed, little doubt that the other Universities will in due course follow the example set by Cambridge and London, by adding Acoustics to their curriculum for musical degrees. Unless the demands of the other Universities should hereafter be very much more severe than those of Cambridge and London, the Student may, after he has conscientiously worked out the questions and problems at the end of the book, confidently sit down to work any paper likely to be set in Acoustics.

There is, moreover, in addition to students preparing for examination, a large and increasing public which takes a deep interest in the natural basis of music. To say that Acoustics forms no part of the necessary mental furniture of a musician, or that

the great masters would not have composed better music had they been acquainted with the scientific phenomena of sound, is altogether beside the question. Cultured people want nowadays to know as much as possible about the causes of things; and in writing this book I have kept in mind the latter class' and have endeavoured to place in their hands reliable information from the works of the best authorities. To secure this end I have consulted and quoted from the following books :—

"On the Sensations of Tone as a Physiological Basis for the Theory of Music." By H. L. F. Helmholtz. (Translated from the German by A. J. Ellis, B.A.) Price 36s. Longmans.

Stainer and Barrett's "Dictionary of Musical Terms." Price 16s, Novello & Co.

"The Philosophy of Music." By Dr. Wm. Pole. Price 10s. 6d. Trübner & Co.

"On Sound and Atmospheric Vibrations." By Dr. G. B. Airy, Astronomer Royal. Price 9s. Macmillan & Co.

"On Sound." By Dr. Tyndall. Price 10s. 6d. Longmans.

"Sound and Music." By Sedley Taylor, M.A. Price 8s. 6d. Macmillan & Co.

"Elementary Lessons on Sound." By Dr. W. H. Stone. Price 3s. 6d. Macmillan & Co.

It would, of course, have been an easy matter to re-write these quotations, and give their meaning in my own words; but I have judged it better to support my own statements by the undeniable authority of well-known names, rather than ask the reader to

take everything upon trust. The object of the present book is to give, in one volume, a good general view of the subject to those who can neither spare time to read nor money to buy a number of large and expensive works. Readers who exhaust this work and resolve to dig for further treasure will find, in the authors above named, ample reward for their labour.

J. B.

WEST VIEW, HADLEY GREEN,
November 1880.

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CHAPTER I.

SENSATION AND EXTERNAL CAUSE OF SOUND.

1. SOUND is "something heard." The ear is the organ by which what we call "sound" is made palpable to the brain, or the channel through which the cause of sound is carried to that part of the organism in which we are made conscious of sound. The cause of sound is without us; the sensation of sound is within us; the ear is the medium which enables the external cause to become a sensation.

2. The sensation of sound is beyond our power to analyse. We know it because we experience it; but although we can trace the motions which produce it, and the machinery which transmits those motions from without to within, we cannot at present tell how the latter become a part of our consciousness, any more than we can say why the material we call "brain" can see, or feel, or think, or know anything at all. In speaking, therefore, of the "sensation of sound," we speak of what is within the knowledge of all persons not stone-deaf, but which is to be dealt with by psychology much more than by

physiology, and by acoustics not at all. Not being able to trace it, we cannot describe it, but can only define it by saying that "the sensation of sound is what is experienced when we hear anything."

"Sensations result from the action of an external stimulus on the sensitive apparatus of our nerves. Kinds of sensation differ partly with the organ of sense excited, and partly with the kind of stimulus employed. Each organ of sense produces peculiar sensations, which cannot be excited in any other; the eye gives sensations of light, the ear sensations of sound, the skin sensations of touch. Even when the same sunbeams which excite in the eye sensations of light, impinge on the skin and excite its nerves, they are felt only as heat, not as light. In the same way the vibration of elastic bodies heard by the ear can also be felt by the skin, but in that case produce only a whirring, fluttering sensation, not sound. The sensation of sound is, therefore, a species of reaction against external stimulus, peculiar to the ear, and excitable in no other organ of the body, and is completely distinct from the sensation of any other sense. The various nerves of the human body have their origin in the brain, which is the seat of sensation. When the finger is wounded, the sensor nerves convey to the brain intelligence of the injury, and if these nerves be severed, however serious the hurt may be, no pain is experienced. We have the strongest reason for believing that what the nerves convey to the brain is, in all cases, *motion*. The motion here meant is not, however, that of the nerve as a whole, but of its molecules or smallest particles. Different nerves are appropriated to the transmission of different kinds of molecular motion. The nerves of taste, for example, are not competent to transmit the tremors of light, nor is the optic nerve competent to transmit sonorous vibrations. For

these, a special nerve is necessary, which passes from the brain into one of the cavities of the ear, and there divides into a multitude of filaments. It is the motion imparted to this, the *auditory nerve*, which, in the brain, is translated into sound."—*Tyndall.*

3. The external cause of sound is easier to deal with, seeing that we can examine, experiment, calculate, and reason upon it. It may be defined, broadly, as whatever produces within us the sensation of hearing. Coming to particulars, the external cause of sound is usually the motion of air which is first excited by various causes. The nature and mode of that motion will be discussed later; here it will suffice to show that without some medium—such as air or water—sound could never exist. The causes may exist which under proper conditions would produce sound; but without some medium or mode of communication between the object producing the sound and the ear which conveys it to the brain, there could be no such thing as sound, properly so called.

4. That some medium is necessary to transmit the motions of the air which produce sound may be demonstrated. The well-known experiment with the "exhausted receiver" will here recur to the mind of every student of natural philosophy. A small bell is suspended in a glass chamber, and the chamber connected with an air-pump. This bell, which is connected with clock-work, is so placed that it can be rung when the whole or any part of the air has been

pumped out of the chamber. If the bell is rung before air is removed, it gives out its ordinary sound; the more air is taken from the chamber, the less intense is the sound; and when the chamber is exhausted of air, the bell is not heard. The gradual return of the air increases the volume of tone, and when the chamber is filled, the bell sounds as at first.

5. The displacement of air is, primarily, the cause of sound. Whatever be the mechanical means by which sound of any kind is produced, it is the displacement of air, and the impact of the effect of that displacement upon the nerves of the ear, which enable such causes to be translated into sound. The string of the violin or piano, the reed of the harmonium, the current of air in an organ pipe, the lips of the player of a trumpet or horn, and the vocal chords in the larynx intercepting air from the lungs, all produce sound by first of all giving motion to air.

"Applying a flame to a small collodion balloon which contains a mixture of oxygen and hydrogen, the gases explode, and every ear in this room is conscious of a shock, which we name a sound. How was this shock transmitted from the balloon to our organs of hearing? Have the exploding gases shot the air-particles against the auditory nerve as a gun shoots a ball against a target? No doubt in the neighbourhood of the balloon there is to some extent a propulsion of particles; but no particle of air from the vicinity of the balloon reaches the ear of any person here present. The process was this: When the flame touched the mixed gases they combined chemically, and their union

was accompanied by the development of intense heat. The heated air expanded suddenly, forcing the surrounding air violently away on all sides. This motion of air close to the balloon was rapidly imparted to that a little farther off, the air first set in motion coming at the same to rest. The air at a little distance passed its motion on to the air at a greater distance, and came also in its turn to rest. Thus each shell of air, if I may use the term, surrounding the balloon took up the motion of the shell next preceding, and transmitted it to the next succeeding shell, the motion being thus propagated as a *pulse* or *wave* through the air."—*Tyndall.*

6. Seeing that in the great majority of cases sound reaches the ear through the air, it is not necessary to enter here into any elaborate discussion of other modes of transmitting sound; but it should be mentioned that air is not, of course, the only medium through which sound-impulses can travel. Water, wood, steel, and other things may be named. A loud noise will startle fishes, and prove that they hear sound which can reach them in no other way than through the water. A thin rod of wood, attached to the sound-board of a pianoforte, has conveyed tones to a room where they could not possibly have been heard without such connection: and, as a more familiar instance of the sound-carrying power of wood, a gentle tapping at one end of a felled tree may be heard at the other end if the ear be firmly pressed against it in a still and otherwise silent atmosphere. Even a piece of common cotton or thread will convey sound considerably over a hundred

yards, if it is attached at each end to the centre of a small disc of parchment for the receipt and delivery of the message, as in the following figure, where A, A, represent the discs of parchment, through the centre of which the cotton, B, is passed, and secured by a knot on the inner side; C, C, being narrow rims, or flat rings of wood or tin over which the parchment is fastened:—

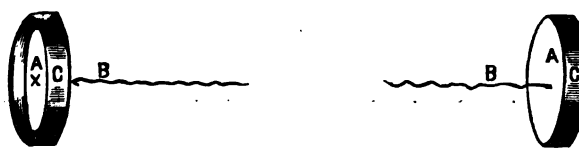


Fig. 1.

If this "toy telephone" is kept with the cotton stretched, and a message is spoken into one disc, a listener at the other will hear it quite distinctly. The telegraph wire, too, which has so long been transmitting signs only, is now being utilised for the transmission of sound; and verbal messages, prayers, hymns, sermons, and musical performances of various kinds have been heard through many miles of wire. These, and other instances which will suggest themselves, prove that air is not the only medium through which sound-impulses can pass. But as other media are, as yet, comparatively seldom used for practical purposes, the present work refers, except as otherwise stated, to air alone as a transmitter of sound.

7. We are led at the outset to connect sound with

motion. The terrific explosion of gunpowder on the Regent's Park canal, some years since, broke innumerable windows in the neighbourhood of the disaster—not by hurling masses of *débris* against the glass, but from no other cause than the violent motion imparted to the air. This motion is greater in its force in proportion to the intensity of the sound, but it is connected with all sound, and is, in fact, the only means by which sound is transmitted in air at all. The longer strings of a pianoforte vibrate visibly, and thus the eye can see the cause while the ear notes the effect. Even where it is almost too fine to be seen by the eye—as at the upper end of the pianoforte—it is yet motion which informs the ear that the wire has been struck; and when there is no visible vibrating body, it is impossible to arrive at any other conclusion than that a moving body moves the air, which in its turn moves the aural nerve, and so makes us conscious of sound

8. These motions may be regular or synchronous, *i.e.*, vibrating equal numbers in equal periods; or they may be irregular and uncertain. The first of these results in a musical tone; the second in a noise. These two classes comprise all possible sounds. A sound which, when produced singly, remains a noise, becomes a musical sound when its production recurs a stated number of times per second. If a disc of cardboard, having eight holes pierced at regular intervals round its rim as follows,—

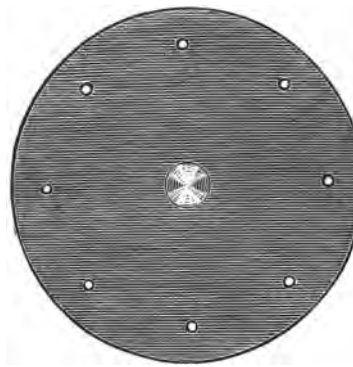


Fig. 2.

be fixed on a spindle so that it can be turned round, and the holes be blown through, as they pass, by the mouth or the end of a pipe, the puffs of wind so made will remain puffs while the disc turns slowly; but when it is turned so that forty or fifty puffs occur in a second, they will unite to form a tone. If the speed be varied—now slow, now fast—a noise will be the result; if the speed be steady, a musical tone will be the result. The student can verify this for himself with ease, and we would advise that, whenever possible, acoustics should be studied experimentally. The trouble involved will be repaid a thousandfold by the results obtained. The following are excellent definitions of sound and noise:—

"The first and principal difference between various sounds experienced by our ear, is that between *noises* and *musical tones*. The soughing, howling, and whistling of the wind, the splashing of water, the rolling and rumbling of carriages, are examples of the first kind, and the tones of all musical

instruments of the second. Noises and musical tones may certainly intermingle in very various degrees, and pass insensibly into one another, but their extremes are widely separated. The nature of the difference between musical tones and noises can generally be determined by attentive aural observation without artificial assistance. We perceive that generally a noise is accompanied by a rapid alternation of different kinds of sensations of sound. Think, for example, of the rattling of a carriage over granite paving stones, the splashing or seething of a waterfall, the waves of the sea, or of the rustling of leaves in a wood. In all these cases we have rapid, irregular, but distinctly perceptible alternations of various kinds of sounds which crop up fitfully. When the wind howls the alternation is slow, the sound slowly and gradually rises and then falls again. It is more or less possible to separate restlessly alternating sounds from the greater number of other noises. We shall hereafter become acquainted with an instrument called a resonator, which will materially assist the ear in making this separation. On the other hand, a musical tone strikes the ear as a perfectly undisturbed uniform sound which remains unaltered as long as it exists, and it presents no alternation of various kinds of constituents. To this then corresponds a simple regular kind of sensation, whereas in a noise many various sensations of musical tone are irregularly mixed up, and, as it were, tumbled about in confusion. We can easily compound noises out of musical tones, as, for example, by striking all the keys contained in one or two octaves of a pianoforte at once. This shows us that musical tones are the simpler and more regular elements of the sensations of hearing, and that we have consequently first to study the laws and peculiarities of this class of sensations. Then comes the question: On what difference in the external means of excitement does the difference between noise and

musical tone depend? The normal and usual means of excitement for the human ear is atmospheric vibration. The irregularly alternating sensation of the ear in the case of noises leads us to conclude that for these the vibration of the air must also change irregularly. For musical tones, on the other hand, we anticipate a regular motion of the air, continuing uniformly, and in its turn excited by an equally regular motion of the sonorous body, whose impulses were conducted to the ear by the air. Those regular motions which produce musical tones have been exactly investigated by physicists. They are *oscillations*, *vibrations*, or swings, that is, up and down, or to and fro, motions of sonorous bodies, and it is necessary that these oscillations should be regularly *periodic*. By a *periodic motion* we mean one which constantly returns to the same condition after exactly equal intervals of time. The length of the equal intervals of time between one state of the motion and its next exact repetition, we call the *length of the oscillation*, vibration, or swing, or the *period* of the motion. The kind of motion of the moving body during one period is perfectly indifferent. As illustrations of periodical motion, take the motion of a clock pendulum, of a stone attached to a string and whirled round in a circle with uniform velocity, of a hammer made to rise and fall uniformly by its connection with a water wheel. All these motions, however different be their form, are periodic in the sense here explained. The length of their periods, which in the cases adduced is generally from one to several seconds, is relatively long in comparison with the much shorter periods of the vibrations producing musical tones, the lowest or deepest of which makes at least thirty in a second, while in other cases their number may increase to several thousands in a second. Our definition of periodic motion then enables us to answer the question proposed as follows: *The sensation of a musical tone is due to a rapid*

periodic motion of the sonorous body; the sensation of a noise to non-periodic motions."—Helmholtz.

"Noise effects us as an irregular succession of shocks. We are conscious while listening to it of a jolting and jarring of the auditory nerve, while a musical sound flows smoothly and without asperity or irregularity. How is this smoothness secured? *By rendering the impulses received by the tympanic membrane perfectly periodic.* A periodic motion is one that repeats itself. The motion of a common pendulum, for example, is periodic, but its vibrations are far too sluggish to excite sonorous waves. To produce a musical tone we must have a body which vibrates with the unerring regularity of the pendulum, but which can impart much sharper and quicker shocks to the air.

"Imagine the first of a series of pulses following each other at regular intervals, impinging upon the tympanic membrane. It is shaken by the shock; and a body once shaken cannot come instantaneously to rest. The human ear, indeed, is so constructed that the sonorous motion vanishes with extreme rapidity, but its disappearance is not instantaneous; and if the motion imparted to the auditory nerve by each individual pulse of our series continue until the arrival of its successor, the sound will not cease at all. The effect of every shock will be renewed before it vanishes, and the recurrent impulses will link themselves together to a continuous musical sound. The pulses, on the contrary, which produce noise are of irregular strength and recurrence. The action of noise upon the ear has been well compared to that of a flickering light upon the eye, both being painful through the sudden and abrupt changes which they impose upon their respective nerves.

"The only condition necessary to the production of a musical sound is that the pulses should succeed each other

in the same interval of time. No matter what its origin may be, if this condition be fulfilled the sound becomes musical. If a watch, for example, could be caused to tick with sufficient rapidity—say one hundred times in a second—the ticks would lose their individuality and blend to a musical tone; and if the strokes of a pigeon's wings could be accomplished at the same rate, the progress of the bird through the air would be accompanied by music. In the humming-bird the necessary rapidity is attained; and when we pass on from birds to insects, where the vibrations are more rapid, we have a musical note as the ordinary accompaniment of the insects' flight. The puffs of a locomotive at starting follow each other slowly at first, but they soon increase so rapidly as to be almost incapable of being counted. If this increase could continue up to fifty or sixty puffs a second, the approach of the engine would be heralded by an organ peal of tremendous power."—*Tyndall.*

9. The puffs of air through the cardboard disc, or vibrational motions of any kind which disturb the air so as to produce a continuous sound, will not result in a tone if below a certain number per second, though what that number is has been variously stated by different authors. Helmholtz puts it at sixteen per second. The student will notice, immediately he hears a note at all, that to increase the rate of rotation of the disc will make the note higher, and, *vice versâ.* This will convince him of three important points, from which he must set out in his investigation of the science of acoustics as connected with music, and which must be the foundation on which his temple of acoustic learning will be built:—

a. Sound is produced by motions, or vibrations, imparted to the atmosphere by some moving body.

b. When such motions or vibrations occur regularly at a given number per second, a musical sound is produced.

c. When the number of these vibrations is increased, the sound is raised in pitch—i.e., a higher, or shriller, note is produced.

How many vibrations per second can be recognised as a sound has not been precisely determined, as the hearing capacity, both for high and low notes, varies with different persons. For all the ordinary purposes of practical music, the limits of sound may be fixed between 40 and 4000 vibrations per second; other sounds can be heard and used, but they are only with difficulty recognised as tones by an ordinary ear, and by some people not at all. This limit gives a range of about seven octaves, the extent of a modern pianoforte; and any notes beyond this limit can only serve the purpose of strengthening notes within it, and only well-practised ears can recognise as musical sounds even the extreme notes of such seven octaves, which do not really extend the compass of the pianoforte for musical purposes. It must be understood, therefore, that there is a limit on either side—though from the nature of things, and the varied powers of hearing in different people, it is not a very clearly defined limit—beyond which vibrations no longer coalesce to form musical tones.

Savart 8 per sec.

"The ear's range of hearing is limited in both directions. Savart fixed the lower limit at eight complete vibrations a second; and to cause these slowly-recurring vibrations to link themselves together, he was obliged to employ shocks of great power. By means of a toothed wheel and an associated counter, he fixed the upper limit of hearing at 24,000 vibrations a second. Helmholtz has recently fixed the lower limit at sixteen vibrations, and the higher at 38,000 vibrations, a second. By employing very small tuning-forks, the late M. Depretz showed that a sound corresponding to 38,000 vibrations a second is audible. Starting from the note sixteen and multiplying continually by two, or more compendiously raising two to the eleventh power, and multiplying this by sixteen, we should find that at eleven octaves above the fundamental note the number of vibrations would be 32,768. Taking, therefore, the limits assigned by Helmholtz, the entire range of the human ear embraces about eleven octaves. But all the notes comprised within these limits cannot be employed in music. The practical range of musical sounds is comprised between 40 and 4000 vibrations a second, which amounts, in round numbers, to seven octaves.

"The limits of hearing are different in different persons. While endeavouring to estimate the pitch of certain sharp sounds, Dr. Wollaston remarked in a friend a total insensibility to the sound of a small organ-pipe, which, in respect to acuteness, was far within the ordinary limits of hearing. The sense of hearing of this person terminated at a note four octaves above the middle E of the pianoforte. The squeak of the bat, the sound of a cricket, even the chirrup of the common house-sparrow, are unheard by some people who for lower sounds possess a sensitive ear. A difference of a single note is sometimes sufficient to produce the change from sound to silence. 'The suddenness of the

transition,' writes Wollaston, 'from perfect hearing to total want of perception, occasions a degree of surprise which renders an experiment of this kind with a series of small pipes among several persons rather amusing. It is curious to observe the change of feeling manifested by various individuals of the party, in succession, as the sounds approach and pass the limits of their hearing. Those who enjoy a temporary triumph are often compelled, in their turn, to acknowledge to how short a distance their little superiority extends.' 'Nothing can be more surprising,' writes Sir John Herschel, 'than to see two persons, neither of them deaf, the one complaining of the penetrating shrillness of a sound, while the other maintains there is no sound at all. Thus, while one person mentioned by Dr. Wollaston could but just hear a note four octaves above the middle E of the pianoforte, others have a distinct perception of sounds full two octaves higher. The chirrup of the sparrow is about the former limit; the cry of the bat about an octave above it; and that of some insects probably another octave.' In 'The Glaciers of the Alps' I have referred to a case of short auditory range noticed by myself, in crossing the Wengern Alp in company with a friend. The grass at each side of the path swarmed with insects, which to me rent the air with their shrill chirruping. My friend heard nothing of this, the insect-music lying beyond his limit of audition."—*Tyndall.*

[The student is strongly recommended to turn to the "Examination Questions" at the end, and work carefully through those referring to any given chapter before he reads the next.]

CHAPTER II.

TRANSMISSION OF SOUND.

10. WE have now to consider how vibrations of air produced in one place are transmitted to another. That this is the case needs no proof, seeing that every day's experience demonstrates it a thousand times over. If it were not so, no sound would be heard whose vibrations were not made on the tympanic membrane of the ear itself. Every vibration must travel and reach some ear before it can become sound at all; and we will now try to see how it travels. The question of softness or loudness, height or depth, smoothness or roughness, does not concern us here. We have to inquire the process by which *all* sounds alike travel from place to place.

11. The vibratory motions of the air are not carried from one point to another by a current or rush of wind; nor is the whole body of air between the ear and the vibrating cause pushed aside to allow of the passage of the actual particles of air first affected. We can hear in the stillness of the summer evening the croak of the frog in the pool at our feet, the screech of the night-owl in the wood a quarter of a

mile away, and the shriek of the railway whistle two or three miles off, and though these sounds may come to us in different directions, we can hear them at the same moment, even though the atmosphere is in a state of absolute calm, and when, as is usually said, "not a breath of air is stirring." Sounds can be heard even when a current of air is moving from us directly towards the place whence they proceed. The most delicate notes of a violin solo can be heard in a concert-room at the farthest point from the player, even if a draught of air is proceeding straight from the listener to the instrument; and a series of cross-currents, by which the air is whirled hither and thither in all directions, will not prevent the voice of a singer from being heard all over a vast hall or cathedral. We must therefore seek some other explanation.

12. It must be remembered, to begin with, that sound is not something solid and tangible, which is carried bodily from place to place, but is the result of a certain condition of the atmosphere, and that it is not transmitted by anything solid moving in the air, but by the motion excited in any portion of air being imparted to some other portion. Air is elastic; it can be compressed or rarefied, and with an air-pump so much air may be forced into a glass vessel that it will burst from the pressure within, or, on the other hand, so much air may be pumped out of it that it will burst by the pressure from without. A given portion of air will therefore occupy a greater or less

space according to the nature and extent of the influence brought to bear upon it. This elasticity of particles of air may be familiarly illustrated. If a number of billiard balls be placed in a straight line, thus—

Fig. 3.

and A be gently driven against C, B will receive the motion imparted to A without any visible motion on the part of the intervening balls; and on a perfectly smooth slate table, B will be propelled to *b*, the same distance as from A to *a*. The force of A is imparted to the first ball at rest; the first imparts it to the second; the second to the third, and so on through the series, until B having no ball to resist or take up its motion, leaves its place with as much force as the cue gave to A. The balls are elastic, and there is motion from end to end of the series without any perceptible motion of the whole series itself. In an exactly similar way, particles of air, being set in motion by a vibrating body, impart motion to each other, until the drum of the ear being affected by the motion of the particles next to it, the sensation of sound is received by the brain. It will be sufficient at present if the student contents himself with regarding sound as coming direct from its cause to his ear, without taking into account the spreading of the vibratory motion in every direction. If a thousand

listeners be at the same distance from a vibrating body, and the conditions under which the vibrations are transmitted (such as temperature, &c.) are equal in all these cases, the operation which sends the vibration along, and translates it into sound, will in each instance be exactly alike.

"Scientific education ought to teach us to see the invisible as well as the visible in nature; to picture with the vision of the mind those operations which entirely elude bodily vision; to look at the very atoms of matter in motion and at rest, and to follow them forth, without ever once losing sight of them, into the world of the senses, and see them there integrating themselves in natural phenomena. With regard to the point now under consideration, we must endeavour to form a definite image of a wave of sound. We ought to see mentally the air particles when urged outwards by the explosion of our balloon crowding closely together; but immediately behind this condensation we ought to see the particles separated more widely apart. We must, in short, be able to seize the conception that a sonorous wave consists of two portions, in the one of which the air is more dense, and in the other of which it is less dense than usual. A condensation and a rarefaction, then, are the two constituents of a wave of sound."—*Tyndall.*

13. These then, are important truths to learn in trying to form an idea of the actual transmission of sound:—

a. Sound is not something tangible or solid, propelled through the air.

b. Sound is caused by the passing of an altered condition of a portion of air from one point to another.

c. This altered condition may pass along without any perceptible motion of the whole body of air affected by it.

The determination of the velocity of sound must depend upon the medium through which it travels, and, if it is air, upon the temperature of that air. The following table is from Professor Airy's work on "Sound":—

Authority.	Experimenter.	Distance in feet.	Velocity in feet.	Temp. Fahren.
Phil. Trans. 1823	Goldingham	29,547	1089.9	32°
		13,932	1079.9	32
Conn. des Temps, 1825	Arago and others	61,064	1086.1	32
Vienna Jahrbuch, Vol. vii.	Myrbach and Stamfer	32,615	1092.1	32
Phil. Trans. 1824.	Moll and Van Beek	57,839	1089.4	32
Camb. Trans. Vol. ii.	Gregory	13,440	1097	33
			1085.8	64
		9,874	1117	66
Parry's 3d Voyage	Parry and Forster	12,893	1014.4	− 38.5
			1010.3	− 37.5
			1029.0	− 37.0
			1021.0	− 24.5
			1026.6	− 21.5
			1039.3	− 18.0
			1037.3	− 9.0
			1040.5	− 7.0
			1098.3	+ 33.5
			1118.1	35
Mem. de l'Académie, tome 37	Regnault	Various	1085.0	32

This table represents the most trustworthy experiments made since the year 1820.

"In the greatest part of the experiments, the observations have been those of the flash and the report of a distant cannon. The flash, and the first disturbance of air by the

emission of gas, occur so nearly or exactly at the same instant, that no sensible error arises from the difference in the nature of these two phenomena. The same observer observes both phenomena with the same watch or clock; and if the distance of the gun be several miles, there is ample time for the observer to write down the observation of the flash before preparing himself for the observation of the sound. All these circumstances are very advantageous. The gun is usually pointed towards the observer, and it seems probable that this circumstance may slightly accelerate the pulse of air in the beginning of its course, but possibly by a few feet only, corresponding to an imperceptible error of time.

"But there is a physiological circumstance, the effects of which have hitherto escaped notice, but which probably produces a sensible error; it is, that two different senses (sight and hearing) are employed in the observation of the two phenomena, and we are not certain that impressions are received on them with equal speed. Indeed we believe that the perception of sound is slower by a measurable quantity, perhaps $0^{s}\cdot2$, than the perception of light; and this may affect the result with an error amounting to some hundreds of feet.

"We should much prefer a plan of observation in which two observers observed, in the same manner, the time of the sound passing two stations. By using signals given reciprocally from two stations beyond both the observing stations, it will be easy to obtain a result for the time of passage of the sound, independent of the habits of each observer, independent of the difference of the indications of their time-keepers, and independent of the velocity of the wind. (The reader will verify this without difficulty, by putting algebraical symbols for the different elements just mentioned; when it will be found that, on taking the mean

of the two apparent times occupied by the passage of sound, according as the gun at first station or at second station is used, those elements disappear.) A process of this kind is employed in the measurement of higher velocities, as the velocity of the galvanic current in a telegraph-wire.

"Difficulties have sometimes been experienced, by persons not familiar with astronomical practices, in the estimation of fractions of a second of time. To avoid these, a time-piece was employed in the Dutch experiments to be mentioned below (perhaps, on the whole, the best which had been made before those of M. Regnault) in which the motion, being regulated by a pendulum revolving in a conical form, was free from the jerks of a common clock, and the index could be stopped at any fraction of a second.

"A most elaborate series of experiments by M. V. Regnault is published in the *Mémoires de l'Académie des Sciences*, tome XXXVII., occupying 575 pages. The most important were made in tubes prepared for conveyance of gas and water in the neighbourhood of Paris: these tubes varied in diameter from 0·108 mètre to 1·10 mètre, and in length from 961 to 4886 mètres. The general principle in all was, to cover the near end of the tube with a firm plate (except, in some early experiments), in which was a hole through which a pistol barrel was thrust, and a charge of powder, or sometimes a large percussion cap, was fired. The distant end of the tube was covered with a sheet of caoutchouc, which was made to tremble by the shock of the air-wave: sometimes it produced a reflexion to the firm plate, and from it to the caoutchouc again, &c. The pistol-explosion broke a galvanic circuit, and the trembling of the caoutchouc restored it: and these galvanic effects were registered upon a revolving barrel, on which were also registered the

beats of a clock and the vibrations of a tuning-fork. In some experiments, laminæ of caoutchouc were applied to apertures in the sides of the tube at different distances. Finally, experiments were made in the same way without tubes, using the explosion of a heavy cannon. Experiments were also made on the velocity of sound through air of different densities, and through various gases. These, we believe, are the only experiments in which there has been no reference to human nerves."—*Airy.*

The next table represents the theoretical velocity of sound at different temperatures Fahrenheit:—

Temp.	Velocity.	Temp.	Velocity.
	feet		feet
−40°	1014.1	32°	1099.5
−30	1026.4	40	1108.6
−20	1038.5	50	1119.9
−10	1050.5	60	1131.1
0	1062.4	70	1142.2
10	1074.1	80	1153.0
20	1085.7	90	1163.8
30	1097.2	100	1174.5

Experiments on the velocity of Sound through gases.

"It is impossible for us to form an atmosphere of hydrogen gas or of carbonic acid, extending several miles, and therefore it is impossible for us to experiment on the velocity of sound through gas in the same way as through air. To explain the process which has been successfully used, we must here anticipate the results of a subsequent section. It must be understood, then, that when an organ-pipe is sounded in the usual way, the frequency of sound-waves which it produces depends upon the time occupied by a wave's travelling from one end of the pipe to the other (or, in certain cases, travelling twice the length of the pipe);

and it must also be understood that every definite frequency of sound-waves produces a definite musical note to the ear. Consequently, with a given organ-pipe, the musical note produced will depend on the velocity of the wave's travel; and the accurate observation of the musical note will give accurate information on that velocity. It is only necessary therefore to inclose an organ-pipe in an atmosphere of the gas upon which it is desired to experiment, and to adapt to it apparatus for blowing the gas in the same manner in which air is blown for the ordinary sounds of the organ-pipe, and to remark the note which it produces; the relation of this note to the note which the same pipe produces in air, interpreted with reference to the theory of musical tones, which we shall explain in a subsequent section, gives the proportion of the frequencies of sound-waves in the pipe, and the proportion of the velocities of the wave's progress in the gas and in air.

"Thus the experimental numbers in the following table have been obtained (Dulong, *Mémoires de l'Institut*, tome x.). Instead of giving the actual velocity of sound in each gas, it has appeared more convenient to give the proportion of each velocity to the velocity in air at the same tempera-

Name of Gas.	Proportion of specific gravity to that of air.	Theoretical proportion of sound-velocity in gas to that in air.	Observed proportion of sound-velocity in gas to that in air.
Oxygen gas	1·1026	0·9523	0·9525
Hydrogen gas	0·0688	3·8125	3·8123
Carbonic acid	1·524	0·8100	0·7855
Oxide of carbon	0·974	1·0133	1·0132
Oxide of azote	1·527	0·8092	0·7865
Olefiant gas	0·981	1·0096	0·9439

ture. The theoretical proportions of velocities are computed by the formula of Article 60.*

* In Airy's work

"The defect in the observed velocity in carbonic acid, oxide of azote, and olefiant gas, indicates a value of n (Articles 16, 21), smaller for those gases than for atmospheric air. This circumstance is connected with a chemical theory of 'specific heat,' for which we refer the reader to treatises on Modern Chemistry.

"M. Regnault found from direct experiments in tubes,

Hydrogen	3·801
Carbonic Acid	0·7848 0·8009
Oxide of Azote	0·8007
Ammoniacal gas sp. grav. 0·596	1·2279

Experiments on the velocity of Sound through solid bodies.

"It is easy to perceive the difference in the velocities of sound, as transmitted by the air, or as transmitted by metals (where the portions of metal are united by solder, &c., so as to form a continuous piece of great length, or where their parts are forced into firm contact by considerable tension). We have remarked, for instance, that a strong chain, lying upon a long and steep incline of a railway, transmits sound well. If the chain is struck at one end, and an observer at the other end applies his ear to it, he will perceive two sounds; the first conveyed by the metal, the second (which travels more slowly) transmitted by the air. It was in this manner that Biot (*Traité de Physique*) made experiments on a length of 951 mètres of cast iron pipes, from which he concluded that the velocity of sound in iron is 10·5 × velocity of sound in air; and Wertheim (Poggendorf, *Ergänzungsheft* III.), using 4067·2 mètres of telegraph wires, found a velocity of 3485 mètres per second, which differs little from Biot's.

"Attempts, however, have been made to measure the velo-

city by experiments on a small scale (see Wertheim, *Annales de Chimie*, 3me série, t. 12). Without going into details of complicated apparatus, we shall state that, by reference to a musical note (as will be mentioned in a subsequent Section), the rapidity of vibration of a tuning-fork is known; and that this can be exhibited by scratches which it makes on a glass surface moving under it, slightly covered with lamp-black. Transversal vibrations of a given bar, treated in the same manner, were made comparable with the tuning-fork-vibrations; and longitudinal vibrations, in like manner, were made comparable with the transversal vibrations. The time occupied by a longitudinal vibration was held, as that of air in an organ-pipe (hereafter to be mentioned), to be the time in which a wave passed through the double length of the bar. Thus the velocities, as compared with that in air, were found for different metals; the highest being that of iron, 15.108; the lowest that of lead, 3.974. Experiments were made at the same time on the extensibility of the metals; these, interpreted by the theory of Article 61, gave for velocities, in iron, 15·472, in lead, 3·561; differing little from the former.

"A more remarkable method, however, has lately been introduced (see Kundt, Poggendorf's *Annalen*, Vol. 127). If a bar of metal, &c., be chafed, it is put into longitudinal vibration; and if its end carry a light piston, which nearly fits without touching the inside of a glass tube, vibrations of the same period will be excited in the air within the tube, and the lengths of the waves of these vibrations may be made visible by scattering a very light dry dust in the interior of the tube, which dust collects in little heaps in those parts of the tube where the air has no motion (Article 58), and the corresponding length of the wave of air is therefore known. In this manner, the length of wave in the metal, &c. (which is the double length of the bar), is immediately

comparable with the length of wave of the same period in air; and when the periods are equal, the velocities of the transmission of waves are in the same proportion as the lengths of the waves (Article 30). Thus the following proportions of the velocities of sound-waves to the velocities in air were found:—

in steel,	15·34;
in glass,	15·25;
in copper,	11·96;
in brass,	10·87.

'The velocity of sound in a stretched wire, confined at its ends, may be found by chafing it longitudinally and observing the musical tone which it produces; the number of waves corresponding to that tone being known (Article 94, below), the double length of the wire must be multiplied by the number of waves.

"The velocity of sound through wood has been found in the same way. Along the fibre, it varies from 10900 to 15400 feet per second; transversally across the rings, from 4400 to 6000; and transversally along the rings, from 2600 to 4600 (Wertheim, *Mémoires*).

Experiments on the velocity of Sound through fluids.

"A most important series of experiments was made by MM. Colladon and Sturm, for ascertaining by direct observation the time occupied by sound in passing through the water of the lake of Geneva (*Annales de Chimie*, tome 36). The method adopted was, to suspend a large bell in the water, and to strike it with a hammer; at the place of observation, a tube was inserted in the water, having a large spoon-shaped orifice at its lower end, turned towards the origin of sound, and having a conical form at the upper end terminating in a small hole to which the observer's ear was

applied. The sound of a bell, weighing 500 kilogrammes ($\frac{1}{2}$ ton), sunk 3 mètres (10 feet) deep in the water, and struck by a man with a hammer weighing 10 kilogrammes (22 lbs.), was heard very well at the distance of 35000 mètres (nearly 22 miles). But the experiments which they were enabled to carry to the greatest extent were made with a bell only 7 decimètres high, suspended 1 mètre deep in the water, with a striking apparatus so arranged that at the instant of striking the bell it fired some gunpowder; the observer was stationed at a distance of 13487 mètres (44250 feet, or more than 8 miles). The velocity found was 1435 mètres or 4708 feet per second. The temperature of the water was 8°·1 centigrade.

"Experiments made by the same philosophers on the compressibility of water gave, for the compression produced by the weight of one atmosphere, 49·5 millionth parts of the whole. From this, using formulæ similar to those of Article 62, they inferred a theoretical velocity of 1428 mètres, agreeing well with that which was observed.

"Wertheim (*Annales de Chimie*, 3rd series, Vol. XXIII.) has attempted an experimental determination of the velocity of sound in water, by the same method which we have described above (Article 68) as applied to various gases, namely, by immersing an organ-pipe in water, and forcing the water through it in the same manner as air; a sort of musical tone was produced, sufficiently good to have its pitch recognised. The velocities found for water of the Seine varied from 1173 mètres to 1480 mètres per second; all much inferior to those found by the direct experiment. To reconcile them, Wertheim supposed that the velocities in a column of water and in an unlimited space of water are not the same; an idea which we do not accept. Among possible causes of the difference, we might suggest the yielding of the sides of the tube when pressed by the vibra-

tions of a dense liquid; the yielding would be insensibly small when the vibrating mass was air or any other gas."—*Airy.*

The remaining tables are from Tyndall "On Sound":—

VELOCITY OF SOUND IN GASES AT THE TEMPERATURE OF 0° C.

	Velocity
Air	1,092 feet.
Oxygen	1,040 ,,
Hydrogen	4,164 ,,
Carbonic acid	858 ,,
Carbonic oxide	1,107 ,,
Protoxide of nitrogen	859 ,,
Olefiant gas	1,030 ,,

TRANSMISSION OF SOUND THROUGH LIQUIDS.

Name of Liquid.	Temperature.	Velocity.
		feet.
River water (Seine)	15° C.	4,714
,, ,,	30	5,013
,, ,,	60	5,657
Sea water (artificial)	20	4,768
Solution of common salt	18	5,132
Solution of sulphide of soda	20	5,194
Solution of carbonate of soda	22	5,230
Solution of nitrate of soda	21	5,477
Solution of chloride of calcium	23	6,493
Common alcohol	20	4,218
Absolute alcohol	23	3,804
Spirits of turpentine	24	3,976
Sulphuric ether	0	3,801

VELOCITY OF SOUND THROUGH METALS.

Name of Metals.	at 20° C.	at 100° C.	at 200° C.
Lead	4,030	3,951	—
Gold	5,717	5,640	5,619
Silver	8,553	8,658	8,127
Copper	11,666	10,802	9.690
Platinum	8,815	8,437	8,079
Iron	16,822	17,386	15,483
Iron wire (ordinary)	16,130	16,728	—
Cast steel	16,357	16,153	15.709
Steel wire (English)	15,470	17,201	16,394
Steel wire	16,023	16,443	—

VELOCITY OF SOUND IN WOOD.

Name of Wood.	Along Fibre.	Across Rings.	Along Rings.
Acacia	15,467	4,840	4,436
Fir	15,218	4,382	2,572
Beech	10,965	6,028	4,643
Oak	12,622	5,036	4,229
Pine	10,900	4,611	2,605
Elm	13,516	4,665	3,324
Sycamore	14,639	4,916	3,728
Ash	15,314	4,567	4,142
Alder	15,306	4,491	3,423
Aspen	16,677	5,297	2,987
Maple	13,472	5,047	3,401
Poplar	14,050	4,600	3,444

"We shall take this opportunity of adding another system of numerical elements corresponding to the different notes. We have shown that

$$\text{Length of wave} = \frac{\text{Velocity of wave}}{\text{Frequency of wave}},$$

where velocity and frequency are to be referred to the same unit of time; it is indifferent what the unit may be. On examining the numbers in Article 65, it will be seen that, at the temperatures at which it is interesting to examine musical notes, the velocity scarcely differs from 1100 English feet in a second of time, or 1000 English feet in $\frac{10}{11}$ths of a second of time. Consequently the length of wave for each note, in English feet, will be found by dividing 1000 by the number of vibrations in $\frac{10}{11}$ths of a second of time."—*Airy.*

"Two conditions determine the velocity of propagation of a sonorous wave; namely, the elasticity and the density of the medium through which the wave passes. The elasticity of air is measured by the pressure which it sustains or can hold in equilibrium. At the sea-level this pressure is equal to that of a stratum of mercury about thirty inches high. At the summit of Mont Blanc the barometer column is not

much more than half this height; and, consequently, the elasticity of the air upon the summit of the mountain is not much more than half what it is at the sea-level.

"If we could augment the elasticity of air, without at the same time augmenting its density, we should augment the velocity of sound. Or, if allowing the elasticity to remain constant, we could diminish the density, we should augment the velocity. Now, air in a closed vessel, where it cannot expand, has its elasticity augmented by heat, while its density remains unchanged. Through such heated air sound travels more rapidly than through cold air. Again, air free to expand has its density lessened by warming, its elasticity remaining the same, and through such air sound travels more rapidly than through cold air. This is the case with our atmosphere when heated by the sun. The velocity of sound in air, *at the freezing temperature*, is 1090 feet a second. At all lower temperatures the velocity is less than this, and at all higher temperatures it is greater. The late M. Wertheim has determined the velocity of sound in air of different temperatures, and here are some of his results:—

Temperature of air.	Velocity of sound.
0.5° centigrade . .	1089 feet
2.10 ,, . .	1091 ,,
8.5 ,, . .	1109 ,,
12.0 ,, . .	1113 ,,
26.6 ,, . .	1140 ,,

"At a temperature of half a degree above the freezing point of water the velocity is 1089 feet a second; at a temperature of 26·6 degrees, it is 1140 feet a second, or a difference of fifty-one feet for twenty-six degrees, that is to say, an augmentation of velocity of nearly two feet for every single degree centigrade."—*Tyndall.*

"We are now prepared to appreciate an extremely beautiful experiment, for which we are indebted to Sir Charles Wheat

stone. In a room underneath this, and separated from it by two floors, is a piano. Through the two floors passes a tin tube 2½ inches in diameter, and along the axis of this tube passes a rod of deal, the end of which emerges from the floor in front of the lecture table. The rod is clasped by india-rubber bands, which entirely close the tin tube. The lower end of the rod rests upon the sound-board of the piano, its upper end being exposed before you. An artist is at this moment engaged at the instrument, but you hear no sound. When, however, a violin is placed upon the end of the rod, the instrument becomes instantly musical, not, however, with the vibrations of its own strings, but with those of the piano. When the violin is removed, the sound ceases; putting in its place a guitar, the music revives. For the violin and guitar we may substitute a plain wooden tray, which is also rendered musical. Here, finally, is a harp, against the sound-board of which the end of the deal rod is caused to press; every note of the piano is reproduced before you. On lifting the harp so as to break the connection with the piano, the sounds vanish, but the moment the sound-board is caused to press upon the rod, the music is restored. The sound of the piano so far resembles that of the harp that it is hard to resist the impression that the music you hear is that of the latter instrument. An uneducated person might well believe that witchcraft or 'spiritualism' is concerned in the production of this music. What a curious transference of action is here presented to the mind! At the command of the musician's will, the fingers strike the keys; the hammers strike the strings, by which the rude mechanical shock is converted into tremors. The vibrations are communicated to the sound-board of the piano. Upon that board rests the end of the deal rod, thinned off to a sharp edge to make it fit more easily between the wires. Through the edge, and

afterwards along the rod, are poured with unfailing precision the entangled pulsations produced by the shocks of those ten agile fingers. To the sound-board of the harp before you the rod faithfully delivers up the vibrations of which it is the vehicle. This second sound-board transfers the motion to the air, carving it and chasing it into forms so transcendently complicated that confusion alone could be anticipated from the shock and jostle of the sonorous waves. But the marvellous human ear accepts every feature of the motion, and all the strife and struggle and confusion melt finally into music upon the brain." * — *Tyndall.*

* An ordinary musical box may be substituted for the piano in this experiment.

CHAPTER III.

NATURE OF WAVE MOTION IN GENERAL.

14. THE extract from Professor Tyndall's work "On Sound," given in the last chapter, spoke of "a wave of sound." In this chapter we are to consider the theory of waves, and to learn how the form of a wave can progress without any actual progression of the particles which from time to time make the wave. This wave-theory once understood, its application to the travelling of sound (see Chapter IV.) will be easy. The study of waves of water will be a good introduction.

15. The following figures will convey the idea of wave-motion :—

Fig. 4.

In Figure 4 the line A B represents water at rest, and the dots the drops of water, spaces being left between the drops for the sake of clearness of description :—

In Figure 5 some of the drops are raised vertically

and others depressed, A and B still representing the level of the water line :—

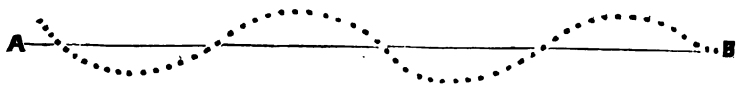


Fig. 5.

If a line be drawn connecting these drops with each other, the following wave-form will result :—

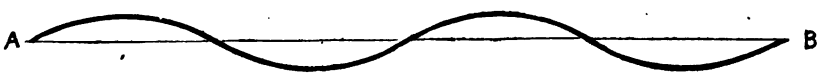


Fig. 6.

Let us now trace, step by step, the formation of this wave. Beginning at A, if from A to the point where the curve meets the level line, the drops *rise* one after another to the height shown in the second of the above figures, the result will be this form :—

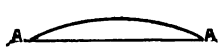


Fig. 7.

Then if the drops in the next section *fall* to the positions shown, this form will follow :—

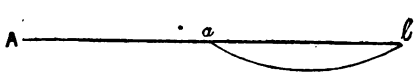


Fig. 8.

The first of these is a ridge, the second a hollow, and the two form a complete wave, the drops at *b* being at the same level as those at A.

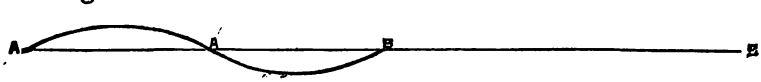


Fig. 9.

"Though the sound-impulse thus advances with a steady and very high velocity, the medium by which it is transmitted clearly does not share such a motion. Solid conductors of sound remain, on the whole, at rest during its passage, and a slight yielding of their separate parts is all that their constitution generally admits of. In fluids, or in the air, a rapid forward motion is equally out of the question. The movement of the particles composing the sound-conveying medium will be found to be of a kind examples of which are constantly presenting themselves, but without attracting an amount of attention at all commensurate with their interest and importance. An observer who looks down upon the sea from a moderate elevation, on a day when the wind, after blowing strongly, has suddenly dropped, sees long lines of waves advancing towards the shore at a uniform pace and at equal distances from each other. The effect to the eye, is that of a vast army marching up in column, or of a ploughed field moving along horizontally in a direction perpendicular to the lines of its ridges and hollows. The *actual* motion of the water is, however, very different from its *apparent* motion, as may be ascertained by noticing the behaviour of a cork, or other body floating on the surface of the sea, and therefore sharing its movement. Instead of steadily advancing, like the waves, the cork merely performs a heaving motion as the successive waves reach it, alternately riding over their crests and sinking into their troughs, as if anchored in the position it happens to occupy. Hence, while the waves travel steadily forward horizontally, the drops of water composing them are in a state of swaying to-and-fro motion, each separate drop rising and falling in a vertical straight line, but having no horizontal motion whatever. Thus, when we say that the waves advance horizontally, we mean, not that the masses of water of which they at any given instant consist,

advance, but that these masses, by virtue of the separate vertical motions of their individual drops, *successively arrange themselves in the same relative positions*, so that the curved shapes of the surface, which we call waves, are transmitted *without their materials sharing in the progress.*"—*Taylor.*

If the student does not yet realise how a wave is formed and carried along by drops having only a vertical and not a forward motion, he should read the definition again, as further elaboration would only confuse him at present.

If the above description is now understood, the following extracts from Airy and Helmholtz will help to illustrate the wave-theory:—

"The theory of the transmission of sound through the air (as well as through other bodies) is essentially founded upon the conception of the transmission of waves, in which the nature of the motion is such, that the movement of every particle is limited, while the law of relative movement of neighbouring particles is transmitted to an unlimited distance, either without change or with change following a definite law. Now if the places of the points in these lines * represent the positions of the particles at successive equal intervals of time, it is plain that we have states of condensation and states of rarefaction travelling on continually without limit, in one direction; while the motion of every individual particle is extremely small, and is alternately backwards and forwards. And this is the conception of a wave as depending on the motion of particles in the same line as that in which the wave travels; this is the kind of

* This and similar references are to figures drawn on the same principle as those given above.

wave which we shall consider as explaining the transmission of sound.

"But there are other kinds of movements of particles which are equally included under the conception of wave. For instance, in Figure 7, the motion of the particles is entirely transverse to the horizontal lines of the diagram; and here it is not states of condensation and rarefaction that travel continually in the same direction, but states of elevation and depression that so travel. (This is the kind of wave which is recognised as applying to polarised light.) In Figure 8, the motion of the particles consists of a combination of the two motions in Figure 6 and Figure 7; the vertical displacement of the particles so accompanying the horizontal displacements, that the places where the particles are most condensed in the horizontal direction are the places where they are most elevated in the vertical direction. (This is the character of waves of water.)

"But in all these there is one general character; that a *state of displacement* travels on continually in one direction, without limit; while the motion of each individual particle is, or may be, small and of oscillatory character. And this is the general conception of a wave. It will be remembered that the special character of the waves of air applying to the problem of sound is, that the displacements of the particles are in the same direction (backwards and forwards) as that in which the wave travels."—*Airy.*

"We must, however, here distinguish between the motion of the individual particles of air—which takes place periodically backwards and forwards within very narrow limits—and the propagation of the sonorous tremor. The latter is constantly advancing by the constant attraction of fresh particles into its sphere of tremor. This is a peculiarity of all so-called *undulatory motions.* Suppose a stone to be thrown into a piece of calm water. Round the spot struck

there forms a little ring of wave, which, advancing equally in all directions, expands to a constantly increasing circle. Corresponding to this ring of wave, sound also proceeds in the air from the excited point, and advances in all directions as far as the limits of the mass of air extend. The process in the air is essentially identical with that on the surface of the water. The principal difference consists in the spherical propagation of sound in all directions through the atmosphere which fills all surrounding space, whereas the waves of the water can only advance in rings or circles on its surface. The crests of the waves of water correspond in the waves of sound to spherical shells where the air is condensed, and the troughs to shells of rarefaction. On the free surface of the water, the mass on compression can slip upwards and so form ridges, but in the interior of the sea of air, the mass must be condensed, as there is no unoccupied spot for its escape. The waves of water, therefore, continually advance without returning. But we must not suppose that the particles of water of which the waves are composed advance in a similar manner to the waves themselves. The motion of the particles of water on the surface can easily be rendered visible by floating a chip of wood upon it. This will perfectly share the motion of the adjacent particles. Now, such a chip is not carried on by the rings of wave. It only bobs up and down, and finally rests on its original spot. The adjacent particles of water move in the same manner. When the ring of wave reaches them they are set bobbing; when it has passed over them they are still in their old place, and remain there at rest, while the ring of wave continues to advance towards fresh spots on the surface of the water, and sets new particles of water in motion. Hence the waves which pass over the surface of the water are constantly built up of fresh particles of water. What really advances as a wave is only the

tremor, the altered form of the surface, while the individual particles of water themselves merely move up and down transiently, and never depart far from their original position. The same relation is seen still more clearly in the waves of a rope or chain. Take a flexible string of several feet in length, or a thin metal chain, hold it at one end and let the other hang down, stretched by its own weight alone. Now, move the hand by which you hold it quickly to one side and back again. The excursion which we have caused in the upper end of the string by moving the hand, will run down it as a kind of wave, so that constantly lower parts of the string will make a sidewards excursion while the upper returns again into the straight position of rest. But it is evident that while the wave runs down, each individual particle of the string can have moved only horizontally backwards and forwards, and can have taken no share at all in the advance of the wave. The experiment succeeds still better with a long elastic line, such as a thick piece of india-rubber, or a brass wire spiral spring, from eight to twelve feet in length, fastened at one end, and slightly stretched by being held with the hand at the other. The hand is then easily able to excite waves, which will run very regularly to the end of the line, be there reflected and return. In this case it is also evident that it can be no part of the line itself which runs backwards and forwards, but that the advancing wave is composed of continually fresh particles of the line. By these examples the reader will be able to form a mental image of the kind of motion to which sound belongs, where the material particles of the body merely make periodical oscillations, while the tremor itself is constantly propagated forwards."—*Helmholtz.*

16. A field of corn, viewed from an adjacent hill on a windy day, will enable the student to see for

himself a practical proof that the form of a wave may go from end to end of a field while the ears of corn have but a limited motion to and fro.

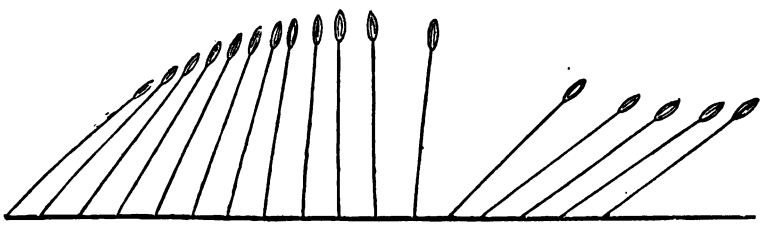


Fig. 10.

As the ears, bent down by the wind, rise again through their own elasticity, the wave (caused by the swaying to and fro of the masses over which the wind passes) may be followed by the eye the whole length of the field, though it is of course obvious to the meanest capacity that the ears have no motion but the backward and forward swing.

17. It must not be forgotten that the various llustrations just given to explain wave-motion are only approximate ideas, and that the real motion of air when sound travels in it is that of elastic balls, like the bagatelle balls shown in Figure 3. It will be the object of the next chapter to show what connection there is between wave-motion and the vibratory motions which produce sound.

"These air-waves have the property of moving forward, in a manner analogous to the motion of waves on the surface of water. The sudden impulse of a vibrating body on the adjoining particles of air, forming, say, a momentary

compression therein, is transmitted forward to the next adjoining particles, from these to the next, and so on, so as to produce a propagation or travel of the compressive action, without any necessary motion of the general body of air.

"Now, we happen to know by observation the rate at which such compressive action will be propagated, this being another name for the *velocity of sound.* It varies with the state of the atmosphere as regards temperature and density; but it is, under ordinary circumstances, about 1100 feet per second, *i.e.*, the impulse given by the motion of any vibrating body, will propagate itself through the air at this velocity.

"Knowing this, we can determine, with some precision, the magnitude of the air-waves corresponding to certain sounds. An example will best show this. Suppose the sounding body to be one which makes 256 complete vibrations in a second, corresponding, as will be explained in the next chapter, to about the note called 'middle C,' the result will be that the sounding body will give a series of regular impulses to the air, occurring at intervals of $\frac{1}{256}$ of a second. Supposing, then, one of these blows to be given, causing a compression in the air adjoining, this compression will be propagated through the air at the rate of 1100 feet in a second, and consequently, when a repetition of the blow comes, in $\frac{1}{256}$ of a second, it will have advanced $\frac{1100}{256}$, *i.e.*, 4.28 feet. By the repetition of the blow a new wave will be originated, precisely similar to the former one, and thus the sounding body will go on generating a series of waves, each 4.28 feet long, and all flying off into space, with the velocity known as the velocity of sound. This length of 4.28 feet is therefore the *length of air-wave* corresponding to the note 'middle C.'"—*Pole.*

"Since the crests of the waves are raised above the ordinary level of the sea, the troughs must necessarily be

depressed below it, just as, in a ploughed field, the earth heaped up to form the ridges must be taken out of the furrows. Each crest being thus associated with a trough, it is convenient to regard one crest and one trough as forming together one complete wave. Thus each wave consists of a part raised above, and a part depressed below, the horizontal plane which would be the surface of the sea were it not being traversed by waves."—*Taylor*.

18. We will now see what are the features which distinguish waves one from another. Remembering that a raised part and a depressed part form together a complete wave, it will be seen that waves may vary in width, that is to say, in the height to which they rise and the depth to which they sink, while the other features of the wave remain the same. A series of figures will best illustrate this.

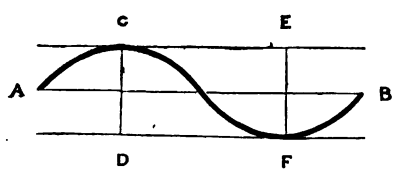


Fig. 11.

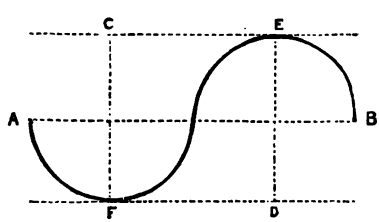


Fig. 12.

The distance from A to B (that is, the *length* of the

waves) is in each of these cases the same, but the space between C D, E F, (that is, the *width* of the waves) is much greater in the second figure than in the first. This is, of course, because the one wave rises higher and falls lower than the other. The distance between the extreme points, C D, E F, is called the width, or *amplitude* of the wave.

19. Again, waves may be of the same width but of various *lengths*. Thus, in the next two figures the width from C to D, E to F is alike, while the length of wave is greater in the second wave than in the first:—

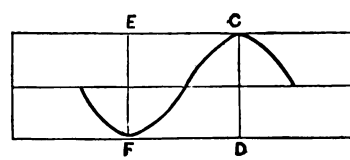


Fig. 13.*

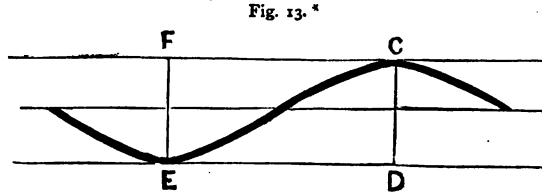


Fig. 14.

20. A third difference is when length and width are alike but the waves vary in *form;* that is to say, the points at which the wave ceases to rise or fall, and also the shape it takes in reaching those points, may be different in waves of the same length and width.

* The engraver has reversed some of the figures, but the result is not affected.

The next three figures will illustrate this, and with careful study will be their own best comment:—

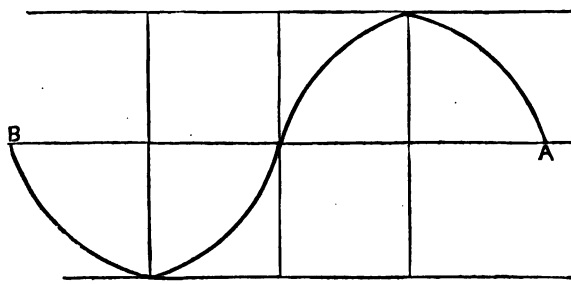


Fig 15

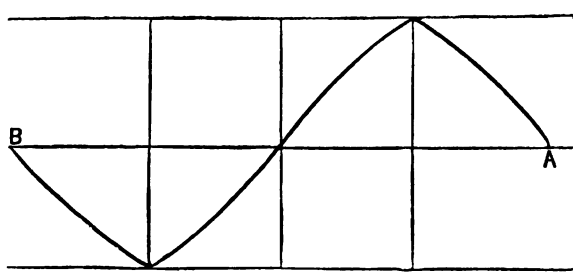


Fig. 16.

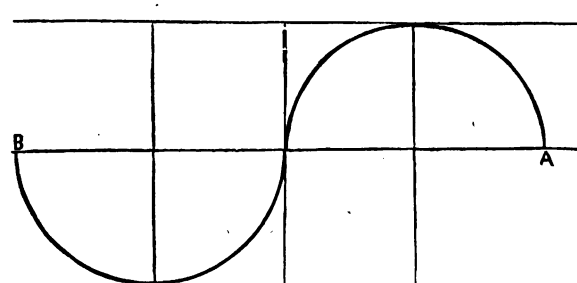


Fig. 17.

21. The three varieties of "wave-quality" are therefore these:—

1. Length;
2. Amplitude (width);
3. Form.

And it will be demonstrated later on that these features are connected with different varieties of sounds, the connection being as follows:—

1. *Length* of wave corresponds with *pitch* (height or depth of sound);
2. *Amplitude* of wave corresponds with *loudness* of sound;
3. *Form* of wave corresponds with *quality* of sound.

Sounds cannot possibly have any further variety than of pitch, force, or quality.

"Having thus spoken of the principal division of sound into Noise and Musical Tones, and then described the general motion of the air for these tones, we pass on to the peculiarities which distinguish such tones one from the other. We are acquainted with three points of difference in musical tones, confining ourselves in the first place to such tones as are isolatedly produced by our usual musical instruments, and excluding the simultaneous sounding of the tones of different instruments. Musical tones are distinguished:—

"1. By their *force* or *loudness*.

"2. By their *pitch* or *relative height*.

"3. By their *quality*.

"It is unnecessary to explain what we mean by the force and pitch of a tone. By the quality of a tone we mean that peculiarity which distinguishes the musical tone of a violin from that of a flute or that of a clarinet, or that of the human

voice, when all these instruments produce the same note at the same pitch.

"We have now to explain what peculiarities of the motion of sound correspond to these three principal differences between musical tones.

"First, We easily recognise that the *force* or loudness of a musical tone increases and diminishes with the extent or so-called *amplitude* of the oscillations of the particles of the sounding body. When we strike a string, its vibrations are at first sufficiently large for us to see them, and its corresponding tone is loudest. The visible vibrations become smaller and smaller, and at the same time the loudness diminishes. The same observation can be made on strings excited by a violin bow, and on the reeds of reed-pipes, and on many other sonorous bodies. The same conclusion results from the diminution of the loudness of a tone when we increase our distance from the sounding body in the open air, although the pitch and quality remain unaltered; for it is only the amplitude of the oscillations of the particles of air which diminishes as their distance from the sounding body increases. Hence loudness must depend on this amplitude, and none other of the properties of sound do so.*

"The second essential difference between different musical tones consists in their *pitch* or relative height. Daily experience shows us that musical tones of the same pitch can be produced upon most diverse instruments by means of most diverse mechanical contrivances, and with most diverse degrees of loudness. All the motions of the air

* Mechanically the force of the oscillations for tones of different pitch is measured by their *vis viva*, that is, by the square of the greatest velocity attained by the oscillating particles. But the ear has different degrees of sensibility for tones of different pitch, so that no measure can be found for the intensity of the sensation of sound, that is, for the loudness of sound at all pitches.

thus excited must be periodic, because they would not otherwise excite in us the sensation of a musical tone. But the motion within each single period may be of any kind whatever, and yet if the length of the periodic time of two musical tones is the same, they have the same pitch. Hence: *Pitch depends solely on the length of time in which each single vibration is executed, or,* which comes to the same thing, *on the vibrational number of the tone.* We are accustomed to take a second of time as the unit, and consequently mean by *vibrational number* the number of vibrations which the particles of a sounding body perform in one second of time. It is self-evident that we find the periodic time or *vibrational period,* that is length of time which is occupied in performing a single vibration backwards and forwards, by dividing one second of time by the vibrational number.

. . . . . .

"On inquiring to what external physical difference in the waves of sound the different qualities of tone correspond, we must remember that the amplitude of the vibration determines the force or loudness, and the period of vibration the pitch. Quality of tone can therefore depend upon neither of these. The only possible hypothesis, therefore, is that the quality of tone should depend upon the manner in which the motion is performed within the period of each single vibration. For the generation of a musical tone we have only required that the motion should be periodic, that is, that in any one single period of vibration exactly the same state should occur, in the same order of occurrence as it presents itself in any other single period. As to the kind of motion that should take place within any single period, no hypothesis was made. In this respect, then, an endless variety of motions might be possible for the production of sound."—*Helmholtz.*

22. This fact must be constantly borne in mind—*The form of a wave is produced by the limited swingings of individual drops.* Let this be well understood, and then this second truth must be grasped—*length, amplitude, and form of wave are all caused by varieties of swing in the individual drops which make the wave.* These varieties must now be studied.

23. When a pendulum swings to and fro across a straight line, its point moves within certain limits, and those limits are at the same distance from the centre of the swing. In making this swing to and fro, the point *twice* crosses the central line. Thus if the pendulum is at rest at *a* in Figure 18, and begins

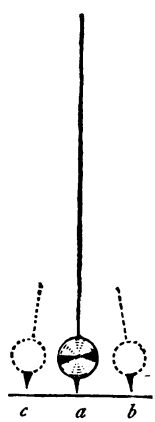


Fig. 18.

to swing towards *b*, it will on reaching *b*, turn back, and go to *a*. It has thus travelled twice over *a b*. Then it goes from *a* to *c*, and from *c* to *a*. It has thus travelled twice over *a c*. (The pendulum would

of course describe a small arc, or portion of a circle; but for our purpose we will presume that for this short distance it travels along the straight line *b c*, and the slight variation from that line need not interfere with our argument.) The pendulum point thus goes twice over the whole length of its course *b c*.

Now let the student imagine this pendulum laid horizontally, without its swing being interfered with (as of course it would be by gravitation): but let us look at this horizontal pendulum as if freed from the control of gravitation. Its swing would then be as follows:—

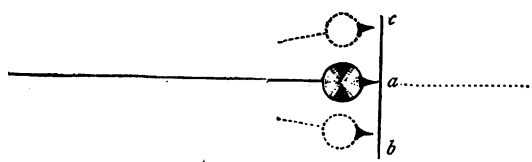


Fig. 19.

The drops of water which move up and down to form the wave, have every one the same motion as the point of this horizontal pendulum. The level of the water is the point at rest at *a*, and the rod, and the dotted line *a e*, represent that level. The drop of water falls to *b*, then rises to *a*, rises to *c*, and falls to *a*. *This is one complete vibration of the drop.* If now the pendulum point has a pencil attached to it, and a sheet of paper, with a dotted line drawn upon it to represent the water-level, be moved along, the pencil, though it only moves in a straight line along *b a c* (Fig. 22), will make a wave-line corresponding to the rate of motion given to the paper, and

whatever variety is made in the form of wave-motion given to the sheet of paper will appear in the form of wave drawn by the pencil moving within that narrow limit between *b* and *c*. The figures here given will illustrate this fact:—

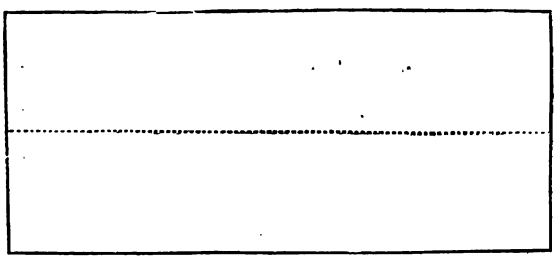


Fig. 20.

This shows the sheet of paper with a dotted line upon it. The next shows the pendulum at rest, the black spot, which shows the position of the drop, being the pencil-point so placed as to mark the paper, and the dotted line being partially covered by the pendulum rod.

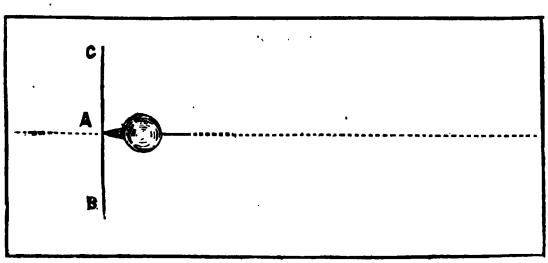


Fig. 21.

The sheet of paper is now moved from right to left, while the pendulum goes once over its path, *a b*,

b a, *a c*, *c a*, and the next figure shows the wave in course of formation, the point of the pendulum being sufficient to allow its progress to be traced :—

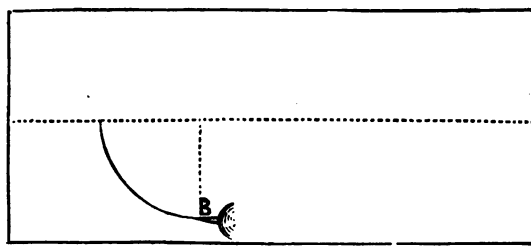


Fig. 22.

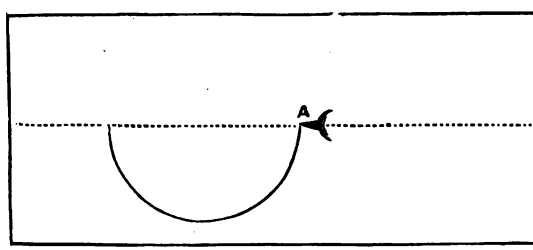


Fig. 23.

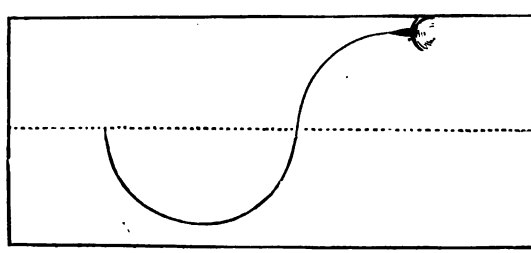


Fig. 24.

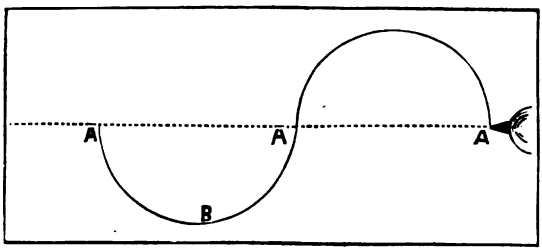


Fig. 25.

It will thus be seen that while the drop has gone over its vertical course *a b, b a, a c, c a,* it has described the wave-form *a b a a. Every time the drop goes once over its own course, the wave goes once through its own whole length:* and consequently, *if the drop occupies just one second in its journey, the wave length will be traversed in one second also.* This is equally true of every drop in the wave's length—its up and down course is made in just the same time as a complete wave is formed. From the resemblance between the motion of the pendulum point and of the drop of water, this kind of oscillation is called pendular vibration, and the *extent* of such vibration makes the *amplitude of the wave.* With a little practice in drawing waves, the student will soon learn that curves of the nature just described must necessarily be formed by a backward and forward motion similar to that of a pendulum point; and this is why such forms are called *the associated waves* of pendular vibration.

24. If waves of *different lengths* are formed in the *same time,* the rate of motion in the drops will vary accordingly. Thus, in this figure, it will be seen that

other things being equal, *the longer the wave, the slower the rate of vibration of the drops which form it.*

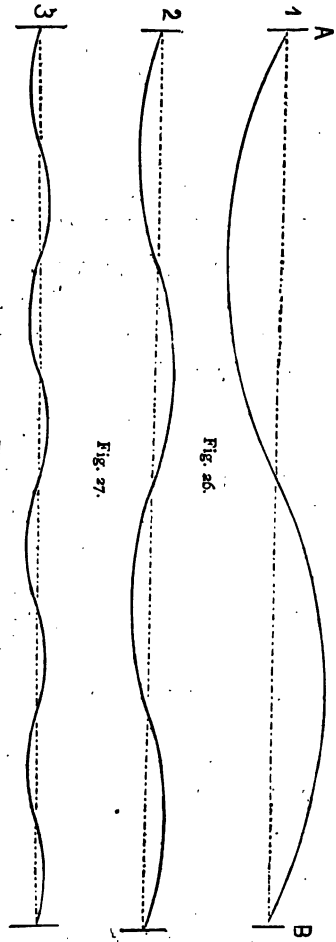


Fig. 26.

Fig. 27.

Fig. 28.

The drops in 1 would vibrate *once* between A and B,

which we will suppose to be in one second; those in 2, *twice;* those in 3, *four times.* The time of vibration is thus connected with the *wave-length.*

25. We have only now to consider how the vibration affects the *form* of the wave. Its *extent* makes the *amplitude of wave;* its *time* the *length of wave;* and its *mode* the *form of wave.* We have hitherto supposed that the drop moved through equal spaces in equal times; but *if it moves through equal spaces in unequal times,* or if, in other words, it moves slowly in one part of its journey and rapidly in another, the form of the curve described will be materially altered. Let these twenty drops be at the water-level, and quiescent:—

Fig. 29.

Then so long as the drops travelled equal distances in equal times, the curve of the wave would be regular, but two examples will suffice to show that if these conditions were altered in any way, the form of the wave would be altered. To prove this, let us suppose that in rising the drops travel much faster than in falling. We shall then have this result:—

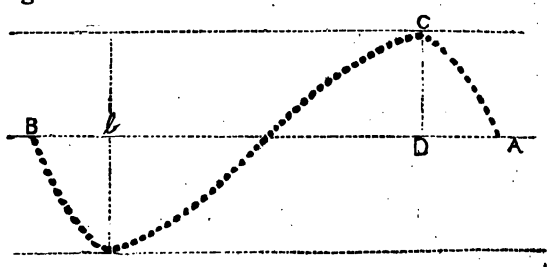


Fig. 30.

The drops, rising rapidly, would reach the height of the crest at *c* early in the wave-length, but, falling more slowly, would traverse that part of the wave-length between D and *b*, and at *b* the ascent would again begin, and being as rapid as from *a* to *c*, the wave-length b B would be traversed in the same time as A D.

Taking the other case, in which the ascent would be slow and the descent rapid, we should have this wave-form as the result :—

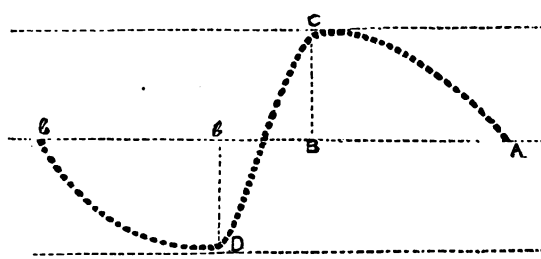


Fig 31.

This instance reverses the conditions of the last, and gives a totally different wave-form. The slow ascent allows A B to be traversed before the crest is reached ; the descent is rapid and sudden, occupying only B b, and the slow ascent occurs between b c, which is equal in length to A B. The difference between the curves resulting from these varied conditions will be better seen if the spaces between the drops are filled up.

26. When it is remembered that length and amplitude, as well as form, may vary, it will be seen that the variety of wave-forms is well-nigh infinite. The

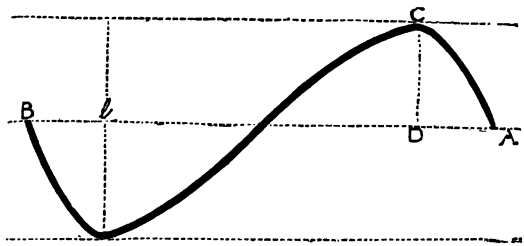


Fig. 32.

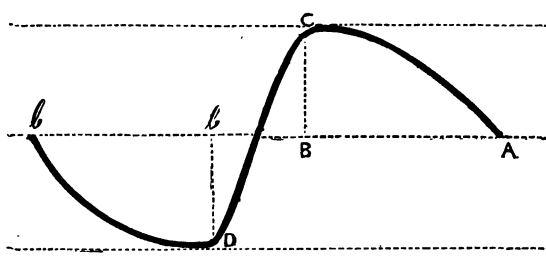


Fig. 33.

student should construct some forms for himself, and the following are offered as suggestions upon which to begin:—

1. Variety of wave-*length* alone.
2. „ *length* and *amplitude* combined.
3. „ *length* and *form* alone.
4. „ *length, form,* and *amplitude.*

5. „ *amplitude* alone.
6. „ *amplitude* and *length.*
7. „ *amplitude* and *form.*
8. „ *amplitude, form,* and *length.*

9. Variety of *form* alone.
10. „ *form* and *length.*
11. „ *form* and *amplitude.*
12. „ *form, length,* and *amplitude.**

In making waves by these directions, care should be taken to use, in each set of four as bracketed, the same number of drops. Any number may be chosen, provided it be sufficient to make a wave when the drops are joined together by short lines between them, but the benefit of the exercise will be greatly enhanced if the same number be employed throughout. In constructing these waves, care should be taken to let each drop rise and fall in a perpendicular line, and until the student can keep this fact in mind, and lift or lower each drop in this way without help, he should draw dotted lines to connect the drop at rest with the positions they occupy when moved to form the wave, as in the following figure, where twelve

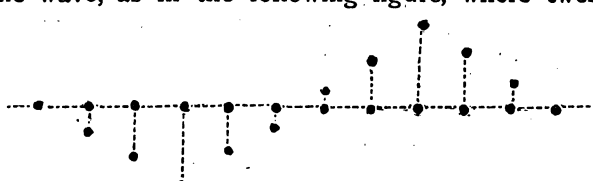


Fig. 34.

drops are employed. It is also useful to connect, by dotted lines, the drops forming the curve:—

* In this list 2 and 6 may seem alike, so also 3 and 10, 7 and 11, and 4, 8, and 12; but in each of these cases the first-named element must remain constant while the others vary, so that while 2 refers to waves of the same length but different width, 6 refers to waves of the same width but different length.

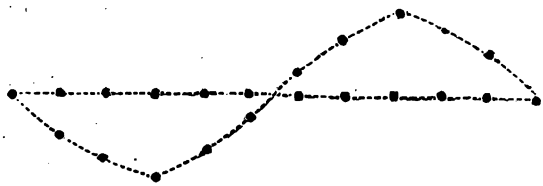


Fig. 35.

Constant practice in wave-drawing, upon the principles here laid down, will soon familiarise the student with a principle which he must fully comprehend if he is to grasp the problems of sound, viz., that *waves are formed by drops which themselves make no forward motion, but only move up and down in perpendicular lines.*

"How are we to picture to ourselves the condition of the air through which a musical sound is passing? Imagine one of the prongs of the vibrating fork swiftly advancing; it compresses the air immediately in front of it, and when it retreats it leaves a partial vacuum behind, the process being repeated by every subsequent advance and retreat. The whole function of the tuning-fork is to carve the air into these condensations and rarefactions, and they, as they are formed, propagate themselves in succession through the air. A condensation with its associated rarefaction constitutes, as already stated, a sonorous wave. In water the length of a wave is measured from crest to crest, while in the case of sound, the *wave length* is the distance between two successive condensations. The condensation of the sound-wave corresponds to the crest, while the rarefaction of the sound-wave corresponds to the *sinus*, or depression of the water-wave."—*Tyndall.*

"The *length*, *amplitude*, and *form* of a wave completely

determine the wave, just as the length, breadth, and height of an oblong block of wood, *i.e.*, its three dimensions, fix the size of the block. These three elements of a wave are mutually independent, that is to say, we may alter one of them without altering the other two."—*Taylor.*

"We are acquainted with three points of difference in musical tones, confining ourselves, in the first place, to such tones as are isolatedly produced by our usual musical instruments, and excluding the simultaneous sounding of the tones of different instruments. Musical tones are distinguished:—(1.) By their *force* or *loudness.* (2.) By their *pitch* or *relative height.* (3.) By their quality."—*Helmholtz.*

CHAPTER IV.

APPLICATION OF THE WAVE THEORY TO SOUND.

27. THE waves of water, which have been used as illustrations of wave-motion generally, differ considerably in their construction from the waves of air by which sound is propelled, and the former were referred to for the purpose of illustrating the principle that the particles forming the wave do not move forward as the wave itself does. It will be necessary to investigate now the theory of waves of air, and to consider how these waves act in conveying sound.

The chief difficulty which stands in the way of the student who reads of waves, motions of drops of water, and sound-waves, is to connect in his own mind the actual and visible motion of the water-wave with the equally real but invisible motion of the air-wave. This is a real and not an imaginary difficulty, and the present chapter is intended to enable the student to overcome it.

28. The elasticity of air was explained in Chapter II. by an experiment with billiard balls, and the formation of waves in water was also illustrated by various diagrams; and the problem to be solved is to show how the motion of the water-drops forming

the wave is connected with the elasticity of the particles forming the air-wave.

29. Bearing in mind that condensation of an air particle corresponds with the *crest* of a wave, and rarefaction with the *hollow*, we shall be able to comprehend what follows. Let the next figure represent particles of air at rest:—

Fig. 36.

The fork is now set in motion, and we will suppose it begins with an outward swing. This immediately condenses the particle of air next to it; and as air is elastic, and the fork acting upon it can only alter its condition and not its bulk, the particle is forced, so to speak, out of shape, and made to occupy a less space in the direction of the impulse than before that impulse was received. This compression in what we will call the width of the particle will result in an increase of its length, and it will assume another shape, as in the next figure.

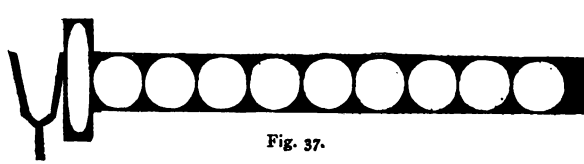


Fig. 37.

This is called the *condensation* of the particle, and the force thus imparted to the first is passed on to each successive one, so that the whole series, beginning at the fork, receive one after another the impulse

of condensation, and are pressed out of shape like the first. When the inward swing begins the first particle resumes its former shape, and also by virtue of its elasticity follows the fork in its retreat, assuming the shape shown in this figure, where its width is increased and its length reduced.

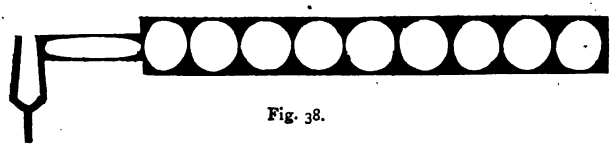


Fig. 38.

This is the *rarefaction* of the air-particle, because the pressure which condensed it is withdrawn, and instead of being compressed it is now expanded, and is thinner or rarer than when at rest. This result is also followed by a like effect upon each particle, the whole series following each other successively into rarefaction, just as they had previously done into condensation. The result of one complete swing of the fork will therefore be to change the shape of the first particle, and of the rest through it, as under :—

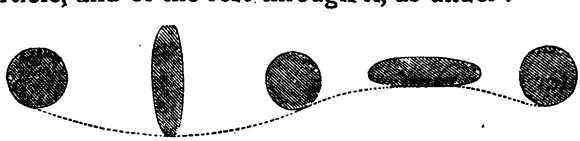


Fig. 39.

Now let the student try to follow the application of this theory of the motion of a single particle to the motion of the whole series, and he will then understand how the theory of wave motion is connected with the successive condensations and rare-

factions of air. It has been said that a condensation of air corresponds with the crest of a wave, and a rarefaction with the trough.

It must be distinctly understood that when a fork begins to swing, or a string to vibrate, the *quantity* of air condensed by one outward swing or vibration (that quantity being represented in our last illustration by what we have called "a particle of air") depends upon the length of the vibrating body which sets the air in motion; in other words, the length of the wave (that is to say, the *pitch* of the sound), other things being equal, depends upon the rate of the vibration setting it in motion. But when the quantity of air so set in motion has been determined by the speed of the vibrating cause, the *extent* to which it is alternately condensed and rarefied depends upon the *force* of the impact; in other words, the *width* of the waves (that is to say, the *loudness* of the sound) depends upon the extent to which the bulk of air forming the wave-length is condensed and rarefied by the vibrating body. This will best be illustrated by diagrams. The darker portions represent condensations, and the lighter ones rarefactions.

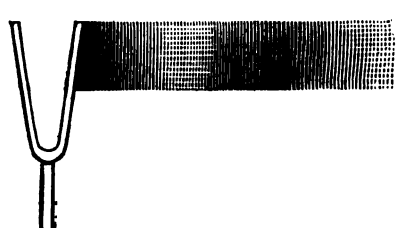


Fig. 40.

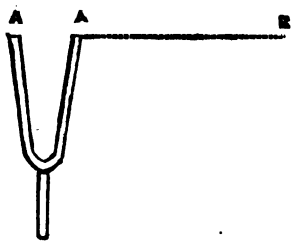


Fig. 41.

Then *a b*, the length of the wave (or the distance between the centre of the two condensations), is decided by the pitch of the fork, and would be the same whether the sound were very loud or scarcely audible. But the width of the wave, as shown in the following figures, is dependent upon the width of the swing, and the more force is imparted to the vibrating body, the wider will be the swing, and consequently the greater the condensation and rarefaction.

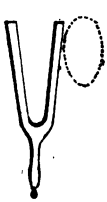


Fig. 42

Fig. 43.

Here the length of fork, and consequent length of wave, is the same; but in No. 1, the swing of the fork is small; in No. 2, the swing is wider and the sound louder, though the pitch is the same.

30. Here, then, we at length reach the true ex-

planation of how wave-forms are made, and waves of sound conveyed, by alternate condensations and rarefactions of air. The fork, commencing, we will say, with an outward swing, condenses (more or less, according to its own force) a bulk of air equal in length to the wave-length of its own pitch. The condensed air rebounds by its elasticity, reaches its first state, and is rarefied; reaches again its first state, and is then again sent through the same stages by the second stroke of the fork. Or, graphically, as follows:—

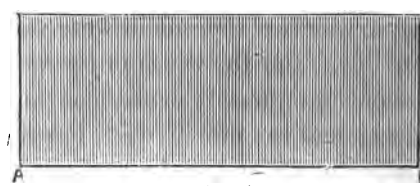


Fig. 44.

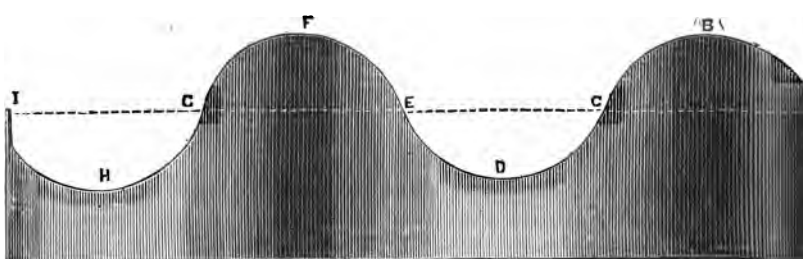


Fig. 45.

In this figure No. 1 represents a quantity of air, of the wave-length A B, at rest. A fork, the pitch of which corresponds to that wave-length (a fork, that is, of such pitch that at each vibration it acts on a

pulse of air of the length A B), is set in vibration. The effect of its outward swing will be to displace by condensation the particles nearest to it, till they reach B, then return to their original position at C, rebound to D (the rarefaction), and return to E, where the second swing of the fork begins to take effect. If a line be drawn along the curve A B C D E, it will be seen that it forms a wave, and that the condensation is greatest at the highest part of the crest, and the rarefaction is greatest at the lowest part of the trough; that is to say, the air is most compressed at B, and most expanded at D.

"The transmission of sound through the air takes place by the formation of ærial elastic *waves*, which are propagated through the fluid medium with great velocity and ultimately enter the ear. These are in some respects analogous to waves formed on water, but with the difference that, whereas the latter are at the surface, ærial waves exist in the mass of the air itself. This is rendered possible by the elasticity of the air, which allows of its being either compressed, or expanded, under slight forces applied. The effect of the slight blows given to the air by the vibrations of the sounding body, is to produce slight compressions and expansions of the circumambient air, and these, being propagated to a distance through the fluid, form the waves. Each wave is of a compound nature, consisting partly of compressed, and partly of expanded air, and one complete wave is produced by each complete vibration of the sounding body."—*Pole.*

"Let us now replace our row of indefinitely numerous points by the slenderest filament of some material whose parts (like those of an elastic string) admit of being compressed, or dilated, at pleasure. When any portion of the

filament is shortened, a larger quantity of material is forced into the space which was before occupied by a smaller quantity. The matter within this space is, therefore, more tightly packed, more *dense*, than it was, *i.e.*, a process of *condensation* has occurred. On the other hand, when a portion of the filament is lengthened, a smaller quantity is made to occupy the space before occupied by a larger quantity. Here the matter is more loosely packed, more *rare*, than it was, *i.e.*, a process of *rarefaction* has taken place.

"Let us now suppose the particles, or smallest conceivable atoms, of the filament, to be thrown into successive vibrations in the manner already so fully explained. Alternate states of condensation and rarefaction will then travel along the filament. It will be convenient to call these states 'pulses'—of condensation or rarefaction as the case may be. A pulse of condensation and a pulse of rarefaction together make up a complete wave."—*Taylor.*

31. For the sake of simplicity, only the upper portion of the body of air affected by each pulse of the fork is here shown; but, dealing only with waves propelled in one direction, the condensation at each pulse will, of course, occur in both an upward and a downward direction, as in the next figure, where the black line is at the centre of the bulk of air comprising the wave-length, and the dotted line shows the level of the air before the impulse is received.

32. A fork which gives "tenor C," or the middle C of a seven-octave pianoforte, swings outward 256 times, and back the same number, every second, making 512 motions, or 256 complete vibrations in that time. When a fork at that pitch, or a piano

wire giving the same note, is set in motion, the wave which it makes by one condensation and one rarefaction is two feet long, an octave lower the wave would be four feet long, an octave higher one foot. The connection between the number of vibrations per

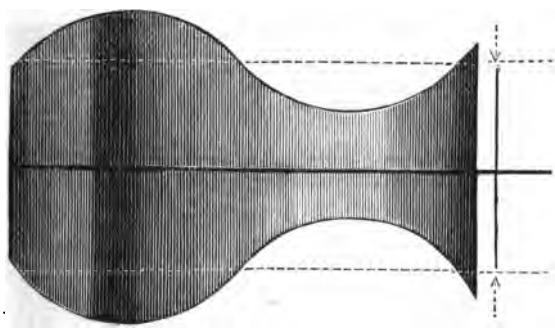


Fig. 46.

second (or pitch) and the length of wave will be dealt with in a subsequent chapter; it will be enough for our present purpose if the student thoroughly comprehends what has here been discussed. He should by this time be able to convey to another person a clear notion of wave-motion in water by perpendicular movements of drops, and also of *wave-motion in air by the rising (condensation) and falling (rarefaction) of air in a direction perpendicular to that in which the sound-wave is travelling, be the latter what it may;* and should further be able to draw diagrams of his own, based on the same principles as those here given, but dealing with waves of different lengths and forms. To vary the form of his waves he has only to increase

the height to which the wave is pushed when condensed and the depth to which it recedes when rarefied. It should further be borne in mind that, as shown by the diagrams in the latter part of Chapter III., the forms of waves vary not only because additional force of swing may cause a greater width of wave, but also because, from different causes, the air forming the wave may condense rapidly and rarefy slowly, or *vice versa*, the diversity of wave-forms being thus practically infinite. The student can adopt no surer means of attaining a permanent knowledge of the way in which alternate condensation and rarefaction makes wave-forms, than by carefully reading this chapter, and then constructing wave-forms of his own on the basis of the diagrams in Chapter III. just referred to.

33. If the reader who has thus far accompanied us be of an inquiring turn of mind, he will start a difficulty with regard to the facts of condensation and rarefaction of air, and will ask what proof there is that air, when compressed by the prong of the fork, so as to form the crest of a wave, will also recede when rarefied so as to form the trough. As we wish every intelligent inquirer to be satisfied, we refer here to what is known as "Mariotte's law," which is a law discovered by a French *savant* of that name. Mariotte demonstrated by experiment that when air is compressed by any force whatever, it will rebound with as much force as was originally applied to it. If force be applied, that is, to the piston of a per-

fectly air-tight cylinder, until the confined air is made to occupy a space three inches less than before, it will, on the force being removed, expand until it occupies three inches more than when at rest. *Force is put into the air by pressure, and exactly that amount of force will be exerted by the air when the pressure is removed.* The more pressure is applied the more dense does the air become; and the more dense it becomes, the more pressure will it exert in an opposite direction when released. Thus Mariotte's law teaches that as the density of air varies so does its pressure; and it will therefore easily be understood that when air has been so displaced by pressure as to be condensed into the crest of a wave, it will, by virtue of the force thus imparted to it, rebound until it is rarefied to the same extent as it was condensed, and the trough is formed.

34. When a vibrating body moves so as to form waves of air and produce sound, those waves are, under ordinary conditions, propelled in all directions with equal force, and as a matter of course, the sound is proportionately weaker at any given distance than when propelled only in a direct line, upon the easily-understood principle, that force scattered in all directions is sooner expended than the same amount of force driven in one direction only. This is why the quieter musical instruments are heard to greater advantage in a room than in the open air, and the same reason accounts for the unpleasantness which arises when the louder brass instruments, which are

tolerable when played out of doors, assault the ear when their tones are confined within four walls.

"It is assumed, and we have no reason to doubt the law, that Air, as a fluid, is subject to the law of Equality of Pressure in all directions. If, for instance, air is forcibly confined in a vessel of glass or other material furnished with closed holes of equal dimensions on different sides, the confined air will exert equal pressures on the stoppers of those holes; and, if one of them is removed, the air will rush out with the same velocity, whichever be the hole selected, and whether the outburst be upwards, downwards, or in a horizontal or any other direction. It is also assumed that this fundamental property of each small volume of air holds when that small volume of air is in motion; although that motion might in some degree derange the laws of pressure in experimental cases like that to which we have alluded. Thus, if the pressure of air within a bottle were produced by the sudden rush of a quantity of air into the neck of the bottle, that part of the shoulder of the bottle near to its neck might not sustain the same pressure (on equal portions of surface) as the base of the bottle, because the inertia of the moving portions of air would produce the largest pressure on that part of the bottle whose resistance brought the air to a state of rest; although the Law of Equal Pressure applied to every small volume of air in motion.

"Indeed, as all our experiments on air are made on air moving through space with great rapidity, it is impossible to deny the application of experimental results derived from air apparently at rest to air which is really in motion.

"It is also assumed, in the following investigations, that the particles or very small volumes of air can move among each other with perfect freedom from friction or viscosity.

It would seem probable that the effect of such friction, &c., would be, not to alter materially the laws of vibration at which we shall arrive, but to produce the rapid extinction of motion.

"But the properties of Air to which it is most important to call attention here are the Laws of Pressure of Air considered as an Elastic Gas. These are three, (I) the law connecting the elastic force of air (or, which is necessarily the same thing, the external pressure that compels the air to occupy only a certain limited space) with the density of the air, at a definite temperature; (II) the variation produced in that law by a permanent change of the temperature of the air; (III) the variation produced in that law by a sudden change of the compression of the air."—*Airy.*

"The constitution of an air-wave, taken between its extreme boundaries, is, as has been said, of a compound nature. In a part of it the air is compressed, in another part of it the air is expanded; and a fair idea may be formed of this constitution by returning to our simile of waves upon water. A complete water-wave consists partly of an elevation above, and partly of a depression below, the normal water level; and if we imagine the former to be analogous to the compression of the air, and the latter to its expansion, we may understand how the different components may be arranged.

"It may also be easily imagined that in different waves the disposition of the compressed and expanded parts may be different, giving rise to different *forms* of wave. It is also clear that, though the form may be the same, the disturbance of the particles, *i.e.*, the degree of compression or expansion, may vary, giving rise to different *intensities* of waves, just as on the water some waves may be high and violent, others low and gentle.

"It will now be intelligible how the ærial waves will

correspond with, and will therefore transmit, the several different properties of the original vibrations of the particles of the sounding body.

"In the first place, it has been already explained how every new vibration will originate a new air-wave, each flying off into space at a certain velocity; so that the *rapidity* of the original vibrations will be represented, inversely, by the length of the wave; double the rapidity of vibration giving half the length of wave, and so on.

"Secondly, as the vibrations are more powerful, they will naturally produce greater disturbance of the particles of the air, so that the *amplitude* of the original vibrations will correspond with the intensity of the disturbance in the air-waves.

"Thirdly, the variations in the *form* of the vibrations will produce corresponding variations of form in the air-waves.

"And in regard to this point there is a principle which, as applied to music, is of much importance. It is, that a simple musical sound (*i.e.*, a sound not compounded of several, as most musical sounds are) always gives rise to a certain form of air-wave, only varying in its length and its intensity. The more complex forms of waves are produced by the combination of several simple sounds together. It is true that each sound will tend to produce (just as if it were sounding alone) its own particular form of wave; but since these cannot exist separately in the same mass of air, they will combine, and their combination will constitute a complex form of wave. And conversely, according to a certain mathematical law called *Fourier's theorem*, it is demonstrable that any regular periodic form of compound air-wave may be resolved into a number of simple ones, which may be discovered, and their properties identified."
—*Pole.*

The difficulty mentioned at the beginning of this chapter—as to applying the visible motion of water-waves to illustrate the invisible motion of air-waves—will be a difficulty no longer if, after carefully reading what has just been said, the student weighs well what follows. At the sea-level air and water meet, but as the density of water is so much greater, and its mobility so much less than that of air, the atmosphere above the water gives way at the slightest motion of the heavier body beneath. To one who hears a sound, it matters not that waves are propelled in all directions by the sound-producing body; if he can understand the process by which the sound reaches his own ear, he can disregard the thousands of similar processes going on at the same time in all directions. Let the student, therefore, regard the sound-waves coming to him, as produced on a body of water which is like a layer between two masses of air, one above, the other below. If a long glass tube, say of six-inch bore, were filled with water, and laid in a horizontal position between the ear and the sound-producing body, and it were possible to take away the tube and leave the water just where it was, *that body of water would serve to represent the body of air which we wish the student to picture in his mind as conveying waves of sound from the vibrating body to his ear.* Any force acting upon such a body of water so as to produce waves at one end, would cause such waves to be propelled along the whole mass until they reached the other end, while the water

as a body would remain quiescent. The air above and below would yield as the water-waves rose, and would follow them as they fell, forming, as it were, an elastic covering which would conform exactly to the varying shapes taken by the surface of the water.

An exactly similar operation goes on when sound is produced in air. The cause of the waves—whatever it be—vibrates in the air so as to produce, by alternate condensations and rarefactions, alternate risings and fallings of the surfaces of that portion of air which goes from the vibrating body to the ear which hears it; and just as the air around the water gives way when a wave rises, and follows the surface when a hollow is formed, so does the body of air around the particular mass of air which conveys the sound to the ear; and the student will find it easier to comprehend the application of the wave theory to sound, if he will try to regard only the motions of the particular body of air which conveys the air waves from their originating cause to his own ear.

CHAPTER V.

ELEMENTS OF A MUSICAL SOUND.

35. IN Chapter III. it has been pointed out that a sound can only have three elements, viz., force, pitch and quality; and remembering that the difference between a musical and a non-musical sound is that the former is produced by regular and the latter by irregular vibrations, we will now discuss these different elements. It is hardly necessary to demonstrate that these three elements do exist in every musical sound, and that no others can exist. The same note on a violin bowed now gently, and now vigorously, differs only in *loudness*. A higher or a lower note on any instrument, while its loudness may be exactly the same as in either of the two cases just mentioned, would differ from them in height or depth—or, as it is called, in *pitch*. But if a note were played upon a violin, and a note at precisely the same pitch and of the same force played on a clarionet, the most uncultivated ear would perceive a difference of *quality*. No other differences can possibly exist between sounds than these of force, pitch, or quality. A note may be

louder or softer than another, or it may be higher or lower, or it may be of different quality, though varying neither in force nor pitch ; but if there be any difference at all between any two notes, it is necessarily in one of these three particulars.

Let us first consider the element of force, or

(*a.*) LOUDNESS AND EXTENT OF VIBRATION.

36. It is not difficult to get a visible illustration of the fact that loudness depends on the extent of the vibration, or on the *width* of the wave, usually called its amplitude. The string of a violin, gently pulled aside by the finger, and released, gives but a faint sound. The g string, which moves slowest of the four, is the best to experiment upon. When vigorously sounded its vibrations are wider, and give a louder tone, which gradually dies away as the vibrations grow narrower, *i.e.*, as the wave's amplitude decreases. The bass strings of a pianoforte furnish similar proof, if the front is taken off to expose them to view. In organ pipes, a pipe of a large "scale," by which organ builders mean a wide pipe, gives a much louder tone than a narrower one of the same length. As a sound travels in the open air, the reason of its decreased power at increased distance is that its force is more and more expended every yard it moves, or its width of wave grows less until, by the resistance of the atmosphere, it dwindles to nothing and becomes inaudible.

(*b.*) PITCH AND RAPIDITY OF VIBRATION.

37. It is not easy to demonstrate to the eye the fact that the *pitch* of a sound depends on the *rate* at which the vibrating cause moves. By pitch is meant the height or depth of the sound. If the middle C of a pianoforte is struck, any note on the right hand of that note will be higher than it, and any note to the left will be lower; and the further the note is removed to the right or left of that C, the higher or lower will the sound be—the sound will be "pitched" higher or lower.

If each individual vibration of a sound-producing body could be followed by the eye, the fact referred to in paragraph 36, and that now being stated, would be equally plain to the sense, and would require less effort of the mind to comprehend. But while the one fact is to be seen, the other must be apprehended by some other process. If a string could be made to register its vibrations per second, or if some machine could be devised by which it could be *shown* that so many vibrations per second produced a given sound, that a less number made a lower sound, and a greater number a higher, proof of a conclusive nature would then be afforded. Such a machine has been constructed. The disc of cardboard referred to in Chap. I., and illustrated by Fig. 1, is an elementary form of a machine which places it beyond dispute that *the pitch of a sound depends upon the rapidity of the vibrations which produce it*, and also that *the greater*

is the number of vibrations in a given period of time the higher is the pitch of the resulting sound. The following paragraphs will make these facts plain.

38. This machine, of which many kinds have been made, is called *The Syren.* Why a name so romantic is applied to a thing so matter-of-fact does not appear, nor does its name concern us so much as its facilities for demonstrating the point we are discussing. Take a round piece of common tin, 12 inches in diameter (a), and pierce, close to the edge, 16 holes at equal distances from each other, thus:

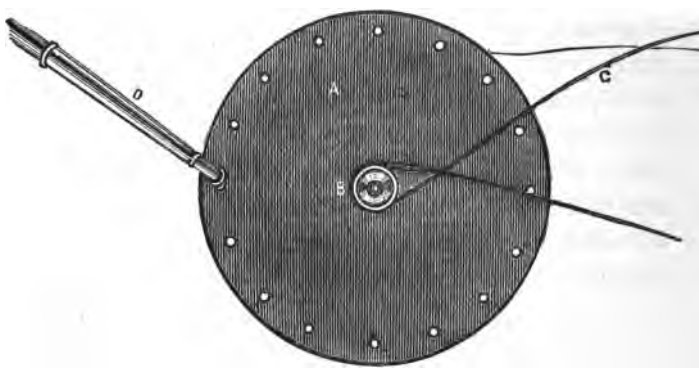


Fig. 47.

Fix it upon a spindle with a centre piece (b), round which a cord (c) can pass. Carry this cord over a large wheel which will turn the disc a given number of times at each of its own revolutions. A handle fixed to the wheel will enable it to be turned at a uniform speed. If air be blown through the pipe, and the handle be turned very slowly,

puffs of air will be made through each hole as it passes by the end of the pipe. So long as these puffs are made with extreme slowness, they will be heard as puffs, but when with more rapid turning they lose their individuality, and become merged into one, they will produce *a note.* Its tone will be weak and its quality poor; but so long as the motion of the wheel is steady and regular, an unmistakable musical tone will result. Increase the speed, and the pitch of the note will rise; decrease the speed, and the pitch will be lowered. If now the wheel be turned so as to give just 16 revolutions per second to the pierced disc, the sound will tally very nearly with the middle C of the pianoforte. But 16 holes and 16 revolutions give 16 times 16 puffs in a second; $16 \times 16 = 256$; and it is proved, therefore, that *the note in question is the result of vibrations at the rate of* 256 *in a second.*

Now pierce another row of eight holes, nearer the centre of the spindle, thus:

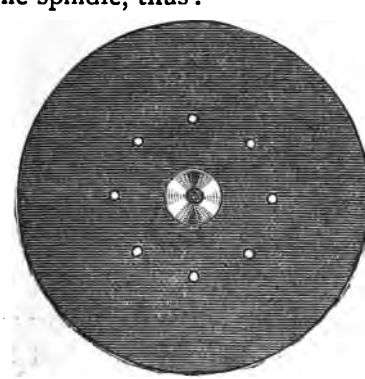


Fig. 48.

Blow through these eight holes as they pass by the end of the pipe, and turn the wheel, as before, so as to give 16 turns in a second to the disc; $16 \times 8 = 128$, and if a note eight notes below the "middle C" is now struck (that is, the C which is technically spoken of as "an octave" lower than middle C), it will be found that the 128 puffs per second of the syren give a tone of exactly the same pitch as that given by the piano. It follows, therefore, that *whatever be the vibration-number of any note, the note an octave above it is produced by a vibration-number twice as great*, so that if 140 vibrations produce a given note, 280 will give the octave above it, 560 the octave above that, and so on. This holds true of *all* notes and their octaves.

Now pierce in the disc another row of twelve holes, between the other two rows, as in Figure 49.

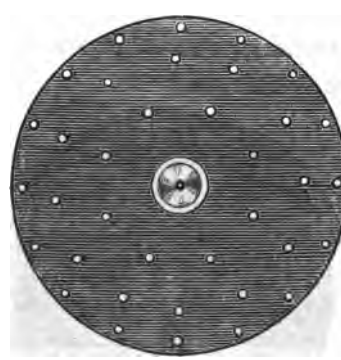


Fig. 49.

If the pipe is applied to this middle row and 16 turns per second given to the disc, we shall have a tone

produced by 16 × 12 vibrations; 16 × 12 = 192; and this will agree with the G between the two Cs last named—the note, that is, which is a fourth below the upper C and a fifth above the lower one. The octave is thus seen to bear to the lower tone the proportion of 2 to 1, that is, the upper octave vibrates twice to every one vibration of the lower; and the fifth bears to the lower tone the proportion of 3 to 2, or, the fifth vibrates three times while the lower octave vibrates twice.

These experiments, if carried out for each note of the scale, will give the following results:—The proportion of the upper octave to the lower is as 2 to 1; of the fifth, 3 to 2; of the fourth, 4 to 3; of the major third, 5 to 4; of the major sixth, 5 to 3; of the major second, 9 to 8; and of the major seventh, 15 to 8. If the vibration-number of the lower note of the scale (called the tonic) be taken as 24, that of the upper octave will therefore be 48; of the fifth, 36; of the fourth, 32; of the major third, 30; of the major sixth, 40; of the major second, 27; and of the major seventh, 45. These complete the diatonic scale, and the following table shows at one view the notes referred to, their intervals from the tonic, their vibration-numbers related to 1 as a tonic, the relation of the various notes of the scale to the tonic, calculated, for simplicity's sake, in low numbers; and the actual numbers per second of the vibrations made by the scale commencing at "tenor C":—

1. Name of Note.								
2. Description.		Second.	Major Third.	Fourth.	Fifth.	Major Sixth.	Major Seventh.	Octave.
3. Vibration number relative to Key-note.	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
4. Vibrations per Second, say	24	27	30	32	36	40	45	48
5. Actual Vibration Numbers.	1	9:8	5:4	4:3	3:2	5:3	15:8	2
	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512

39. To render these results the more definite, it has been proved by Helmholtz, Tyndall, and other investigators, that it is the number of holes in the row, combined with the precise rate of revolution, which alone determines the pitch of the note. The force of wind may vary, but that would only increase or decrease the loudness of the note given. There may be, say, two rows of sixteen holes instead of one row, and two pipes may supply them with wind, but the pitch would still be invariable so long as the number of revolutions of the disc per second remained the same. But the slightest variation in the speed of the rotating disc changes the pitch of the tone, upwards

if the speed be increased, downwards if it be decreased. A penknife held so that the holes catch its point as they pass, produces a musical tone, different in quality from that resulting from blowing through the pipe, but of precisely the same pitch. The *number* of puffs, and not their force or method, is therefore the element which determines their pitch.

40. Whatever may be the form of syren constructed, or the minuteness of detail connected with its operation, the principle involved is always the same,—air is blown through holes pierced in a rotating disc. The syren made by the French philosopher Caignard de la Tour was provided with double discs, pierced with many rows, containing different numbers of holes, and furnished with tooth-wheels and a dial, by which the number of rotations made by the disc in a second could be recorded with strict accuracy. Seebeck's syren was of the comparatively simple form referred to in Fig. 1; while the syren of Dove (pronounced Dō-vay) was "polyphonic," or capable of sounding several notes together, and furnished with the means of opening or closing, at pleasure, the supply of air to various rows of holes.

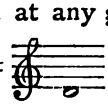
"This instrument received the name of syren from its inventor, Caignard de la Tour. The one now before you is the syren as greatly improved by Dove. The pasteboard syren, whose performance you have already heard, was devised by Seebeck, who gave the instrument various interesting forms, and executed with it many important experiments. Let us now make the syren sing. By pressing the key *m*, the

outer series of apertures in the cylinder c is opened, and by working the bellows, the air is caused to impinge against the disc. It begins to rotate, and you hear a succession of puffs which follow each other so slowly that they may be counted. But as the motion augments, the puffs succeed each other with increasing rapidity, and at length you hear a deep musical note. As the velocity of rotation increases the note rises in pitch: it is now very clear and full, and as the air is urged more vigorously, it becomes so shrill as to be painful. Here we have a further illustration of the dependence of pitch on rapidity of vibration. I touch the side of the disc and lower its speed; the pitch falls instantly. Continuing the pressure the tone continues to sink, ending in the discontinuous puffs with which it began.

"Were the blast sufficiently powerful and the syren sufficiently free from friction, it might be urged to higher and higher notes, until finally its sound would become inaudible to human ears. This, however, would not prove the absence of vibratory motion in the air; but would rather show that our auditory apparatus is incompetent to take up and translate into sound vibrations whose rapidity exceeds a certain limit. The ear, as we shall immediately learn, is in this respect similar to the eye."—*Tyndall.*

41. The student should now calculate for himself what the syren is capable of teaching him. Beginning with simple problems, let him calculate, for instance, the vibration-numbers of the sounds produced by rows of 8, 10, 12, 14, 16, 18, and 20 holes, rotating at a uniform rate of 12 revolutions per second; then the problem may be varied by stating the vibration-number required, and finding what rapidity of rotation would be needed to make that

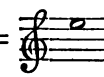
note with rows of 6, 8, 10, 12, &c., holes in a row. Then let him deduce, with the help of the table given above, the vibration-numbers of the fifth, fourth, and other intervals of the scale, when the number of vibrations made by the tonic is given; or, given the vibration-number of the fifth or any interval, he should be able conversely to find how many times in a second the tonic vibrates. By these means he will learn the utility of the syren in determining the vibration-number of any note. Its chief uses are these:—(1.) A note being sounded on a tuning fork or other instrument, to determine by producing the same note and counting the puffs required, how many vibrations in a second it makes. (2.) To determine what relation the notes of the scale bears to the tonic or to each other. (3.) To decide the cause which produces "beats" (see the chapter in which that subject is treated), and to demonstrate the limits within which beats are possible. (4.) It shows that below a certain number per second vibrations do not unite to make a musical tone recognisable by the human ear.

We give here a few calculations illustrating the uses of the syren referred to above. (1.) *To determine the vibration-number of any given sound.* Let a string be sounded at any given pitch—say the D-string of a violin, =  Turn the disc of the syren, and apply the air-pipe to the row containing 16 holes. Turning more and more rapidly, the tone gradually approaches that of the string, until 18 turns

per second are reached, when they will exactly agree. Now 18 turns at 16 puffs per turn, is equal to

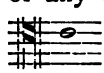
$$18 \times 16 = 288,$$

and on referring to the table given above, the theoretical vibration number of D will be seen to be 288.

Now sound the first string E =  To reach this note the syren must be turned much more rapidly; but on reaching *forty* revolutions per second the tone of the syren will correspond with that of the violin. But—

$$40 \times 16 = 640 = 320 \times 2$$

and as the E in the table gives 320 vibrations, its octave (the E of the first violin string) gives twice 320=640.

(2.) *To determine the relation of the notes of the scale to the tonic, or to each other.* Thus if the number of any note which is the tonic of a scale—say C  vibration-number 256, the theoretical vibration-number of its fifth G is, according to the table, in the same relation to 256 as 3 is to 2. Thus—

$$256 \times 3 = 768 \div 2 = 384,$$

and that number is the theoretical vibration-number of this G. The syren proves this relationship in an unmistakable manner, for if the row of 8 holes is revolving 32 times in a second, the tenor C is pro-

duced; but if the air-pipe is put to the 12 hole row, without the speed being altered, the note G, a fifth above it, is immediately produced. Thus—

$$8 \times 32 = 256; \text{ and}$$
$$12 \times 32 = 384.$$
$$\text{But } 8 : 12 :: 2 : 3; \text{ or } 8 \times 3 \div 2 = 12; \text{ and}$$
$$256 : 384 :: 2 : 3; \text{ or } 256 \times 3 \div 2 = 384.$$

Further, if when the relation of the tonic to its fifth ($1 : \frac{3}{2}$) has been demonstrated, the student wishes for similar proof of the relationship of any other note of the scale, the syren will afford such proof. It is desired, for instance, to prove that the tonic is to the major third as $1 : \frac{5}{4}$. Then take the same tonic C = 256 (any other will serve the purpose equally well), and calculate the theoretical vibration-number of its major-third E as follows:—

$$\frac{256 \times 5}{4} = \frac{1280}{4} = 320;$$

and if the 8-hole row be revolved 40 times per second, E, the major-third of C, will be produced. But—

$$8 \times 32 = 256$$
$$\text{and } 8 \times 40 = 320.$$
$$\text{Also } 32 : 40 :: 4 : 5, \text{ and therefore}$$
$$256 : 320 :: 4 : 5.$$

Proceeding in this way, the syren will not only decide theoretically what the vibration-number of any note should be, but the note being sounded, the instrument will demonstrate that such note *is* produced by a certain number of vibrations.

(3.) If the two wires of a pianoforte which formed tenor C are tuned so as to produce four beats, or throbs, per second *—that is to say, if one wire is in tune and the other is sharpened till four beats per second are the result, theory asserts that one of the wires is making more vibrations per second than the other. This is demonstrated by the syren thus: the 8 hole row, revolving 32 times in a second, produces tenor C, as has been shown, but if a 10 hole row, on another syren, be revolved 26 times, so as to make 260 vibrations, and the two are sounded together—the 256 and the 260—they will produce just four beats or throbs per second. The cause of this is explained in Chapter XII.; it is enough for the student now to know how to prove it by means of the syren.

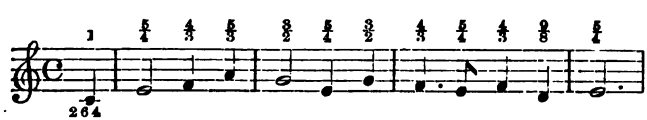
(c.) MEASURES OF ABSOLUTE AND RELATIVE PITCH.

Absolute pitch is the actual vibration-number which any note makes, without regard to any other note. *Relative pitch* is the proportion which the vibration-number of a given note bears to that of some other given note—usually the tonic, or key-note. The string of a violin may be tuned to any note we please: we may make the *absolute pitch* of A either 400, 410, 420, 430, or any other number of vibrations per second, according to the pitch of the instrument with which we play. A violinist, we will suppose, spends three "musical evenings" a week in different families, and

* See Chapter XII.

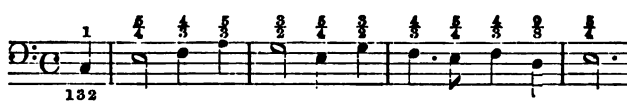
each evening he may find it necessary to tune his A string differently; the first night the piano is somewhat old, and is tuned at a low pitch; the next night he finds the piano at the house of the second friend is slightly higher, and he must tune accordingly; while the third evening he has to play to a "grand" at "concert pitch," probably nearly a full tone higher than that with which he played on the first evening. On each night, therefore, he alters the *absolute pitch* of his violin; but the relative pitch of the four strings must be preserved intact, and whatever be the pitch of the A string (to which a violinist tunes the other strings) the E will be a fifth above it, the D a fifth below it, and the G a fifth below the D. There will be different As, Es, Ds, and Gs at each different tuning; but there will always be a fifth between each string and its nearest neighbour. The *absolute pitch* varies, but not the *relative pitch.*

Take, as a further illustration of this subject, the first strains of the tune "Home, Sweet Home."

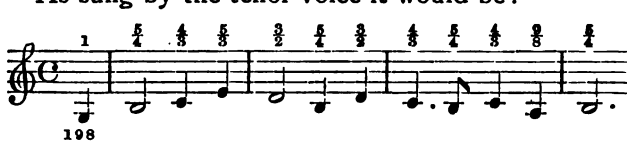


A bass voice, a tenor voice, and a soprano voice may sing this familiar air at various *pitches.* The strain is here given in the key of C, and the first note vibrates 264 times in a second; but a bass voice may sing it an octave lower, starting with a note vibrating

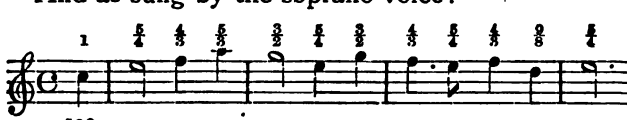
only 132 times in a second; the tenor may sing it a fourth lower, and start with a note whose vibration-number is 198; while the soprano could take it a complete octave higher, commencing upon C, vibrating 528 times per second. There is a wide difference in the *absolute pitch* at which the melody would be sung in these three cases, but the *relative pitch* would remain the same; *each note of the air would have, in each case, a vibration-number bearing the same proportion to the key-note as the fractions over the notes in the above figure bear to* 1. The air sung by the bass voice would have this notation, and the vibration-numbers of the notes would be as figured:—



As sung by the tenor voice it would be:—



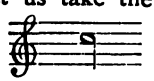
And as sung by the soprano voice:—



The vibration-numbers are thus seen to be always in the same proportion to the tonic, though the vibration-number of that tonic may vary to any extent. *Absolute pitch* may be at any vibration-number;

relative pitch must bear the same proportion to a given standard.

"Since every possible musical sound corresponds to a definite and known velocity of vibration, it may be thought that any note used in music ought to be at once positively and unmistakably connected with its proper vibration-number. This, however, is not so, the reason being that at present there is no definite standard, no general agreement among musicians, as to the exact place in the infinite scale of possible tones where any nominal musical note should stand.

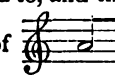
"An example will render this clear. Let us take the note treble C, indicated in music by the sign  and let us suppose that some competent inquirer wished to ascertain for himself what was the vibration-number corresponding to this note. If it were sounded to him by an old organ or an old pitch-pipe, he would probably find it something near 500: Westminster Abbey organ would give him 518; a new piano of Broadwood's would give him 526; and a modern opera band, on a hot summer evening, would give about 545. The highest of these examples would be about a semitone and a half different from the lowest, and he would find, in different cases, all sorts of intermediate values for this note called treble C.

"This opens the question of the *Standard of Pitch*, one which has been much debated.

"It stands to reason and common sense that there ought to be some common agreement among the musicians of the world as to what musical note should be denoted by a certain musical sign; but unfortunately there is no such agreement, and the question is therefore still undetermined.

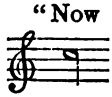
"There is reasonable evidence that from the time of

Handel down to the death of Beethoven, during which period the greatest musical works extant were written, the pitch ordinarily used was (always referring to the treble C) somewhere between 500 and 520. Since that time it has risen considerably, and the modern English orchestras are tuned to about 540.

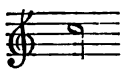
"This rise, however, has been objected to, and the French have established by law a standard of  = 435, which gives treble C (by equal temperament) = 517. This pitch is now also generally adopted in this country for church and cathedral organs, and for all vocal music when unaccompanied by an orchestra.

"Although, however, practice differs so much in regard to the pitch, it is desirable, on philosophical grounds, to determine on some standard by which the position of musical notes can be theoretically defined; and fortunately there is a very easy and very satisfactory means by which this can be done.

"Let us inquire what note would be given by the simplest rate of vibration, *i.e.*, one double vibration per second. We cannot tell this by actual experiment, for the reason that the sound would be inaudible. But since there is a well-known harmonic law (to be hereafter explained), that by doubling the vibration-number we raise any sound exactly an octave, we have only to go on doubling several times, and we get sounds that can be perfectly identified. For example, at the ninth doubling we get 512 vibrations per second, and we know that the note given by this will be exactly the same note as by one vibration, only nine octaves higher.

"Now this note almost exactly corresponds with the  used in the time of Handel, Mozart and Beeth-

oven; and is very nearly the same as the French legalised pitch and the vocal pitch in England. And as the note C is the simplest note in our modern musical system, and the one generally used for a standard, we find that on this pitch *the simplest rate of vibration produces the simplest musical note.*

"We thus obtain a reasonably good standard of pitch, *i.e.*,  = 512. This has acquired the name of the *philosophical pitch*, and, as already stated, it corresponds fairly well with the actual pitch in the best musical times. It will be adopted throughout this work whenever the absolute pitch of notes has to be referred to.*"—*Pole.*

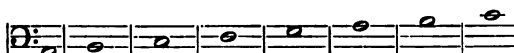
The notes of the scale in "just intonation" always have vibration-numbers bearing the same proportion to the tonic. They are here repeated, to enable the student to familiarise himself with the method of calculating vibration-numbers; the problem is now worked out for the scale whose tonic is C vibrating 132 times per second.

In practice slight variations are made from these fractions, to meet the requirements of "temperament." This subject is treated further on.

* By this standard the vibration number for C will always be some power of 2; so that 2^n *vibrations per second will always give the note C.*

Helmholtz uses a higher standard pitch, namely, C = 528, but nothing of importance turns on this, and therefore I have adhered to the standard I have always advocated.

It is worthy of remark that the philosophical pitch is given almost exactly by the Westminster bells, the key-note of which, "Big Ben," sounds an F of about 170 vibrations per second.

Notation.								
Interval from Tonic.		Major Second.	Major Third.	Perfect Fourth.	Perfect Fifth.	Major Sixth.	Major Seventh.	Octave.
Technical Name.	Tonic.	Upper Tonic.	Mediant.	Sub-dominant.	Dominant.	Sub-mediant.	Leading Note.	Octave of Tonic.
Numerical Relation to Tonic.	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Actual Vibration Number.	132	148½	165	176	198	220	247½	264

"It follows from what has been said that the number of possible notes all differing from each other in pitch, is theoretically unlimited, inasmuch as any difference in the vibration-number will certainly give rise to a different sound. Practically, also the number is very large, depending only on the sensitiveness of the ear for minute differences of pitch; a skilful pianoforte tuner, for example, is obliged, in the exercise of his art, to distinguish between a true and an equally tempered fifth, the difference being only about *one-fiftieth of a semitone*, which would give 600 distinguishable sounds in the octave! The pianoforte has only 12 sounds in the octave, but the intervals between them may be easily divided into several parts, and it is usual to estimate that from 50 to 100 sounds in the octave may be distinguished by ordinary ears."—*Pole.*

(d.) MODES OF MEASURING PITCH.

These are very clearly and admirably defined in Dr W. H. Stone's "Sound," as follows:—

I. Mechanical methods.
   1. Savart's toothed wheel.
   2. Caignard de la Tour's syren.
   3. Perronet Thompson's weighted monochord.
   4. Duhamel's vibroscope.
   5. Leon Scott's phonautograph.
   6. Edison's phonograph.

II. Optical methods.
   1. Lissajous' method.
   2. Helmholtz's vibration microscope.
   3. Kœnig's manometric flames.
   4. McLeod and Clarke's cycloscope.

III. Photographic methods.
   Prof. Blake's experiments.

IV. Electrical methods.
   1. Meyer's electrical tonometer.
   2. Lord Rayleigh's pendulum experiment.

V. Computative methods.
   1. Chladni's rod tonometer.
   2. Scheibler's *Tonmesser* with tuning-forks.
   3. Appunn's tonometer with free reeds.

Some of these methods shall be here very briefly described:

Savart's toothed wheel was simply an apparatus for testing the height at which sounds were audible by the human ear.

The syren first invented by Caignard de la Tour in 1819 was a much more scientific method of determining the pitch of sounds. It has been already alluded to in this work. Its simplest form is that made by Seebeck, consisting merely of a disc pierced with rows of circular holes, through which wind could be blown by means of bellows. It is curious that the name of this instrument is a misnomer. Dr. Stone, in his book on Sound, states that the name is said to have been derived "from its power of sounding under water," a gift which Homer's Σειρήνεος were not endowed with.

The teeth of Savart's wheel are represented by the openings in the circular plate of the syren. The two following figures represent respectively the syren of Caignard de la Tour and Seebeck—

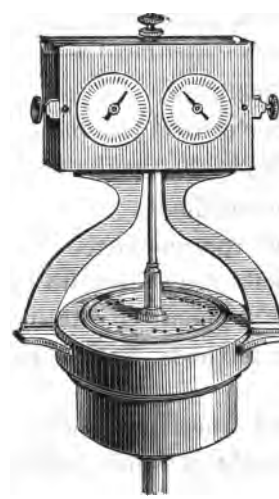


Fig. 50.

Fig. 51.

A more complicated instrument, however, is the double syren of Helmholtz, which is represented in Figure 52, on page 100.

The principle on which the syren is made, however complicated may be its construction, is that of the passage of air through a number of holes, so as to produce a note of any given pitch. With Helmholtz's double syren the theory of beats can also be very clearly demonstrated and explained. Its real use is in showing the materials of which the sound is formed,

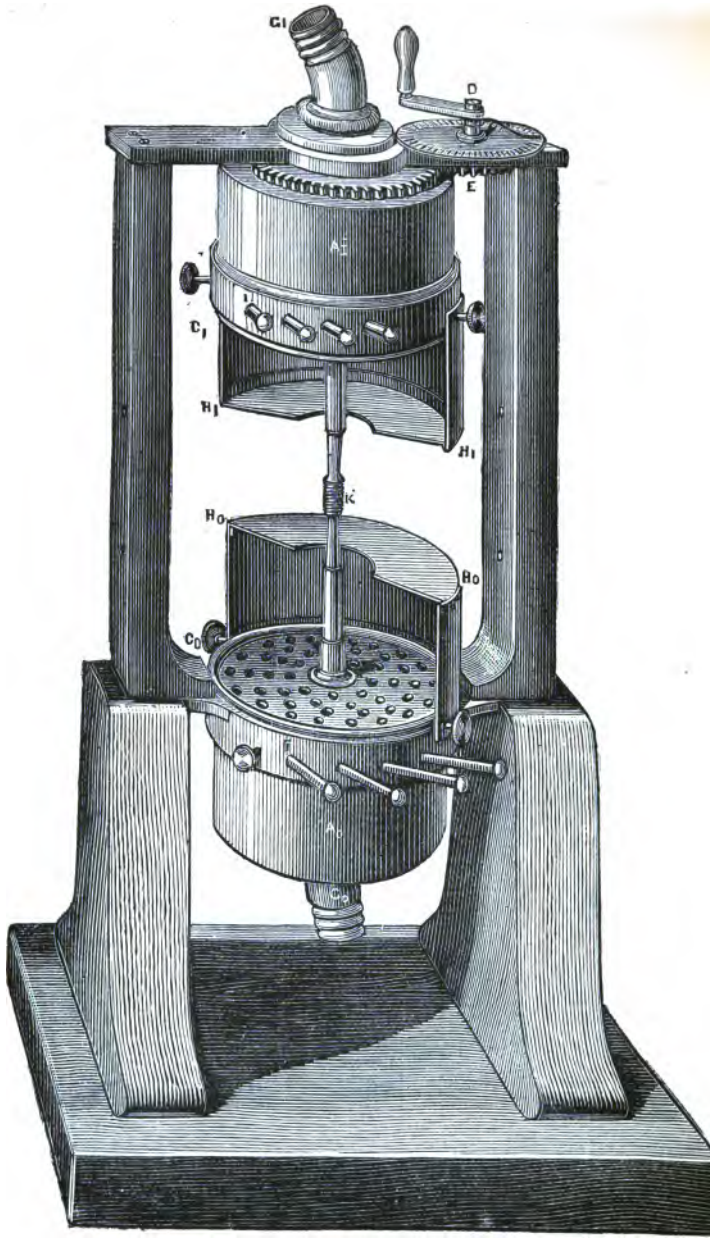


Fig. 52.

and the ratio of the vibration numbers forming the principal consonant and dissonant intervals.

Thompson's monochord is a method of demonstrating the pitch by adding different weights to a stretched string, as its name implies. The string is turned to a given note, and its vibrations determined by calculations taking into account the stretching weight, the weight of the wire itself, and its length. This method is of little value in the determination of pitch, in consequence of the difficulties by which it is attended. Scheibler says, writing on this subject, "I became convinced that a mathematical monochord could not be constructed."

Another method of measuring pitch is the vibroscope, which, as its name implies, is a method of making vibrations visible. A fine point is attached to one of the prongs of a tuning fork, and is then brought into contact with a cylinder rotating uniformly and registering the number of vibrations which it makes. By this means vibrations of various forms can be registered, and some of the forms are very curious indeed. Here are a few different kinds:—

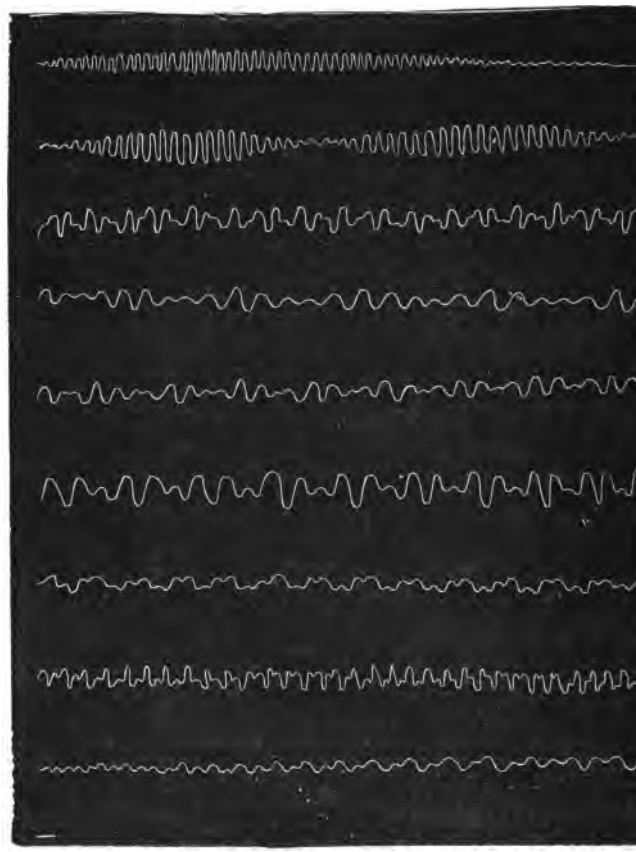


Fig. 53.

Still better methods of registering pitch have, however, been invented.

M. Lissajous has adopted a mode which may be briefly described thus: A grain of starch is fixed to a tuning fork, or to a violin string, as the case may

be, and a strong light is directed upon it. This light, seen through a microscope, is reflected by means of lenses upon a screen, and the vibration makes upon that screen a figure corresponding to the quality of the tone with which it is connected.

The curves which result from the combination of various tones and qualities of tone belong rather to the romance of the science of Acoustics than to its practical application to the purposes of music; and the same remark applies to the manometric flames —the latter method consisting in transmitting the effects of sound waves from a membrane to a reservoir of gas in connection with a flame. The flame flickers up and down and coincides with the vibration. The flickerings are reflected in a revolving mirror, and are thus kept distinct one from the other. If the flame burns steadily, a band of light is the result, but when it is affected by the vibrations of the membrane, a series of flames varying in height and frequency are thrown upon the screen. The value of this method consists in the fact, that when the tuning fork varies, however slightly, from its theoretical vibration number, the variation can be at once detected by its aid.

Scheibler's tone measurer is a method of measuring pitch by means of the beats of tuning forks. His mode was by obtaining a correctly-tuned scale of tuning forks, the vibration numbers of which were already known, to sound another fork besides these, and, by counting the beats, to determine between which of the tones of his scale it lay, and how many

vibrations it was sharper than the one and flatter than the other.

Appunn's reed tonometer is a mode of measuring the pitch by means of harmonium reeds. The quality of those reeds is very rich in upper partials,* and the tones are somewhat harsh, which constitutes its chief advantage in the determination of the pitch, as any, however slight, difference of tone can be detected by beats. Dr. Stone thus describes the arrangement of this instrument: "65 reeds are arranged in a long rectangular box and excited by a steady wind pressure. The reeds each act in a separate chamber controlled by a wire which opens a valve, fully, or to any amount. By pushing in the valve the note is flattened to about 2½ vibrations in the second. The reeds are so tuned that each beats exactly four times a second more than either of the adjacent reeds. The lowest is No. 0 and the highest 64, consequently the highest is 4 times 64, or 256 vibrations higher than the lowest. The lowest and highest sounded together make a perfect octave. The difference between the numbers of vibrations being 256, it follows, from what has been shown above, that the lowest reed makes 286 and the highest 512 vibrations in the second. Unfortunately this apparatus is materially influenced by a power which the reeds, when vibrating strongly, have of influencing one another."

Dr. Stone gives the following facts from a paper

* See Chapter VII.

read before the Society of Arts by Mr. A. Ellis about three years ago:—

"*Variation of Standard Pitch.*—The best testimony to be obtained on this subject is from organs and authentic tuning-forks. Mr. Ellis, in a laborious paper read before the Society of Arts, has collected a very large number of examples, and has 'compiled a complete scale of variation of standard pitch proceeding from Handel to the present day, with three older and much lower isolated pitches.' The following is an analysis of his researches. It may be noticed that his initial determination of the French normal pitch by means of Appunn's tonometer is still open to some doubt, on account of discrepancies due to temperature and the mutual influence of the reeds on one another. He states that instead of 435 or 870, the normal *A* really has a vibration-number of 439 or 878. This statement has been attacked by M. Rudolph Kœnig. But there is no reason to suppose that the relative pitches of the various standards examined suffer from this initial difficulty, which may be easily remedied by a small constant correction to be afterwards applied.

"Mr. Ellis makes five principal groups, as follows:—

"I. *Ancient Low Pitch*, C *below* 500.—The first thing that strikes us is the great flatness of the older pitches. Dr. R. Smith's D, 262, in 1755, taken an octave higher, as D 524, gives nearly the present French normal for C. Hence his pitch was almost exactly a whole tone flatter than the present French C, and a tone and a quarter flatter than Broadwood's present high pitch, which we may take to represent 'concert pitch.' But rejecting this as in all probability wrongly ascertained, we have the fork tuned to Father Schmidt's C pipe at Hampton Court before the organ was reconstructed, and this is more than a semitone

flatter than C 512, and $\frac{7}{8}$ of a tone flatter than 'concert pitch.' It follows that vocal music composed a hundred years ago ought to be transposed a whole tone, if sung at the present pitch, to produce its proper effect.

"II. *The Handel Pitch*, C 500 *to* 513.—C 512, which was insisted on so strongly by Sir John Herschel at the meeting of the Society of Arts to consider the Report of the Committee on Pitch in 1860, was in favour 50 to 100 years ago. Wieprecht gives a Berlin pitch of that amount, but the measurement may be doubted. We find, however, the fork to which Mr. Peppercorn tuned pianos for the Philharmonic concerts in 1815 was about 511, and reckoning by the old tuning, a fork used at the Plymouth Theatre about 1800, gives nearly the same, while Handel's fork of 1751 gave C = 510. We may take then the dawn of modern pitch to be C 512, which would be fully a semitone flatter than the present concert pitch. Hence vocal music of Handel's time should be transposed a semitone lower than it is written when played at concert pitch. The same remark applies to the music of Mozart, and probably of Haydn and Beethoven.

"The value of C 512, which appears to have been aimed at about the period of this group, is entirely arithmetical. It has no other particular advantage. Arithmeticians can deal with any other C with equal ease by means of decimals. In measuring pitch, it is never necessary to consider more than two places of decimals, and even the last place is used only to prevent an accumulation of error.

"III. *French Normal Pitch*, C 514 *to* 527.—About forty years ago there was a French pitch in use almost coincident with that theoretically established in Paris in 1859; one fork from a good maker measured by Scheibler in 1834 actually gave A 434·9 or practically = C 517. The pitch, however, must have risen rapidly to about A 452, and the

object of the French Commission was to regain this older pitch. This modern version of the older fork in the Paris Opera and Conservatoire was preceded in England by the almost identical but flatter pitch of Sir George Smart, and Broadwood's vocal pitch, and also by the very slightly sharper pitch of Scheibler in Germany, which being chosen by him as the mean pitch of Vienna grand pianofortes, represents the Vienna pitch of the time. From having been accepted by a congress of German physicists, who met at Stuttgart in 1834, it is commonly known as the Stuttgart pitch. Altogether, this group, which is comprised within about a quarter of a tone, represents that most in vogue now on the Continent, and consequently has the greatest claims on our attention, although its highest forks are $\frac{3}{8}$ of a semitone below our present high pitch.

"IV. *Medium Pitch*, C 520 *to* 536.—The interval of about $\frac{1}{8}$ of a tone between the French normal and high pitch is not well marked. We have indeed within this group a fork from Leipzig, purporting to be the Dresden low pitch; one from Vienna, measured by Scheibler, but differing materially from the other Vienna forks; one from the Liceo Musicale at Bologna, in 1869; the medium pitch empirically adopted by Messrs. Broadwood and in the organ of St. Paul's. There were also several foreign forks in this group. The theoretical fork of the Society of Arts, which begins it, was never really made, and Griesbach's A, like Hullah's C, were accidental errors. It would seem that the whole of this group is not generally satisfying; it is both too sharp and too flat, and can only be regarded as a neutral medium pitch.

"V. *Modern High Pitch*, C *above* 536.—The highest group contains the moderately high pitch which the French Commission found so excessive, and the still sharper English concert and military pitch of the present day, with the high

pitch of Brussels, strongly advocated in a report of a committee to the Belgian Minister in 1863.* We find that there was a tuning-fork in Paris in 1826 giving A 445 = C 529·2 for the French Opera, another giving A 449·5 = C 534·6 for the Italian Opera, and another A 452 = C 537·5 for the Opera Comique. The two first belong to the preceding group. Many operas were composed to the last pitch, which was afterwards raised to A 455 = C 541. The report mentions that when the French Commission was appointed the Opera pitch was A 453 = C 538·7, and that Lissajous wished to lower it to A 449·5 = C 534·6, but that a contrary opinion prevailed. The Committee say that to this high pitch belongs the tuning-fork of the Brussels Conservatoire, one in use at Ghent, an old fork of the Paris Opera Comique in 1820, the tuning-fork of the Philharmonic Society of London, that of the Berlin Opera in 1861, and lastly, that of the Choral Society of Cologne.

"We have thus, according to Mr. Ellis's observations, a rise for C from 467 to 546, or 80 vibrations = 2·6 semitones in 130 years, or, if the early observations be rejected as possibly erroneous, from the undoubtedly authentic fork of Handel, which gives 507·4 to the vibration number of 546·5 at which the band of the Belgian Guides were playing in 1859. The writer can state from his own careful observations made at the Handel Festival of 1877, during the performance of the Israel in Egypt, on an extremely hot day in June, the thermometer being nearly 80° under the dome of the orchestra, that the pitch of A rose to 460, which is equivalent to a C of 547, and is higher than any previously recorded."—*Stone.*

Mr. A. J. Ellis, whose researches in the department of Musical Acoustics are well known, and whose

* The pitches given should be corrected by subtracting 4 for the error which Mr. Ellis attributes to the French Normal.

writings on the subjects of pitch and temperament are worthy of all respect, read a most valuable and exhaustive paper before the Society of Arts, on March 3rd, 1880, entitled, "On the History of Musical Pitch," and from that paper—which was printed in the Society's *Journal* for the following week—we have made quotations at the end of this work, which will interest the student, but the reading of which he had better defer until he reaches a more advanced stage of the subject. (See Appendix C.) It may be remarked, however, that the pitch of A has varied at different times from 374·2 to 505·8, or more than a fourth; and on this point Mr. Ellis says:—

"Now, these two pitches, A 374·2 and A 505·8, are a comma more than an equal Fourth apart, and it requires considerable faith to believe that sounds so extremely different could ever have been conceived and written as the same note A. But a key to the mystery is furnished by Arnold Schlick, 1511, who says (chap. 2):—'The organ is to be suited to the choir [or church *chor*], and properly tuned for singing, for where this is not considered, persons are often forced to sing too high or too low, and the organist has to play the chromatics, which is, however, not convenient for every one. But what is the proper length of the pipes for this purpose, and convenient to the choir to sing to, cannot be exactly defined, because people sing higher or lower in one place than in another, according as they have small or great voices. However, if the longest pipe, the F below the Gamma-ut in the pedal [that is 8 F below 8 G, the

Gamma-ut being the name given by Guido Aretino to 8 G, the note on the last line of the bass staff], has its body down to the [beginning of the] foot, sixteen times the length of the annexed line [which the editors say is 4 and 7-8 Rhenish inches = 127·5 mm. long, so that the full pipe is 6½ Rhenish feet, or 2,040 mm. long], I think it will be a suitable length for the choir. But if you build an organ a fifth larger, then you must make C in the pedal [that is, 8 C, the lowest note of the violoncello] of this length. . . . The reason is that a greater part of church music ends in *grambus* [a word which puzzles Schlick's editor, who says, however, that it can only mean the transposition of the *tonus primus* by a Fourth], than in the first tone.' And he proceeds to show its advantage, and especially the advantage of the first pitch (with 6½ Rhenish feet to 8 F) in other ecclesiastical tones. Assuming that when Schlick's pipe was taken two octaves higher, its diameter would be one-thirteenth of its length, which is not improbable, although, perhaps, it is rather narrow, I had such a pipe made, and then measured it, and found its pitch to be V 301·6. If we assume this as 2 F, and, taking 2 A a perfect major Third above (and Schlick's directions for tuning made this major Third very nearly perfect), we obtain as the pitch A 377, which is practically the same low pitch as Delezenne's Hospice Comtesse A 374·2. But if, as in Schlick's greater organ, we assume the 6½ Rhenish feet for 8 C, we obtain A 504·2, which is practically identical with the Halberstadt A 505·8, and is parallelled by the actually existing organ of St. James's Church (*S. Jacobikirche*), at Hamburg, A 494·5, as now tuned, but formerly A 489·2. Hence, we have both the very sharp and the very flat pitch recommended by the same writer, at the same date, and on the same grounds, namely, the accommodation of the singer and the organist in playing the old ecclesiastical tones."—*Journal of the Society of Arts.*

CHAPTER VI.

RESONANCE.

WE may best define resonance as "the strengthening or reinforcing of sound." Stretch a violin string over two bridges fixed on a solid block of wood, and tune it to A, and then tune the A string of a violin to the same note, and compare the force of the two sounds. The one merely moves the air by its own unaided swing, which, owing to the thinness of the string, it can only do to a very limited extent, and consequently emits but a feeble sound; the other gives forth a full round tone, with which the first will not bear comparison. The instrument, by the vibration of its belly, sound-post ("soul," the French rightly name it), and back, reinforces the weak tone of the string, and the peculiar quality which secures this end is called "resonance."

"When a sounding body causes another body to emit sound, we have an instance of a very remarkable phenomenon, called *resonance.* The German term for it, 'co-vibration' (*Mitschwingung*), possesses the merit of at once indicating its essential meaning, namely, the setting up of vibrations in an instrument, not by a blow or other immediate action upon it, but indirectly as the result of

the vibrations of another instrument. In order to produce the effect, we have only to press down very gently one of the keys of a pianoforte, so as to raise the damper, without making any sound, and then sing loudly, into the instrument, the corresponding note. When the voice ceases, the instrument will continue to sustain the note, which will then gradually fade away. If the key is allowed to rise again before the sound is extinct, it will abruptly cease. A similar experiment may be tried, as follows, on any horizontal pianoforte which allows the wires to be uncovered. Each note is, it is well known, produced by two, or by three, wires. Having, as in the previous case, raised one of the dampers without striking the note, twitch *one* of the corresponding wires sharply with the finger-nail, and then wait a few seconds. The vibrations will, in this interval, have communicated themselves to the other string, or strings, belonging to the note pressed down: if, now, the first wire be stopped by applying the tip of the finger to the point where it was at first twitched, the same note, produced by these transmitted vibrations, will continue to be sustained by the remaining wire or wires.

"A more instructive method of studying resonance is to take two unison tuning-forks, strike one of them, and hold it near the other, but without touching it. The second fork will then commence sounding by resonance, and will continue to produce its note though the first fork be brought to silence. It is essential to the success of this experiment that the two forks should be rigorously in unison. If the pitch of one of them be lowered by causing a small pellet of wax to adhere to the end of one of its prongs, the effect of resonance will no longer be produced, even though the alteration of pitch be too small to be recognised by the ear. Further, the phenomenon requires a certain appreciable length of time to develop itself; for, if the silent fork be

only *momentarily* exposed to the influence of its vocal fellow, no result ensues. The resonance, when produced, is at first extremely feeble, and gradually increases in intensity under the continued action of the originally-excited fork. Some seconds must elapse before the maximum resonance is attained. The conditions of our experiment show, directly, that the resonance of the second fork was due to the transmission, *by the air*, of the vibrations of the first, the successive air-impulses falling in such a manner on the fork as to produce a *cumulative effect*. If we bear in mind the disproportionate mass of the body set in motion compared to that of the air acting upon it,—steel being more than six thousand times as heavy as atmospheric air, for equal bulks,—we cannot fail to regard this as a very surprising fact."—*Taylor*.

A tuning-fork which sounds C on the ledger line between the treble and bass staves (tenor C, or middle C) vibrates, at the present standard of organ pitch, 256 times in a second. If sounded alone, its tone will be weak and inaudible a few feet away. Take a narrow glass jar, eighteen inches deep, and hold the vibrating fork over its mouth. The result will as yet be practically the same; but if water be poured into the jar until a certain height is reached, the tone of the fork will suddenly be strengthened. If water is still poured in, the tone will be reduced again, but on emptying out water until the depth is reached at which the tone grew stronger, the same result will follow. This is resonance, and is capable of a simple explanation.

The quantity of air set in motion by the fork—the

wave length of its tone, that is—is much greater than the actual swing of the fork itself. The fork in question (giving tenor C 256 vibrations) swings but a short distance, and while travelling that distance

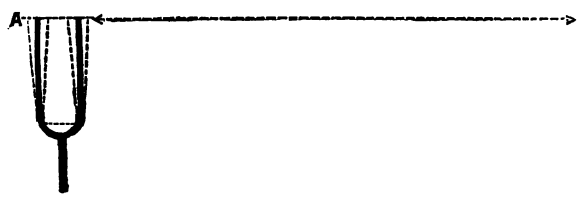


Fig. 55.

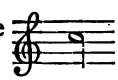
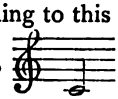
the prong makes half a wave, and the other half is made on the return of the prong. The length of wave made by a note of this pitch has been proved by experiment to be 52 inches; but if the depth of the jar, from the top to the surface of the water, be measured, it will be found to be 13 inches. This is just one-fourth the wave length of the tone given by the fork; hence this general law of resonance, that in a case like this, where one end of the vessel containing the body of air is closed, *a column of air, to reinforce a sound, must be one-fourth of that sound's wave length.* This law holds good of all sounds.

"Thus disciplined we are prepared to consider the subject of organ-pipes, which is one of great importance. Before me on the table are two resonant jars, and in my right hand and my left are held two tuning-forks. I agitate both, and hold them over this jar. One of them only is heard. Held over the other jar, the other fork alone is heard. Each jar

selects that fork whose periods of vibration synchronise with its own. And instead of two forks suppose several of them to be held over the jar; from the confused assemblage of pulses thus generated, the jar would select and reinforce that one which corresponds to its own period of vibration. When I blow across the open mouth of the jar—or, better still, for the jar is too wide for this experiment, when I blow across the open end of a glass tube, of the same length as the jar, a fluttering of the air is thereby produced, an assemblage of pulses at the open mouth of the tube being generated. And what is the consequence? The tube selects that pulse of the flutter which is in synchronism with itself, and raises it to a musical sound. The sound, you perceive, is precisely that obtained when the proper tuning-fork is placed over the tube. The column of air within the tube has, in this case, virtually created its own tuning-fork; for, by the reaction of its pulses upon the sheet of air issuing from the lips, it has compelled that sheet to vibrate in synchronism with itself, and made it thus act the part of the tuning-fork. Selecting for each of the other tuning-forks a resonant tube, in every case, on blowing across the open end of the tube, a tone is produced identical in pitch with that obtained through resonance. When different tubes are compared, the rate of vibration is found to be inversely proportional to the length of the tube. These three tubes are 24, 12, and 6 inches long, respectively. I blow gently across the 24-inch tube, and bring out its fundamental note; similarly treated, the 12-inch tube yields the octave of the note of the 24-inch. In like manner the 6-inch tube yields the octave of the 12-inch. It is plain that this must be the case; for the rate of vibration depending on the distance which the pulse has to travel to complete a vibration, if in one case this distance be twice what it is in another, the rate of vibration must be twice as slow. In general terms, the rate of vibration is

inversely proportional to the length of the tube through which the pulse passes."—*Tyndall.*

"With an open pipe it is found that the length of the sound-wave produced will be double the length of the pipe; and hence, assuming the velocity of sound to be 1100 feet per second, if a pipe be l feet long the length of the sound-wave will be = twice l, and the pipe ought to produce a note vibrating $\frac{1100}{2l}$ times per second. Or conversely, if v = the number of vibrations per second corresponding to any given note, the length of the pipe in feet should be $= \frac{550}{v}$.

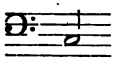
"Take, for example, the note  which corresponds to 512 vibrations; the length of an open organ-pipe sounding this should be $\frac{550}{512} = 1.07$ feet. In practice the rule is somewhat modified by the diameter of the pipe;* but it is a fact that the pipe corresponding to this note is about one foot long. For the C below,  having 256 vibrations, the pipe will be about two feet long; for

* Mr. Ellis ("Nature," p. 172, 26th December 1878) has shown that the influence of the diameter on the pitch of open organ-pipes may be expressed by the following formula: Let L = length from lower lip to open end; and D = internal diameter; both in inches. Then, at 60° Fahr.,

$$\left.\begin{array}{c}\text{Double vibrations per}\\ \text{second}\end{array}\right\} = \frac{20080}{3\,L + 5\,D}$$

$$\text{or } L = \frac{6693}{v} - \frac{5}{3}\,D.$$

This shows that increasing the diameter (which gives a more powerful tone) slightly diminishes the length necessary to produce a given note.

 of 128 vibrations it will be four feet long, and so on, doubling for every octave the note descends. Now, as C is considered the most important note of an organ, the different octaves of this note have acquired, in organ nomenclature, the names of one-foot C, two-foot C, four-foot C, and so on, down to 16-foot C, which is the lowest note in ordinary organs, and 32-foot C, which is the lowest in very large ones.

If the pipe is *stopped* at the top, the effect is to make the length of the sound-wave four times the length of the pipe, or, in other words, to make a stopped pipe speak an octave lower than an open one, the theoretical length of the pipe, measured up to the stopper, being $\frac{275}{v}$."—*Pole.*

It follows, then, that to reinforce a tenor C tuning-fork, we must employ a column of air 13 inches in length. A box of this length will help to bring out the tone of the fork well, and it may be fixed either on the top as in *a*, fig. 56, or held against the open end, as in *b*. In either case the originally weak tone of the fork will be powerfully reinforced by the resonance of the column of air contained in the box. If *both* ends of the box are open, it must be *one half* the wave length to reinforce the tone. The connection between the closed pipe and one fourth the wave length, and between the open pipe and one half the wave length, will be explained when dealing with the question of open and closed organ pipes.

The pianoforte will furnish striking proof of the

effects of resonance. Take off the front, and hold down any note with the finger. If that note be sung near the wire which if struck would produce it, the wire will immediately begin to sound. A tuning fork will, by its vibrations, cause another at exactly the

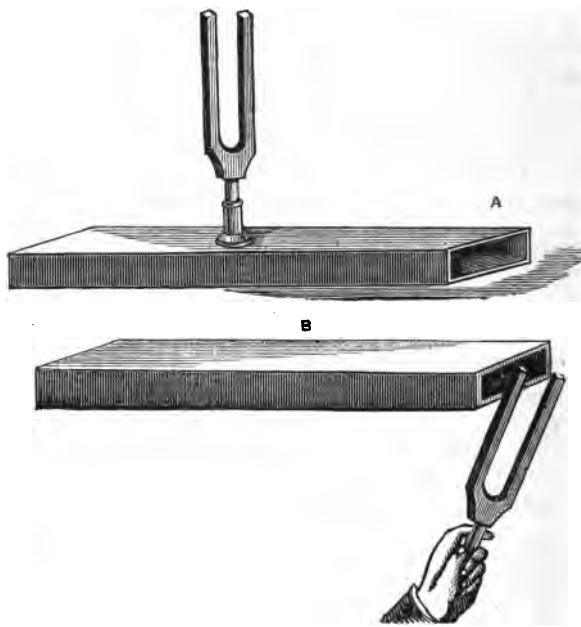


Fig. 56.

same pitch to vibrate too, and the second will again start the first after the latter has been damped with the finger.

The sound-board of a pianoforte, the belly and back of a violin, and the tube of a reed pipe, are familiar instances of the strengthening of tones by

bodies having the power of vibrating synchronously with such tones.

The mechanical causes of resonance are not difficult of discovery. What one impulse could not accomplish alone, a great number in quick succession can do; and although one puff of air could not move a piece of stiff steel, 500 puffs in a second will urge it into powerful vibration.

"Let us examine the mechanical causes to which it is due. Suppose a heavy weight to be suspended from a fixed support by a flexible string, so as to form a pendulum of the simplest kind. In order to cause it to perform oscillations of considerable extent by the application of a number of small impulses, we proceed as follows. As soon as, by the first impulse, the weight has been set vibrating through a small distance, we take care that every succeeding impulse is impressed *in the direction in which the weight is moving at the time.* Each impulse, thus applied, will cause the pendulum to oscillate through a larger angle than before, and, the effects of many impulses being in this way added together, an extensive swing of the pendulum is the result.

"When the distance through which the weight travels to and fro, though in itself considerable, is *small compared to the length of the supporting string*, the time of oscillation is the same for any extent of swing within this limit, and depends only on the length of the string. My readers will find this important principle illustrated in any Manual of Elementary Mechanics, and I must ask them to take it for granted here. For the sake of simplicity, let us suppose that we are dealing with a *second's* pendulum, *i.e.*, one of such a length as to perform one complete oscillation in each

second, and therefore to make a single forward or backward swing in each half second. It will be clear, from what has been said above, that the most rapid effect will be produced on the motion of the pendulum, by applying a forward and a backward impulse respectively during each alternate half second, or, which is the same thing, administering *a pair of to and fro impulses during each complete oscillation of the pendulum.* We have a simple instance of such a proceeding in the way in which a couple of boys set a heavily laden swing in violent motion. They stand facing each other, and each boy, when the swing is moving away from him, helps it along with a vigorous push."—*Taylor.*

"The same process that we have thus become acquainted with for swings of long periodic time, holds precisely for swings of so short a period as resonant vibrations. Any elastic body which is so fastened as to admit of continuing its vibrations for some length of time when once set in motion, can also be made to vibrate sympathetically, when it receives periodic agitations of comparatively small amounts, having a periodic time corresponding to that of its own tone.

"Gently touch one of the keys of a pianoforte without striking the string, so as to raise the damper only, and then sing a note of the corresponding pitch, forcibly directing the voice against the strings of the instrument. On ceasing to sing, the note will be echoed back from the piano. It is easy to discover that this echo is caused by the string which is in unison with the note, for directly the hand is removed from the key, and the damper is allowed to fall, the echo ceases. The sympathetic vibration of the string is still better shown by putting little paper riders upon it, which are jerked off as soon as the string vibrates. The more exactly the singer hits the pitch of the string, the more

strongly it vibrates. A very little deviation from the exact pitch fails in exciting sympathetic vibration.

"In this experiment the sounding-board of the instrument is first struck by the vibrations of the air excited by the human voice. The sounding-board is well-known to consist of a broad flexible wooden plate, which, owing to its extensive surface, is better adapted to convey the agitation of the strings to the air, and of the air to the strings, than the small surface over which string and air are themselves directly in contact. The sounding-board first communicates the agitations which it receives from the air excited by the singer, to the points where the string is fastened. The magnitude of any such single agitation is of course infinitesimally small. A very large number of such effects must necessarily be aggregated, before any sensible motion of the string can be caused. And such a continuous addition of effects really takes place, if, as in the preceding experiments with the bell and the pendulum, the periodic time of the small agitations which are communicated to the extremities of the string by the air through the intervention of the sounding-board, exactly correspond to the periodic time of the string's own vibrations. When this is the case, the string will really be set by a long series of such vibrations into motion which is very violent in comparison with the exciting cause.

"In place of the human voice we might of course use any other musical instrument. Provided only that it can produce the tone of the pianoforte string accurately and sustain it powerfully, it will bring the latter into sympathetic vibration. In place of a pianoforte, again, we can employ any other stringed instrument having a sounding-board, as a violin, guitar, harp, &c., and also stretched membranes, bells, elastic tongues or plates, &c., provided only that the latter are so fastened as to admit to their giving

a tone of sensible duration when once made to sound." —*Helmholtz.*

"There is an element in stringed instruments, in addition to the strings, which is of such importance as to deserve notice, namely, the *sound-board.* Although the strings are the source of the sound, their tone would be very thin and poor if used alone; it is modified, amplified, and improved in quality by the vibrations being communicated to a much larger resounding body.

"In the violin tribe the whole body of the instrument acts as a sounding-board. The vibrations are communicated from the strings to the bridge, and from the bridge to the body, which then sets up vibrations of its own, corresponding in *rapidity* to those of the strings, but altered in amplitude and form, and it is these vibrations which are transmitted most prominently to the ear.

"In the pianoforte the sounding-board is a sheet of thin firwood placed immediately under the strings, and communicating with them by bridges, as in the violin. The wires give the note; but for the power and quality of tone the sounding-board is responsible. This is a more remarkable case than the violin, on account of the number of different sounds that are given off together. The same board appears, at one and the same time, to form part of any number of systems of vibration, and to vibrate in unison with every note of a chord. Thus any number of notes struck at once will distinctly impress the ear; and as it is certain that every note must have a distinct and separate set of compound vibrations in the sound-board peculiarly belonging to itself, the mind becomes bewildered in trying to imagine the immensely complicated motions that must be going on in that simple-looking sheet of Swiss pine."—*Pole.*

"It has been seen in Article 48 that in a divergent oscillating wave of air, such as we may suppose to be caused by the vibrations of a string, the motion of the particles is of the order of R, whose first term varies as the distance raised to the power $-\frac{1}{2}$. Moreover, the smallness of dimension of a wire makes it impossible that it can communicate great motion to the air which it touches. Hence, it is impossible that a wire can, by immediate action on the air, produce a sound easily audible to a considerable or convenient distance. To make it audible, the wire must be connected with an intermediate substance whose vibrations can produce a stronger effect on the air, and those vibrations must be excited by the vibrations of the wire. The intermediate substance used for this purpose is the sounding-board. In the violin, the wires pass over a bridge which rests by two feet upon the upper board; and under that board, at the place where one foot of the bridge presses, is a little post (known by the name of the 'sound-post' or the 'soul') connecting the upper board with the lower board. Every tremulous motion of a wire of the violin acts directly upon the bridge and upon the upper and lower boards; and the tremors of these produce effective vibrations of the air, and diffuse the sound. We know that everything depends on the elastic properties of these boards; but we know nothing of their precise laws of vibration. In the pianoforte, the general construction is simpler, but the sounding-board is so connected with the supports of the wires that it is made to vibrate by the vibrations of the wires."—*Airy.*

Reverting to the calculations in the earlier part of this chapter, it will be found that a column of air in a pipe, to reinforce a tone by resonance, must be one-fourth the tone's wave length, if the pipe is stopped at one end, and one-half if open at both ends. Roughly

speaking, the wave length of is 2 feet; of 1 foot; and of 6 inches; and if a piece of paper 2 feet long be rolled into a cylinder it will reinforce the first of these tones; a tube 1 foot will reinforce the second, and one of 6 inches long the last. Every note corresponds to a column of air of a definite length, and the same note is always reinforced by its own column length, and can be reinforced by no other.

Professor Airy has treated this subject mathematically in his work on Sound, and readers who wish to investigate that aspect of the subject further are referred to that work. The phenomena of resonance has been utilised by Helmholtz in a remarkable way, as it was chiefly by its means that he traced to their origin the causes which give different qualities to different tones. Knowing that a tube gave great prominence to the particular tone whose vibrations corresponded with its own, he constructed tubes and globes of various sizes, by which means he was able to detect the particular upper partial tone (see next chapter), the presence of which he suspected. As we are to consider, in the next chapter, the analysis of compound sound, the following extract from Helmholtz will form a fitting introduction to that subject:—

"The mass of air in a resonator, together with that in the aural passage, and with the tympanic membrane or drum-

skin itself, forms an elastic system which is capable of vibrating in a peculiar manner, and, in especial, the prime tone of the sphere, which is much deeper than any other of its proper tones, can be set into very powerful sympathetic vibration, and then the ear, which is in immediate connection with the air inside the sphere, perceives this augmented tone by direct action. If we stop one ear (which is best done by a plug of sealing wax moulded into the form of the entrance of the ear), and apply a resonator to the other, most of the tones produced in the surrounding air will be considerably damped; but if the proper tone of the resonator is sound, it brays into the ear most powerfully. Hence any one, even if he has no ear for music or is quite unpractised in detecting musical sounds, is put in a condition to pick the required simple tone, even if comparatively faint, from out of a great number of others. The proper tone of the resonator may even be sometimes heard cropping up in the whistling of the wind, the rattling of carriage wheels, or the splashing of water. For these purposes such resonators are incomparably more sensitive than tuned membranes. When the simple tone to be observed is faint in comparison with those which accompany it, it is an advantage to apply the resonator and withdraw it a little alternately. We thus easily feel whether the proper tone of the resonator begins to sound when the instrument is applied, whereas a uniform continuous tone is not so readily perceived.

"A properly tuned series of such resonators is therefore an important instrument for experiments in which individual faint tones have to be distinctly heard, although accompanied by others which are strong, as in observations on the combinational and upper partial tones, and a series of other phenomena to be hereafter described relating to chords. By their means such researches can be carried out even by

ears quite untrained in musical observation, whereas it had been previously impossible to conduct them except by trained musical ears, and much strained attention properly assisted. These tones were consequently accessible to the observation of only a very few individuals; and a large number of physicists and even musicians had indeed never succeeded in distinguishing them. And again even the trained ear is now able, with the assistance of resonators, to carry the analysis of a mass of musical tones much further than before. Without their help, indeed, I should scarcely have succeeded in making the observations hereafter described, with so much precision and certainty as at present.

"It must be carefully noted that the ear does not hear the required tone with augmented force, unless that tone attains a considerable intensity within the mass of air enclosed in the resonator. Now the mathematical theory of the motion of the air shows that, so long as the amplitude of the vibrations is sufficiently small, the enclosed air will execute pendular oscillations of the same periodic time as those in the external air, and none other, and that only those pendular oscillations whose periodic time corresponds with that of the proper tone of the resonator, have any considerable strength; the intensity of the rest diminishing as the difference of their pitch from that of the proper tone increases. All this has nothing to do with the connection of the ear and resonator, except in so far as the tympanic membrane forms one of the enclosing walls of the mass of air. Theoretically this apparatus does not differ from the bottle with an elastic membrane, in fig. 15 (p. 45), but its sensitiveness is amazingly increased by using the drumskin of the ear for the closing membrane of the bottle, and thus bringing it in direct connection with the auditory nerves themselves. Hence we cannot obtain a powerful tone in

the resonator except when an analysis of the motion of the external air into pendular vibrations, would show that one of them has the same periodic time as the proper tone of the resonator. Here again no other analysis but that into pendular vibrations would give a correct result.

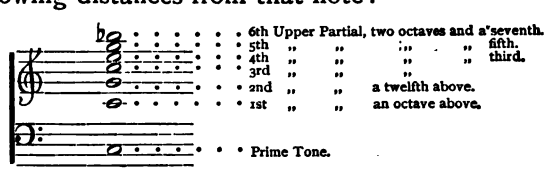
"It is easy for an observer to convince himself of the above-named properties of resonators. Apply one to the ear, and let a piece of harmonised music, in which the proper tone of the resonator frequently occurs, be executed by any instrument. As often as this tone is struck, the ear to which the instrument is held will hear it violently contrasted with all the other tones of the chord."—*Helmholtz.*

CHAPTER VII.

THE ANALYSIS OF COMPOUND SOUNDS.

THE great majority of the musical tones which the human ear can distinguish are not single, but compound sounds. When a note is struck on the pianoforte, the sound heard is called A, or B, or C, as the case may be; but the tone so called consists of something more than the mere sound to which such name is applied. Other sounds have been discovered to be present, and the nature of those sounds, so far as concerns their connection with the fundamental tone, we will now investigate.

They are not irregular or accidental, but occur in an orderly succession. The series consists of a number of sounds, always *above* the prime tone, and at definite intervals. Thus if C on the bass stave is the fundamental or prime tone, the series rises at the following distances from that note:—



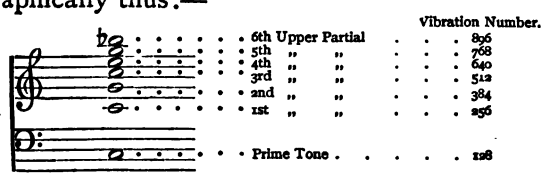
This diagram shows these upper partials as far as

the 6th. They extend far beyond this limit, but this will be far enough for our present purpose, which is to show what is meant by a "compound sound." They have been traced by Helmholtz, in some sounds, beyond the 20th upper partial.

There is a curious law which controls the vibration numbers of the prime tone and its upper partial tones. Whatever may be the number of vibrations per second made by the prime, those of the first upper partial are those of the prime multiplied by 2; the second by 3; the third by 4; the vibration numbers rising in the uninterrupted succession, 1, 2, 3, 4, 5, 6, 7, and so on. Thus the vibration number of the prime tone in the diagram is, say, 128 per second, and the series runs as follows:—

				Vibration numbe per second.
Prime Tone				128
1st Upper Partial (octave above)	128 × 2	=	256
2nd ,, ,, (twelfth ,,)	128 × 3	=	384
3rd ,, ,, (two octaves)	128 × 4	=	512
4th ,, ,, (two oct. and a third)	128 × 5	=	640
5th ,, ,, (,, and fifth)	128 × 6	=	768
6th ,, ,, (,, and seventh)	128 × 7	=	896

Or graphically thus:—



The student, when he comprehends the fact that any note he strikes on the piano is made up of other sounds, must not forget two things which it is highly

important he should remember, viz.:—that upper partials are not contained in every tone, and that where they exist it by no means follows that all of them, or even a regularly succeeding series, will be found. We shall show in the next chapter how the presence, absence, or variety of upper partials in a tone affect its quality or *timbre;* we are now only concerned with the fact that tones are both simple and compound.

It may as well be here explained, that by the word tone is meant a musical sound of any kind; by a simple tone is meant a musical sound in which no upper partials are present; by a compound tone is meant a tone where not only the fundamental tone is present, but where upper partials are found in addition. The prime tone is always the sound which is called by the name which the note bears, as C, B, A, or any other note. This tone is called the prime tone because, as will be shown hereafter, it is always much louder than any of the constituent parts of the sound. It will be well for the student to impress these facts upon his mind now, so that he may understand fully, without having again to refer to their meaning, the terms just explained, which will be used frequently in subsequent chapters.

"The phenomenon of these harmonics, as accompanying fundamental sounds, has been long known. Mersenne, the learned French writer, in his 'Harmonie Universelle,' published in 1636, says, speaking of a question of Aristotle—

"'He seems to have been ignorant that every string

produces five or more different sounds at the same instant, the strongest of which is called the natural sound of the string, and alone is accustomed to be taken notice of, for the others are so feeble that they are only perceptible by delicate ears. . . . Not only the octave and 15th, but also the 12th and major 17th, are always heard; and over and above these I have perceived the major 23d (the 9th partial tone) about the end of the natural sound.'

"Nothing was known at that day of the compound vibrations of strings, and hence Mersenne, having satisfied himself of the fact, endeavoured to explain it by an ingenious hypothesis about the motion of the air, which we now know to be unnecessary.

"The same phenomenon was corroborated and better explained by subsequent writers; as Bernouilli, Riccati, and others. Rameau, in 1722, made it the chief basis of his system of harmony, as we shall see hereafter.

"Chladni devotes much space to the explanation and description of compound sounds; and so widely spread was the knowledge of this phenomenon that he takes pains in many places to correct erroneous inferences that had been drawn from carrying its import too far. He shows that these sounds are not confined to strings, but may and do arise from organ pipes, wind instruments, and other sounding bodies, bells especially.

"Young speaks of 'the superior octave which usually accompanies every sound as a secondary note.'

"Sir John Herschel says (*Encycl. Metrop.*):—

"'It was long known to musicians that, besides the principal or fundamental note of a string, an experienced ear could detect in its sound other notes related to this fundamental one by fixed laws of harmony; and which are called therefore harmonic sounds. They are the very same, which, by the production of distinct nodes, may be in-

sulated, as it were, and cleared from the confusing effect of the co-existent sounds. They are, however, much more distinct in bells and other sounding bodies than in strings, in which only delicate ears can detect them.'

"Mr. Woolhouse, in an excellent little manual published in 1835,* after explaining the modes of vibration of a string first as a whole, and secondly as dividing itself into several equal parts, says—

"'A string very seldom vibrates exclusively either to one or other of these modes of vibration, but generally partakes of both, and often consists of several different modes all in action together. For when the string is vibrating wholly, and producing its fundamental note, it is generally subdivided into various portions, each of which is vibrating at the same time, and producing a harmonic. The mathematical theory of the motion of a musical string shows that any number of different kinds of vibrations, which can be communicated and sustained separately, may be communicated and sustained altogether at the same time; and hence we see the reason why the fundamental notes of large strings, such as those of the harpsichord and violoncello, are usually accompanied with harmonic notes, which are more or less sensible, according to the strength or weakness of the verbal agitation of the portions into which the string has divided itself; they are most readily communicated by a sudden action on the string near to one of its extremities, and therefore almost always accompany the tones of the pianoforte, particularly those of the bass.'"—*Pole.*

"Now if we consider the first of these functions as representing the simple form of that disturbance which produces in the ear the sensation of the *fundamental tone*, then the second of these functions will represent vibrations

* Essay on Musical Intervals, Harmonics, &c. By W. S. B. Woolhouse. London: Souter. 1835. P. 26.

recurring twice as frequently, which we shall find to be very important in music, as representing the *octave above the fundamental tone;* the third will give the *twelfth above the fundamental tone;* the fourth will give the *double octave above the fundamental tone*, &c. These notes have usually been called collectively the *harmonics of the fundamental tone.* (Professor Tyndall has lately advocated the use of the term *overtones*, derived from the German.)

"It appears then that every musical note may be represented by a combination of the fundamental tone and its harmonics in some proportion. It seems that only the note produced by the tuning-fork is confined to the first term or fundamental tone.* It seems probable (from consideration of the mechanical movements), that the ancient Reed requires the supposition of a great departure from the simple law of sines, which implies the introduction of large terms of the higher harmonics, greatly impairing the effect of the fundamental tone. In the case of vibrating wires, we know from the mechanical theory (to be given hereafter), that there is always a mixture of harmonics with the fundamental tone; the character of the mixture depending on the point of the wire at which motion is given to it by the finger or the key.

"The pure fundamental note, as given by the tuning-fork, is sweet, but somewhat inanimate. Richness is given by an admixture of the two or three first harmonics."—*Airy.*

In order to ascertain that musical sounds as a rule are compound tones, raise the cover of a grand pianoforte, or raise the lid and take off the front of a cottage pianoforte. Then strike middle C. If, after that note is silent, the C an octave below it be struck, the sound of the higher of the two tones will be plainly

* It is possible, by an injudicious blow, to produce a different vibration of the tuning-fork, with a very high tone, but it quickly dies away.

heard as a constituent part of the lower tone. That is to say, that when the lower note is struck, not only do we hear that note, but we further hear, as a part of it (and, as we shall show hereafter, affecting its quality to a greater or less extent), its octave. The student will require to listen with great care in order to detect these extra sounds, or as they are technically called, "upper partials." This expression, "upper partials," means that the sounds to which it applies are parts of the prime tone, and also that they are *upper* partials, being higher in pitch than the original fundamental or prime tone. Close attention is required to hear them well, but it can be demonstrated, beyond the possibility of a doubt, that they are actually present. The octave, as has been shown, is the first in the series, and remembering the rule which has been already laid down with reference to the vibration numbers of these upper partials as related to the prime tone, we shall find that the second (when it is present) will be the twelfth above the prime; the third will be two octaves above; the fourth will be two octaves and a third; the fifth two octaves and a fifth; the sixth two octaves and a flat minor seventh—that is, the first upper partial vibrates twice as many times in a second as the prime; the second upper partial three times as often, and so on, in regular series 1, 2, 3, 4, 5, 6, &c.

"The ear when its attention has been properly directed to the effect of the vibrations which strike it, does not hear merely that one musical tone whose pitch is determined by

the period of the vibrations in the manner already explained, but in addition to this it becomes aware of a whole series of higher musical tones, which we will call the *harmonic upper partial tones*, and sometimes simply the *upper partials* of that musical tone, in contradistinction to that first tone, the *fundamental* or *prime partial tone* or simply the *prime*, which is the lowest and generally the loudest of all, and by whose pitch we judge of the pitch of the whole *compound musical tone*, or simply the *compound*. The series of these upper partial tones is precisely the same for all compound musical tones which correspond to a uniformly periodical motion of the air. It is as follows:—

"The first upper partial tone is the upper Octave of the prime tone, and makes double the number of vibrations in the same time. If we call the prime *c*, this upper octave will be *c'*.

"The second upper partial tone is the Fifth of this Octave, or *g'*, making three times as many vibrations in the same time as the prime.

"The third upper partial tone is the second higher Octave or *c''*, making four times as many vibrations as the prime in the same time.

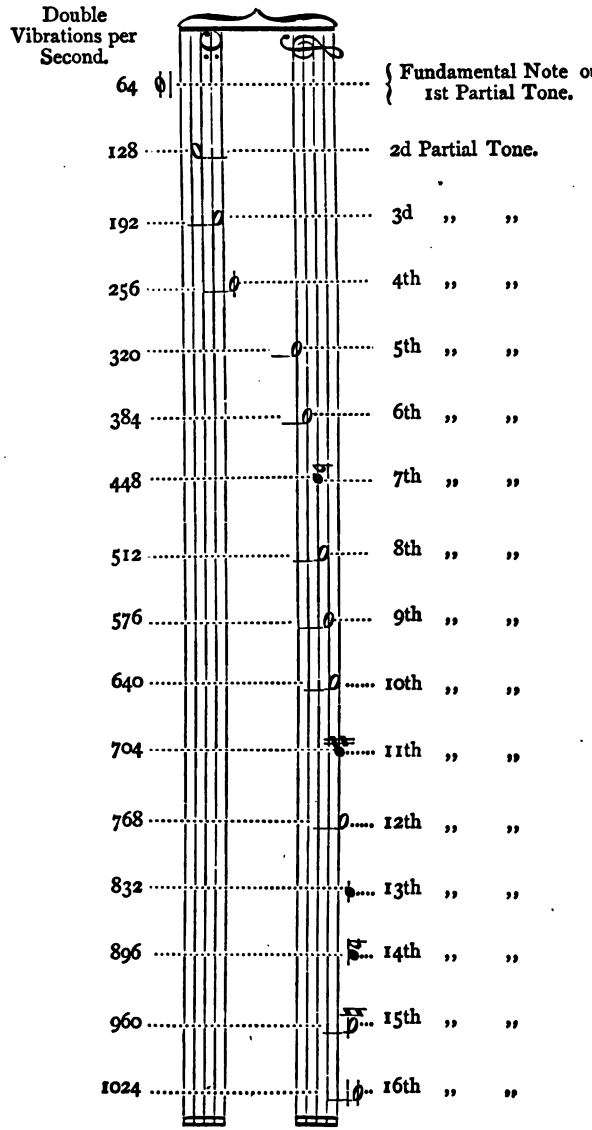
"The fourth upper partial tone is the major Third of this second higher Octave, or *e''*, with five times as many vibrations as the prime in the same time.

"The fifth upper partial tone is the Fifth of the second higher Octave, or *g''*, making six times as many vibrations as the prime in the same time.

"And thus they go on, becoming continually fainter, to tones making 7, 8, 9, &c., times as many vibrations in the same time, as the prime tone."—*Helmholtz.*

The following table is from Dr. Pole's "Philosophy of Music":—

TABLE OF NATURAL HARMONICS OR PARTIAL TONES.



If now G, near the middle of the instrument, be struck gently and released, and C a twelfth below it struck immediately afterwards, close attention will reveal the fact that this G is also present in the prime tone C. The student is recommended to sound the notes which he requires to hear before he tries to hear them, not because it is not possible to hear them without, but because the ear untrained in the art of accurate listening will at first fail to discover the required upper partial, unless, by hearing it thus, it is guided in what direction to listen. The pianoforte contains, generally speaking, upper partials as far as the sixth, at any rate in its middle register, though not so many in the upper notes, because the wires are too short and stiff to vibrate in the numerous segments required for forming the higher upper partials.

Helmholtz found it necessary in his researches to employ artificial aid in order to detect the high upper partials present along with any given prime tone. This artificial aid he found in what he has called "resonators," which are vessels of glass, or metal, of different sizes, made to answer to any given upper partial which it may be required to hear. In order to understand the purpose for which a resonator is used, and the mode in which it reinforces the tone required to be heard, let the student take a deep narrow glass bottle. If now he takes a C tuning fork, he will find on striking it, and holding it over the mouth of the bottle, that the tone varies but slightly. If, however, water be poured into the bottle until it

reaches a certain point, the tone of the fork will be suddenly and powerfully reinforced.* The reason of this is, that the bottle, thus shortened in length by the rising water, reaches at last the exact length which, as in the case of an organ pipe, corresponds with the wave length of the particular sound under examination. In organ pipes, as we shall see hereafter, the wave length of the particular sound is twice the length of the pipe, if it be an open pipe, and four times the length if it be a stopped pipe. The bottle containing the water is virtually a stopped pipe, and the quantity of air which it contains at the precise moment when the tone of the fork is strengthened, corresponds to the wave length of the sound produced by the fork. When it is remembered that no sound can be heard in air unless it reaches the auditory nerve by means of the vibration of the atmosphere, it follows, that if, when the middle C of the pianoforte is struck, and by means of a resonator tuned to C, an octave above, this latter note is heard when the other is struck, and ceases when the other ceases, the second or higher tone must be present in and form a constituent part of the prime tone. In the same way, a resonator tuned to the second upper partial, or twelfth of the prime tone, reveals the presence of that tone in the prime; and so on with all the other upper partials that may be present in any given tone.

Knowing the tones which comprise the harmonic

* See Chapter vi.

series, those which vibrate 2, 3, 4, &c., times as many as the prime, the note required is expected, and a resonator can be made which will powerfully reinforce it, and virtually shut out the rest.

The student will easily understand on consideration that, even though there be but three upper partials besides the prime tone, a great variety of quality is possible. The prime tone is of course the strongest, but the upper partials may vary in their intensity, and it will be an instructive exercise for the reader to calculate for himself how many qualities of tone are possible with, say, three upper partials, each of which has three different degrees of intensity. For instance, let 1 represent the prime tone, and 2, 3, and 4 the upper partials. Then we can have:—

1	2	3	4
1	2	4	3 &c., &c.

Three figures can be placed in six different positions, and the student can thus easily calculate for himself how many varieties of tone are possible within the limits just prescribed.

If the student has apprehended what we have tried to set before him in this chapter, he has reached another stage in his journey, and will comprehend what is meant when it is said that few musical sounds are entirely free from upper partials. These upper partials are present even when it may not be possible to analyse them, though they can be heard sometimes without the aid of resonators. They can be heard in

organ pipes, on the violin, on the piano, and in fact on almost every musical instrument; but although they can be heard without artificial aid, they can be heard much more fully and satisfactorily with it. Where upper partials are not present the tone is dull; where the prime tone is reinforced by the first two or three upper partials, the tone is sharp and full. Where the upper partials—especially the first two or three—are loud in comparison to the prime tone, the quality is nasal, or poor; where the higher upper partials are largely present, the tone is crisp, sharp, and cutting, as in the reed pipes of an organ. It must be remembered in listening for upper partials that they begin when the prime tone begins, that they all commence with the same comparative strength, that as a rule they all increase together, and that when it stops they stop. We shall see later on that it is the presence or absence, the force or weakness, of these upper partials which constitutes the difference between various qualities of tone—a difference which, until the time of Helmholtz, was never adequately explained, but with which, aided by the researches of that profound physicist, most musicians are now perfectly familiar. In listening for the parts of a compound tone, with the unaided ear, in the manner indicated above, it is possible that the impression left upon the mind by the sensation of the required tone when first sounded, may lead the mind to imagine the tone rather than determine by actual sensation that it is present; but when

resonators are used, this is not possible, as the sound is not heard previously, and in fact not heard at all with any accuracy unless a resonator answering to its tone be applied to the ear.

"The analysis of a single musical tone into a series of partial tones depends, then, upon the same property of the ear as that which enables it to distinguish different musical tones from each other, and it must necessarily affect both analyses by a rule which is independent of the fact that the waves of sound are produced by one or by several musical instruments.

"The rule by which the ear proceeds in its analysis was first laid down as generally true by G. S. Ohm. Part of this rule has been already enunciated in the last chapter, where it was stated that only that particular motion of the air which we have denominated a *simple vibration*, for which the vibrating particles swing backwards and forwards according to the law of pendular motion, is capable of exciting in the ear the sensation of a single simple tone. Every motion of the air, then, which corresponds to a composite mass of musical tones, is, according to Ohm's law, capable of being analysed into a sum of simple pendular vibrations, and to each such single simple vibration correponds a simple tone, sensible to the ear, and having a pitch determined by the periodic time of the corresponding motion of the air.

"The simple vibrational form is unalterable and always the same. It is only its amplitude and its periodic time which are subject to change. But we have seen in figs. 11 and 12 (p. 43)* what varied forms the composition of only two simple vibrations can produce. The number of these forms might be greatly increased, even without

* See end of Chapter viii.

introducing fresh simple vibrations of different periodic times, by merely changing the proportions which the heights of the two simple vibrational curves A and B bear to each other, or displacing the curve B by other lengths to the right or left, than those already selected in the figures. By these simplest possible examples of such compositions, the reader will be able to form some idea of the enormous variety of forms which would result from using more than two simple forms of vibration, each form giving an upper partial tone of the same prime, and hence, on addition, always producing other periodic curves. We should be able to make the heights of each single simple vibrational curve greater or smaller at pleasure, and displace each one separately by any amount in respect to the prime,—or, in physical language, we should be able to alter their amplitudes and the difference of their phases; and each such alteration of amplitude and difference of phase in each one of the simple vibrations would produce a fresh change in the resulting composite vibrational form.

"The multiplicity of vibrational forms which can be thus produced by the composition of simple pendular vibrations is not merely extraordinarily great: it is so great that it cannot be greater. The French mathematician Fourier has proved the correctness of a mathematical law, which in reference to our present subject may be thus enunciated: Any given regular periodic form of vibration can always be produced by the addition of simple vibrations, having vibrational numbers which are once, twice, thrice, four times, &c., as great as the vibrational number of the given motion.

"The *amplitudes* of the elementary simple vibrations to which the height of our wave-curves corresponds, and the *difference of phase*, that is, the relative amount of horizontal displacement of the wave-curves, can always be found in every given case, as Fourier has shown, by peculiar methods

of calculation, which, however, do not admit of any popular explanation, so that any given regularly periodic motion can always be exhibited in one single way, and in no other way whatever, as the sum of a certain number of pendular vibrations.

"Since, according to the results already obtained, any regularly periodic motion corresponds to some musical tone, and any simple pendular vibration to a single musical tone, these propositions of Fourier may be thus expressed for the purpose of their application to the theory of sound:

"Any vibrational motion of the air in the aural passages, corresponding to a musical tone, may be always, but for each case only in one single way, exhibited as the sum of a number of simple vibrational motions, corresponding to the partial tones of this musical tone.

"Since, according to these propositions, any form of vibration, no matter what shape it may take, can be expressed as the sum of simple vibrations, its analysis into such a sum is quite independent of the power of the eye to perceive by looking at its representative curve, whether it contains simple vibrations or not, and if it does, what they are. I am obliged to lay stress upon this point, because I have by no means unfrequently found even physicists start on the false hypothesis, that the vibrational form must exhibit little waves corresponding to the several audible upper partial tones. A mere inspection of the figs. 11 and 12 (page 43) will suffice to show that although the composition can be easily traced in the parts where the curve of the prime tone is dotted in, this is quite impossible in those parts of the curves C and D in each figure, where no such assistance has been provided. Or, if we suppose that an observer who had rendered himself thoroughly familiar with the curves of simple vibrations imagined that he could trace the composition in these easy cases, he would

certainly utterly fail on attempting to discover by his eye alone the composition of such curves as are shown in figs. 8 and 9 (page 35). In these will be found straight lines and acute angles. Perhaps it will be asked how it is possible by compounding such smooth and uniformly rounded curves as those of our simple vibrational forms A and B in figs. 11 and 12, to generate at one time straight lines, and at another acute angles. The answer is, that an infinite number of simple vibrations are required to generate curves with such discontinuities as are there shown. But when a great many such curves are combined, and are so chosen that in certain places they all bend in the same direction, and in others in opposite directions, the curvatures mutually strengthen each other in the first case, finally producing an infinitely great curvature, that is, an acute angle, and in the second case they mutually weaken each other, so that ultimately a straight line results. Hence we can generally lay it down as a rule that the force or loudness of the upper partial tones is the greater, the sharper the discontinuities of the atmospheric motion. When the motion alters uniformly and gradually, answering to a vibrational curve proceeding in smoothly curved forms, only the deeper partial tones, which lie nearest to the prime tone, have any perceptible intensity. But where the motion alters by jumps, and hence the vibrational curves show angles or sudden changes of curvature, the upper partial tones will also have sensible force, although in all these cases the amplitudes decrease as the pitch of the upper partial tones become higher.

"The theorem of Fourier here adduced shows first that it is mathematically possible to consider a musical tone as a sum of simple tones, in the meaning we have attached to the words, and mathematicians have indeed always found it convenient to base their acoustic investigations on this

mode of analysing vibrations. But it by no means follows that we are obliged to consider the subject in this way. We have rather to inquire, do these partial constituents of a musical tone, such as the mathematical theory distinguishes and the ear perceives, really exist in the mass of air external to the ear? Is this means of analysing forms of vibration which Fourier's theorem prescribes and renders possible, not merely a mathematical fiction, permissible for facilitating calculation, but not necessarily having any corresponding actual meaning in things themselves? What makes us hit upon pendular vibrations, and none other, as the simplest element of all motions producing sound? We can conceive a whole to be split into parts in very different and arbitrary ways. Thus we may find it convenient for a certain calculation to consider the number 12 as the sum $8+4$, because the eight may have to be cancelled, but it does not follow that 12 must always and necessarily be considered as merely the sum of 8 and 4. In another case it might be more convenient to consider 12 as the sum of 7 and 5. Just as little does the mathematical possibility, proved by Fourier, of compounding all periodic vibrations out of simple vibrations, justify us in concluding that this is the only permissible form of analysis, if we cannot also establish that this analysis has also an essential meaning in nature. That this is indeed the case, that this analysis has a meaning in nature independently of theory, is rendered probable by the fact that the ear really affects the same analysis, and also by the circumstance already named, that this kind of analysis has been found so much more advantageous in mathematical investigations than any other. Those modes of regarding phenomena that correspond to the most intimate constitution of the matter under investigation are also always those which lead to the most suitable and evident theoretical treatment. But it would

not be advisable to begin the investigation with the functions of the ear, because these are very intricate, and in themselves require much explanation. In the next chapter, therefore, we shall inquire whether the analysis of compound into simple vibrations has an actually sensible meaning in the external world, independently of the action of the ear, and we shall really be in a condition to show that certain mechanical effects depend upon whether a certain partial tone is or is not contained in a composite mass of musical tones. The existence of partial tones will thus acquire a meaning in nature, and our knowledge of their mechanical effects will in turn shed a new light on their relations to the human ear."—*Helmholtz.*

"Under the last head it was remarked that when several musical notes sound simultaneously, their combined effect is transmitted through the air in one single wave of a compound form. It might be supposed that the only effect of this on the ear would be a confused noise, something analogous to the impression which the mixture of a number of different-coloured pigments would produce on the eye.

"The reason why this is not so is on account of a wonderful faculty which the human ear has of *analysing* any compound wave submitted to it; of resolving it into its component elements, and of presenting those elements separately and independently to the mind. This is expressed by what is called the *law of Ohm*, which is, that the ear refuses to recognise any sound-wave unless of the simplest form; and that, consequently, the only means it has of treating a compound wave is to find out of what simple elements it consists, and to make these separately audible.

"In order to form an idea of the extent of this power, imagine the case of an auditor in a large music room, say

a promenade concert, where a full band and chorus are performing. Here, speaking first of the music, there are sounds mingled together of all varieties of pitch, loudness, and quality; stringed instruments, wood instruments, brass instruments, instruments of percussion, and voices of many different kinds. And in addition to these there may be all sorts of accidental and irregular sounds and noises, such as the trampling and shuffling of feet, the hum of voices talking, the rustle of dresses, the creaking of doors, and perhaps the gurgling of fountains, and many others.

"Now, be it remembered, the only means the ear has of becoming aware of all these simultaneous sounds is by the peculiar mode of condensation and rarefaction of some particles of air at the end of a tube about the size of a knitting needle, forming a single air-wave, which, though so small, is of such complex structure as to contain in itself some element representing every sound going on in the room. And yet when this wave meets the nerves, they, ignoring the complexity, single out each individual component element by itself, and convey to the mind of the auditor, without any effort on his part, not only the notes and tones of every instrument and class of voice in the orchestra, but the character of every accidental noise in the room, almost as distinctly as if each single sound or noise were heard alone! Truly in the science of acoustics, when we treat of those parts where the human powers come into play, there are depths which are unfathomable!" —*Pole.*

"Upper partial tones, namely, are a phenomenon belonging to pure auditory *sensation.* The aggregation of a series of partial tones into a compound tone, belonging to any determinate musical instrument, is not a process of sensation, but of *perception.* . . . . *Impressions on our senses,* in so far as we become conscious of them only as

conditions of our body, and, in particular, of our nervous apparatus, are called *sensations;* but in so far as we form from them a mental image of external objects, they are termed *perceptions.* When we apprehend a certain sound, as the tone of a violin, we have a perception; we conclude that a certain instrument exists which usually produces tones of that kind. But when we try to resolve this musical tone into its partial tones, this is a matter of pure sensation. No individual partial tone corresponds to any particular resonant body or part of a resonant body. Separated from the accompanying partial tones, any one in particular is nothing but a part of our sensation. Hence, although when for a scientific purpose we institute investigations into our sensations, as in the present book, we may have a great interest in hunting up these partial tones, yet, in the every-day use of our ears, we have no interest of the kind, for then the only value of our sensations is to assist us in comprehending what is going on in the world without us. For this purpose it is enough to apprehend the musical tones correctly. Their resolution into partial tones, even if we were conscious of it, would not only be of no use, but would be a source of extreme disturbance."

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"Now there are many circumstances which assist us first in separating the musical tones arising from different sources, and secondly, in keeping together the partial tones of each separate source. Thus when one musical tone is heard for some time before being joined by the second, and then the second continues after the first has ceased, the separation in sound is facilitated by the succession of time. We have already heard the first musical tone by itself, and hence know immediately what we have to deduct from the compound effect for the effect of this first tone. Even when several parts proceed in the same rhythm in

polyphonic music, the mode in which the tones of different instruments and voices commence, the nature of their increase in force, the certainty with which they are held, and the manner in which they die off, are generally slightly different for each. Thus the tones of a pianoforte commence suddenly with a blow, and are consequently strongest at the first moment, and then rapidly decrease in power; the tones of brass instruments, on the other hand, commence sluggishly, and require a small but sensible time to develop their full strength; the tones of bowed instruments are distinguished by their extreme mobility, but when either the player or the instrument is not unusually perfect they are interrupted by little, very short, pauses, producing in the ear the sensation of scraping, as will be described more in detail when we come to analyse the musical tone of a violin. When then such instruments are sounded together there are generally points of time when one or the other is predominant, and it is consequently easily distinguished by the ear. But besides all this, in good part music, especial care is taken to facilitate the separation of the parts by the ear. In polyphonic music proper, where each part has its own distinct melody, a principal means of clearly separating the progression of each part has always consisted in making them proceed in different rhythms and on different divisions of the bars; or where this could not be done, or was at any rate only partly possible, as in four-part chorales, it is an old rule, contrived for this purpose, to let three parts, if possible, move by single degrees of the scale, and let the fourth leap over several. The small amount of alteration in the pitch makes it easier for the listener to keep the identity of the several voices distinctly in mind.

"All these helps fail in the resolution of musical tones into their constituent partials. When a compound tone commences to sound, all its partial tones commence with

the same comparative strength; when it swells, all of them generally swell uniformly; when it ceases, all cease simultaneously. Hence no opportunity is generally given for hearing them separately and independently. In precisely the same manner as the naturally connected partial tones form a single source of sound, the partial tones in a compound stop on the organ fuse into one, being all struck with the same finger key, and all moving in the same melodic progression as their prime tone.

"Moreover, the tones of most instruments are usually accompanied by characteristic irregular noises, as the scratching and rubbing of the violin bow, the rush of wind in flutes and organ pipes, the grating of reeds, &c. These noises, with which we are already familiar as characterising the instruments, materially facilitate our power of distinguishing them in a composite mass of sounds. The partial tones in a compound have, of course, no such characteristic marks.

"Hence we have no reason to be surprised that the resolution of a compound tone into its partials is not quite so easy for the ear to accomplish, as the resolution of composite masses of the musical sounds of many instruments into their proximate constituents, and that even a trained musical ear requires the application of a considerable amount of attention when it undertakes the former problem.

"It is easy to see that the auxiliary circumstances already named do not always suffice for a correct separation of musical tones. In uniformly-sustained musical tones, where one might be considered as an upper partial of another, our judgment might readily make default. This is really the case. G. S. Ohm proposed a very instructive experiment to show this, using the tones of a violin. But it is more suitable for such an experiment to use simple tones,

as those of a stopped organ pipe. The best instrument, however, is a glass bottle * of the form shown in fig. 46, which is easily procured and prepared for the experiment. A little rod supports a gutta-percha tube in a proper position. The end of the tube, which is directed towards the bottle, is softened in warm water and pressed flat, forming a narrow chink, through which air can be made to rush over the mouth of the bottle. When the tube is fastened by an indiarubber pipe to the nozzle of a bellows, and wind is driven over the bottle, it produces a hollow, obscure sound, like the vowel *oo* in *too*, which is freer from upper partial tones than even the tone of a stopped pipe, and is only accompanied by a slight noise of wind. I find that it is easier to keep the pitch unaltered in this instrument while the pressure of the wind is slightly changed, than in stopped pipes. We deepen the tone by partially shading the orifice of the bottle with a little wooden plate; and we sharpen it by pouring in oil or melted wax. We are thus able to make any required little alterations in pitch. I tuned a large bottle to *b*♭, and a smaller one to *b*′♭ and united them with the same bellows, so that when used both began to speak at the same instant. When thus united they gave a musical tone of the pitch of the deeper *b*♭, but having the quality of tone of the vowel *oa* in *toad*, instead of *oo* in *too*. When, then, I compressed first one of the indiarubber tubes, and then the other, so as to produce the tones alternately, separately, and in connection, I was at last able to hear them separately when sounded together, but I could not continue to hear them separately for long, for the upper tone gradually fused with the lower. This fusion takes place even when the upper tone is somewhat stronger than the lower. The alteration in the quality of tone which takes place during this fusion is characteristic.

* A wide bottle with a narrow neck.

On producing the upper tone first, and then letting the lower sound with it, I found that I at first continued to hear the upper tone with its full force, and the under tone sounding below it in its natural quality of *oo* in *too*. But by degrees, as my recollection of the sound of the isolated upper tone died away, it seemed to become more and more indistinct and weak, while the lower tone appeared to become stronger, and sounded like *oa* in *toad*. This weakening of the upper and strengthening of the lower tone was also observed by Ohm on the violin. As Seebeck remarks, it certainly does not always occur, and probably depends on the liveliness of our recollections of the tones as heard separately, and the greater or less uniformity in the simultaneous production of the tones. But where the experiment succeeds, it gives the best proof of the essential dependence of the result on varying activity of attention. With the tones produced by bottles, in addition to the reinforcement of the lower tone, the alteration in its quality is very evident, and is characteristic of the nature of the process. This alteration is less striking for the penetrating tones of the violin."

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"Another experiment should be adduced. Raise the dampers of a pianoforte so that all the strings can vibrate freely, then sing the vowel *a* in *father*, *art*, loudly to any note of the piano, directing the voice to the sounding-board of the piano; the sympathetic resonance of the strings distinctly re-echoes the same *a*. On singing *oe* in *toe*, the same *oe* is re-echoed. On singing *a* in *fare*, this *a* is re-echoed. For *ee* in *see* the echo is not quite so good. The experiment does not succeed so well if the damper is removed only from the note on which the vowels are sung. The vowel character of the echo arises from the re-echoing of those upper partial tones which characterise the vowels.

These, however, will echo better and more clearly when their corresponding higher strings are free and can vibrate sympathetically. In this case, then, in the last resort, the musical effect of the resonance is compounded of the tones of several strings, and several separate partial tones combine to produce a musical tone of a peculiar quality. In addition to the vowels of the human voice, the piano will also quite distinctly imitate the quality of tone produced by a clarionet, when strongly blown on to the sounding-board."

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"The results of the preceding discussion may be summed up as follows :—

"1. The upper partial tones corresponding to the simple vibrations of a compound motion of the air, are felt, even when they are not always consciously perceived.

"2. But they can be made objects of conscious perception without any other help than a proper direction of attention.

"3. Even in the case of their not being separately perceived, because they fuse into the whole mass of musical sound, their existence in our sensation is established by an alteration in the quality of tone, the impression of their higher pitch being characteristically marked by increased brightness of quality, and apparently greater sharpness of pitch."—*Helmholtz.*

CHAPTER VIII.

HELMHOLTZ'S THEORY OF MUSICAL QUALITY.

QUALITY has been shown to be affected by the number of upper partials present in any given tone, and by the comparative intensity of these upper partials. We have now to consider how far the quality of any given tone depends upon its upper partials. It has been shown, at the conclusion of the last chapter, by the experiment with the two bottles tuned the one an octave above the other, that the addition of a tone an octave above the prime tone (such octave being the first upper partial of the harmonic series) palpably affects the *quality* of the prime, and immediately it is added to the prime tone produces a sound of a quality quite different from that of either of the two alone.

This experiment demonstrates that the addition of one upper partial produces an alteration of quality, and we shall see, as we proceed with our investigations, that difference of quality depends entirely upon the presence or absence, strength or weakness,

of the upper partials present in any tone. The present chapter will show how different combinations of upper partials correspond to different qualities of tone.

"It is well known that this union of several simple tones into one compound tone, which is naturally effected in the tones produced by most musical instruments, is artificially imitated on the organ by peculiar mechanical contrivances. The tones of organ-pipes are comparatively poor in upper partials. When it is desirable to use a stop of incisive penetrating quality of tone and great power, the wide pipes (*principal register* and *weitgedackt*) are not sufficient; their tone is too soft, too defective in upper partials; and the narrow pipes (*geigen-register* and *quintaten*) are also unsuitable, because, although more incisive, their tone is weak. For such occasions, then, as in accompanying congregational singing, recourse is had to the *compound stops*. In these stops every key is connected with a larger or smaller series of pipes, which it opens simultaneously, and which give the prime tone and a certain number of the first upper partials of the compound tone of the note in question. It is very usual to connect the upper Octave with the prime tone, and after that the Twelfth. The more complex compounds (*cornet*) give the first six partial tones, that is, in addition to the two Octaves of the prime tone and its Twelfth, the higher major Third, and the Octave of the Twelfth. This is as much of the series of upper partials as belongs to the tones of a major chord." —*Helmholtz.*

Let us first consider those *musical tones which have no upper partials.* These can be produced by setting

a tuning fork in vibration over the mouth of a "resonance tube," as already described.

"We begin with such musical tones as are not decomposable, but consist of a single simple tone. These are most readily and purely produced by holding a struck tuning-fork over the mouth of a resonance tube, as has been described. . . . . These tones are uncommonly soft and free from all shrillness and roughness. As already remarked, they appear to lie comparatively deep, so that such as correspond to the deep tones of a bass voice produce the impression of a most remarkable and unusual depth. The musical quality of such deep simple tones is also rather dull. The simple tones of the soprano pitch sound bright, but even those corresponding to the highest tones of a soprano voice are very soft, without a trace of that cutting, rasping shrillness which is displayed by most instruments at such pitches, with the exception, perhaps, of the flute, for which the tones are very nearly simple, being accompanied with very few and faint upper partials. Among vowels the *oo* in *too* comes nearest to a simple tone, but even this vowel is not entirely free from upper partials. On comparing the musical quality of a simple tone thus produced with that of a compound tone in which the first harmonic upper partial tones are developed, the latter will be found to be more tuneful, metallic, and brilliant. Even the vowel *oo* in *too*, although the dullest and least tuneful of all vowels, is sensibly more brilliant and less dull than a simple tone of the same pitch."—*Helmholtz.*

It must be remembered that the first six tones of the harmonic series (that is to say, the prime tone and the first five upper partials) constitute a full and

complete major chord; and this fact has doubtless very much to do with the brightness of those sounds which contain only the first five upper partials; the sixth is a flat minor seventh, and when more than the first five are included in the tone, the quality begins to grow more crisp and cutting; but those tones with which we are at present dealing are comparatively dull and free from any "character," and have none of that cutting, shrill quality which attends the tones of some instruments at the same pitch. The tones of the flute are very nearly simple, as they have but very few and faint upper partials. The form of vibration known as "pendular" is that shown by simple tones. Simple tones of the same pitch only differ in loudness and not in quality, that is to say, so long as their pitch remains the same they differ only in *one* of the three elements of which every sound is composed. They cannot differ in quality, because they have no upper partials.

"Since the form of simple waves of known periodic time is completely given when their amplitude is given, simple tones of the same pitch can only differ in loudness and not in musical quality. In fact, the difference of quality remains perfectly indistinguishable, whether the simple tone is conducted to the external air in the preceding methods by a tuning-fork and a resonance tube of any given material, glass, metal, or pasteboard, or by a string, provided only that we guard against any chattering in the apparatus.

"Simple tones accompanied only by the noise of rushing wind can also be produced, as already mentioned, by

blowing over the mouth of bottles with necks. If we disregard the friction of the air, the proper musical quality of such tones is really the same as that produced by tuning-forks."—*Helmholtz.*

Let us next look at some *qualities of tone whose upper partials are inharmonic,* that is to say, tones that have upper partials lying outside the range of the harmonic series. These are not real musical tones, although they are occasionally used for musical purposes, but are of that class which may be described as "clinking" or "tinkling," such as bells, triangles, and instruments of a similar character. This clinking or tinkling is produced by very high upper partials which are not in the harmonic scale, hence they are fitly called "inharmonic upper partials." The quality of tone called "metallic" possesses its peculiar characteristic because of the presence and continuance of high upper partial tones. These peculiar qualities of tone differ according as their upper partials lie close together or wide apart. Stretched membranes, such as drums, have inharmonic upper partials, which rapidly die out and leave the prime tone by itself. These qualities of tone are musical or unmusical according as their upper partials are near to or remote from the pitch of the prime tone.

"Nearest to musical tones without any upper partials are those with secondary tones which are inharmonic to the prime, and such tones, therefore, in strictness, should not be reckoned as musical tones at all. They are exceptionally used in artistic music, but only when it is contrived

that the prime tone should be so much more powerful than the secondary tones, that the existence of the latter may be ignored. Hence they are placed here next to the simple tones, because musically they are available only for the more or less good simple tones which they represent. The first of these are *tuning forks* themselves, when they are struck and applied to a sounding board, or brought very near the ear. The upper partial tones of tuning forks lie very high. In those which I have examined, the first made from 5·8 to 6·6 as many vibrations in the same time as the prime tone, and hence lay between its third diminished Fifth and major Sixth. The vibrational numbers of these high upper partial tones were to one another as the squares of the odd numbers. In the time that the first upper partial would execute $3.3 = 9$ vibrations, the next would execute $5.5 = 25$, and the next $7.7 = 49$, and so on. Their pitch, therefore, increases with extraordinary rapidity, and they are usually all inharmonic with the prime, though some of them may exceptionally become harmonic. If we call the prime tone of the fork *c*, the next succeeding tones are nearly $a''\flat$, $d^{\prime\prime\prime}$, $c^{\vee}\sharp$. These high secondary tones produce a bright inharmonic clink, which is easily heard at a considerable distance when the fork is first struck, whereas when it is brought close to the ear, the prime tone alone is heard. The ear readily separates the prime from the upper tones, and has no inclination to fuse them. The high simple tones usually die off rapidly, while the prime tone remains audible for a long time. It should be remarked, however, that the mutual relations of the proper tones of tuning forks differ somewhat according to the form of the fork, and hence the above indications must be looked upon as merely approximate. In theoretical determinations of the upper partial tones, each prong of the fork may be regarded as a rod fixed at one end."—*Helmholtz.*

We will now consider the *musical tones of strings*, and proceed to analyse what may be called proper musical tones, characterised by harmonic upper partials. Stringed instruments are made to sound either by striking, plucking, or bowing. Amongst the first class are included the pianoforte, the harp, the guitar, and the zither; amongst the latter the whole family of instruments of the violin class. The investigations which Helmholtz made into the nature of the upper partials contained in the tones of strings were very full and complete, and his theories have in all cases agreed with the result of actual experiment. The force of the upper partial tones produced by a string which is struck depends upon three things: *a.* The nature of the stroke; *b.* The point where it falls; *c.* The quality of the string itself. When a string is plucked the number and nature of its upper partials will depend upon the nature of the material with which it is drawn aside. The finger makes a less acute angle in the string than does a sharp point, and for this reason the sharp point will produce more high tinkling upper partials.

"I have already remarked that the strength and number of the upper partial tones increases with the abruptness of the discontinuities in the motion excited. This fact determines the various modes of exciting a string. When a string is plucked, the finger, before quitting it, removes it from its position of rest throughout its whole length. A discontinuity in the string arises only by its forming a more or less acute angle at the place where it wraps itself

about the finger or point. The angle is more acute for a sharp point than for the finger. Hence the sharp point produces a shriller tone with a greater number of high tinkling upper partials, than the finger. But in each case the intensity of the prime tone exceeds that of any upper partial. If the string is struck with a sharp-edged metallic hammer which rebounds instantly, only the one single point struck is set in motion directly. Immediately after the blow the remainder of the string is at rest. It does not move until a wave of deflection arises, and runs backwards and forwards over the string. This limitation of the original motion to a single point produces the most abrupt discontinuities, and a corresponding long series of upper partial tones, having intensities* in most cases equalling or even surpassing that of the prime. When the hammer is soft and elastic, the motion has time to spread before the hammer rebounds. When thus struck the point of the string in contact with such a hammer is not set in motion with a jerk, but increases gradually and continuously in velocity during the contact. The discontinuity of the motion is consequently much less, diminishing as the softness of the hammer increases, and the force of the higher upper partial tones is correspondingly decreased."

. . . . . .

"When we strike with metal, the prime tone is scarcely heard, and the quality of tone is correspondingly *poor*. The peculiar quality of tone commonly termed *poverty*, as opposed to *richness*, arises from the upper partials being comparatively too strong for the prime tone. The prime tone is heard best when the string is plucked with a soft

* When *intensity* is here mentioned, it is always measured objectively, by the *vis viva*, or *mechanical equivalent of work* of the corresponding motion.

finger, which produces a rich and yet harmonious quality of tone. The prime tone is not so strong, at least in the middle and deeper octaves of the instrument, when the strings are struck with the pianoforte hammer, as when they are plucked with the finger."—*Helmholtz.*

The student must remember at this point that strings vibrate in segments, the whole length of the string being divided into 2, 3, 4, 5, &c., segments, as the case may be; and the striking of a string by a sharp point is more favourable to the making of smaller segments (that is, of the higher upper partials) than plucking by the finger. For the same reason, the tone of the string is much more musical when it is struck with the soft hammer of the piano than when it is struck by the sharp blade of a penknife, or any other hard and thin body. When the string is plucked with the soft finger, or struck by its own hammer, it produces a rich, full, and musical quality of sound, because the prime tone in this case predominates.

It should then be remembered, as one of the chief facts in connection with the question of quality of tone, that a string is broken up into very numerous small segments by a blow from a sharp, hard body, making a poor quality of tone, and that a string struck by a soft hammer, or plucked by the finger, makes a more musical quality of tone, because the high upper partials are less prominent and the fundamental tone prevails. Pianoforte makers have long ago discovered in practice the

fact of which Helmholtz has recently revealed the cause, and have covered their hammers with a soft and yielding material calculated to bring out a full, rich, musical quality of tone. The different qualities produced by different makers, and even by the same makers, depends very largely upon the coating with which the hammers are covered. The point where the blow falls has also a great effect on the number of upper partials produced, and the quality of tone can be decidedly altered by changing the point of impact. No upper partial can be heard which has a node* at the point where the blow falls; therefore if the string is struck at the centre the first upper partial (second partial) will not be present in that sound, but the second (third partial) will, because its nodes lie not at half the length of the string, but at one-third and two-thirds respectively. The third upper partial, which is the fifteenth above the prime, will disappear, because its nodes are at one-fourth, one-half, and three-fourths the length of the string, so that when the string is struck in the middle every upper partial which has one of its nodes there will disappear from the tone.†

* See Chapter IX.

† It is unfortunate that some definite nomenclature is not adopted for the designation of these upper partials. Helmholtz treats all the constituent parts of a tone as "partials," and names the prime tone the first; and in speaking of the others he sometimes speaks of the octave of the prime as the first upper partial and at other times as the second partial. This is confusing, and we have endeavoured throughout to speak of the tone itself as the prime tone, and of all its other constituent parts as upper partials; and, where it conduces to clearness, we have put in

This quality of tone is hollow or empty, and nasal. When the blow is given very near the end of the string the quality of the tone becomes tinkling, because the higher upper partials are all strongly developed. The usual point of striking in the pianoforte is between one-seventh and one-ninth of the whole length of the string, and it is especially noticeable that striking at this point causes the disappearance of those upper partials which do not belong to the major chord of the prime tone. Up to the fifth upper partial (sixth partial), two octaves and a fifth from the prime tone, there is no upper partial which is not either an octave, a major third, or a fifth of the prime; the sixth is a flat minor seventh, and the eighth is a major second. These three are not of course a part of the major chord, and it is when the point of striking is at such a distance as to cause the sixth and eighth upper partials to disappear, that the majority of pianos produce a bright and harmonious tone. Their upper partials (rarely more than the first five even in the lower and middle registers, and still less in the higher ones) are all tones of the major common chord. The three first partials of the pianoforte tone (that is, the prime and the first two upper partials) are strong; the fourth and fifth clear, but weak; the sixth, seventh, and eighth are not present at all because of the position of the striking point; and any which may be present beyond the eighth are

parentheses the number in the whole series as well as the number of the upper partial.

so weak as not materially to affect the quality at all.

With regard to the material of the strings themselves, very hard strings do not form any of the higher upper partials, because their material is too rigid to allow of the formation of small segments. This can be manifested to the eye by stretching, say on a violoncello or in any other convenient way, two strings, one very much thinner than the other, and it will be seen that the thinner string divides into many more small segments than the thicker one. With very thin wire a great number of high upper partials can be produced, and this is why wire or other material of extreme tenuity produces a tinkling and non-musical quality of tone. When we get beyond the seventh upper partial (eighth partial) the sounds are less than a tone apart, and beyond the fourteenth they are less than a semitone, so that they become dissonant.* A very thick string, therefore, will be comparatively dull, because it produces too few upper partials, and a very thin string will be tinkling, because it produces too many.

Under this head come the *musical tones of bowed instruments.* Experiments made with violin strings —the nature of which experiments need not in this place be entered into—have brought to light the following facts :—A string played by a bow produces all the upper partial tones of which a string of that

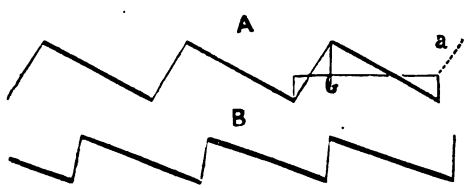
* See chapter on "Consonance and Dissonance."

degree of rigidity is capable, and the higher is their pitch the less will be their intensity. The intensity of these upper partials diminishes, for violin tones, in inverse proportion to that of the prime, that is to say, the first upper partial is one-fourth that of the prime, the second one-ninth, the third one-sixteenth, and so on. Those upper partials which have nodes at the point where the bow comes in contact with the string are of course not present. Those variations in the quality of the tone produced on the violin by bowing nearer to, or further from, the bridge, are caused by different sets of upper partial tones being brought into play; thus, if the bow be drawn too near the finger-board (the end of which should be at one-fifth of the vibrating portion), the fourth and fifth upper partials will be absent, and the tone becomes dull. Students of Spohr's Violin School will remember that he recommends that the bow be used near the finger-board for a soft quality of tone. The usual place of bowing is half-way between the bridge and the string, or at about one-tenth of the entire length of the string, while, as every violinist knows, the bow is carried nearer to the bridge for louder tones, and nearer to the finger-board for softer ones, as has just been explained.

The form of the vibration made by the string varies when bowed in these different places. In an experiment which Helmholtz carried out, a grain of starch was fastened to the string, the position of which was

reflected by a lens, and thrown upon a screen—thus making visible to the eye what was previously known only to the ear. Young violinists will see that a very great deal depends on the use of the bow, and that a steady musical quality of tone, free from scratching, can only be obtained by a steady and practised use of the bow-arm.

The vibrational forms of violin tones are peculiar, and are not by any means simple or pendular. The following figure gives the forms for the middle of a string, when a good full tone is given:—



Figs. 57 and 58.

When bowed nearer the end of the string the form given is A in the above figure, the longer line, represented by a b, being three times the length of the shorter one, represented by the rest of the straight line. Bowed close to the end, the form is as B above.

During the greater part of each vibration the string clings to the bow, and is drawn forward, detaches itself, rebounds, and is seized by the bow and again carried forward. The upper partials are present to about the tenth. The prime is more powerful than

in the pianoforte, the earlier upper partials being weak, but above the sixth they are much stronger, and give to the violin its peculiar cutting tone. They can be easily heard, if the ear is led to expect them by first playing them as harmonics. Touch the string in the middle, and bow it lightly, and the first upper partial will be sounded; then bow the open string, and this tone will be plainly heard. So also with the second, third, &c.

To alter the place of bowing changes the tone by bringing in, or keeping out, certain tones having nodes at the place bowed, but this does not materially alter the angularity of the vibration form given. The peculiar feature of this form is the little crumples, sometimes greater and sometimes less, which mark

Fig. 59.

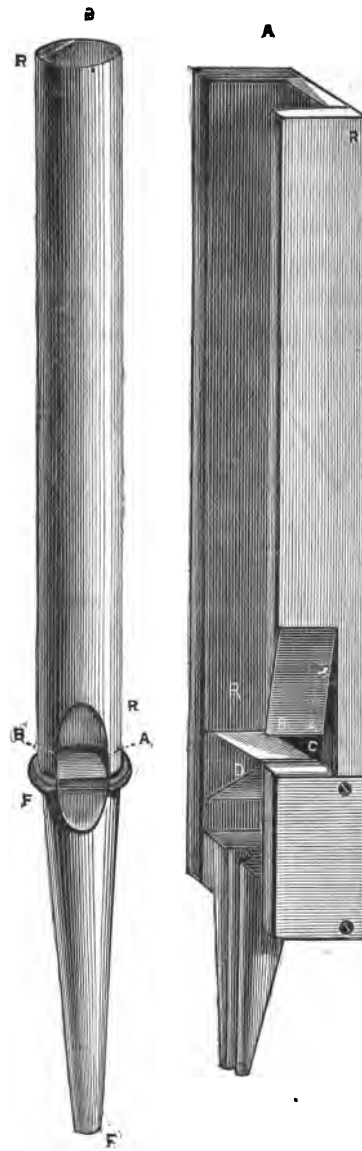
the lines of the figure, which are caused by the dragging forward and rebounding of the string. When the luminous point is at a node of the bowed place, the crumples are not present. The forms given in the last figure but one appear when the string is bowed at one-seventh, two-sevenths, and three-sevenths the length of the string from the bridge. The crumples increase in breadth the farther the bow is removed from the end of the string. If the luminous point is between two

nodes, the form is like that of the last figure. The fourth and fifth upper partials disappear when the bow comes too near the finger-board, and the tone is duller. On bowing lightly and quickly at about one-twentieth the length of the string from the bridge, the first upper partial is sometimes heard *alone*, the prime coming in at once when more weight is laid on the bow, the first upper partial being joined with it. While the pitch thus passes into the higher octave with light bowing, the following form is given :—

Fig. 60.

A crest appears on the front of a wave, increasing in size till the vibration number doubles and the pitch passes into the upper octave.

Let us now look at the quality of the *musical tones of the flue pipes of the organ.* The tone is in this case produced by the impact of rushing air against an opening in a hollow pipe. Flutes and the great majority of organ pipes belong to this class. The following figures will show the construction of flue pipes :—



Figs. 61 and 62.

It is really the edge against which the air is directed which produces the tone, and the body of the air in the pipe only serves to reinforce it, the air chamber of the pipe strengthening the prime and such of the upper partials as correspond with its own proper tone. This is why in the flue pipes of an organ the tone is always accompanied by a rush of wind. The musical quality of the tone of these pipes depends on whether or not the harmonic upper partial of the tone created by blowing corresponds to the proper tone of the pipe with sufficient accuracy to admit of being reinforced at the same time as the prime tone. It is well known that a flute blown gently, with all its holes stopped, gives its lowest note, but by stronger blowing, without removing the fingers, it gives a note an octave above that, and a still stronger blast of wind will produce the twelfth and then the double octave. In narrow organ flue pipes, the upper partials may be heard as high as the fifth. Wide pipes do not so soon produce the harmonics of the prime tone, which latter they give out full and clear without many secondary tones. The high upper partials are not distinctly perceptible in this class of pipes. In pipes which are conically narrowed at the top, such as the salicional and the gemshorn, some of the higher upper partials are made stronger than the lower, which accounts for their peculiarly bright though poor quality. Stopped pipes have tones corresponding to what Helmholtz calls "the uneven partials of the prime, that is, the third (second upper) partial or twelfth, the fifth (fourth upper) partial or higher major third, and so on." (This fifth partial

or higher major third, Helmholtz calls, just below, the fifth *upper* partial or higher major third.) It is the fifth partial, and it is an upper partial, but not the *fifth upper partial.* According to our system of nam ing the upper partials, which does not reckon the prime tone as one, but commences an octave above it, the stopped pipes of an organ produce the second, fourth, &c., upper partials, that is to say, the twelfth higher, major third, &c., of the prime.

Musical Tones of Reeds.—The tones of reeds are produced by the same means as that of the syren, namely, by intermittent puffs of air. In the syren the unperforated part of the plate between the holes temporarily stops up the passage of the wind, and in the same way the tongue of the reed stops up the passage of the air through its aperture. It is the passage of the intermittent puffs of air—in the syren through the holes, in the reed through its aperture—which produces the sound. The reed pipes of organs and the vibrators of harmoniums produce their tones in the same way, namely, by the vibrations of oblong pieces of metal, the mechanism of which is illustrated in the following figures :—

Fig. 63.

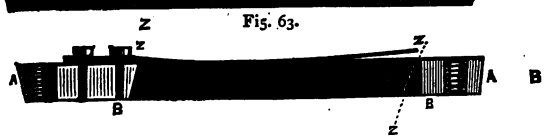


Fig. 64.

In the organ the prime tone of the reed and also some

of its upper partials are strengthened by the pipe, which favours these at the expense of some of the higher upper partials; and were it not for this resonance pipe, the tone of the reeds would be intolerably harsh and cutting, owing to the presence of so many of the higher upper partials. The following figures show the construction of reed pipes :—

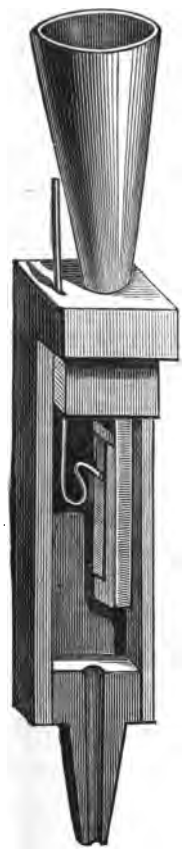


Fig. 65.

Fig. 66.

The tones of the clarionet, oboe, and bassoon are all produced by the vibration of reeds; the clarionet having a single reed, the oboe and bassoon double reeds.

Another modification of the reed is the human voice, in which the tone is produced by the vibrations caused by the passing of the air from the lungs over membranous tongues, called the vocal cords, which are stretched across the windpipe (see chapter on "The Human Voice"). The lips, in blowing brass instruments, also act as reeds.

It has been already remarked that the tone of reeds is produced by intermittent puffs of air, which break through the opening closed by the tongue when at rest. The more intermittent is the motion of the tongue, the greater will be the number of upper partials produced. This is why the syren, the tone of which is discontinuous to a very large extent, has so many upper partials in its tone. When a reed is set in vibration without a pipe to give prominence to its prime tone, its upper partials can be clearly heard, generally as high as the twentieth; and still higher upper partials are present, and could no doubt be heard if the listener were provided with properly-tuned resonators. Whether these upper partials are strong or weak depends upon various conditions, such as the material of which the tongue is made, its position in respect to its frame, the width of the chink round the edges, &c. Hard substances, which do not readily yield to the pressure of the air, make

the tone more discontinuous, and furnish it with a greater number of upper partials, thus rendering the tone more cutting. The tones of the human voice, which we shall consider in another chapter, are softer than any that can be produced by a manufactured reed.

The following quotation from Helmholtz will show the results of his investigations on this subject:—

"From the examples adduced to show the dependence of quality of tone from the mode in which a musical tone is compounded, we may deduce the following rules:—1. *Simple tones*, like those of tuning-forks applied to resonance chambers and wide-stopped organ pipes, have a very soft pleasant sound, free from all roughness, but wanting in power, and dull at low pitches. 2. *Musical tones*, which are accompanied by a moderately loud series of the lower upper partial-tones, up to about the sixth partial, are more harmonious and musical. Compared with simple tones they are rich and splendid, while they are at the same time perfectly sweet and soft if the higher upper partials are absent. To these belong the musical tones produced by the pianoforte, open organ pipes, the softer piano tones of the human voice and of the French horn. The last-named tones form a transition to musical tones with high upper partials, while the tones of flutes and of pipes on the flute stops of organs with a low pressure of wind approach to simple tones. 3. If only the uneven partials * are present (as in narrow-stopped organ pipes, pianoforte strings struck in their middle points, and clarionets), the quality of tone is *hollow*, and, when a large number of such upper partials

* That is to say, the 1st, 3d, 5th, &c., of the entire series, which would be the prime, and the 2d, 4th, 6th, and other even *upper* partials.

are present, *nasal.* When the prime tone predominates, the quality of tone is *rich* and *full;* but when the prime tone is not sufficiently superior in strength to the upper partials, the quality of tone is *poor* or *empty.* Thus the quality of tone in the wider-open organ pipes is fuller than that in the narrower; strings struck with pianoforte hammers give tones of a fuller quality than when struck by a stick or pulled by the finger; the tones of reed pipes, with suitable resonance chambers, have a fuller quality than those without resonance chambers. 4. When partial-tones higher than the sixth or seventh are very distinct, the quality of tone is *cutting* and *rough.* The reason for this will be seen hereafter to lie in the dissonances which they form with one another. The degree of harshness may be very different. When their force is inconsiderable, the higher upper partials do not essentially detract from the musical applicability of the compound tones; on the contrary, they are useful in giving character and expression to the music. The most important musical tones of this description are those of bowed instruments and of most reed pipes, oboe (hautbois), bassoon (fagot), physharmonica (harmonium, concertina, accordion), and the human voice. The rough braying tones of brass instruments are extremely penetrating, and hence are better adapted to give the impression of great power than similar tones of a softer quality. They are consequently little suitable for artistic music when used alone, but produce great effect in an orchestra. Why high dissonating upper partials should make a musical tone more penetrating will appear hereafter."

It is to be observed that every wave, whatever its form—that is to say, every sound, whatever may be the ingredients of which it is made up—can be traced by analysis to the simple waves of which it is

composed. If two waves of different form start at the same point, the result will depend upon their relative position with regard to each other; thus, let the left of the following figure be the starting-point of the two waves, both of which, it need hardly be said, are travelling in the same direction; then, if the lesser wave and the larger wave be added together, the form of the resultant wave will be that of the darker line, and the sound produced by these two waves will

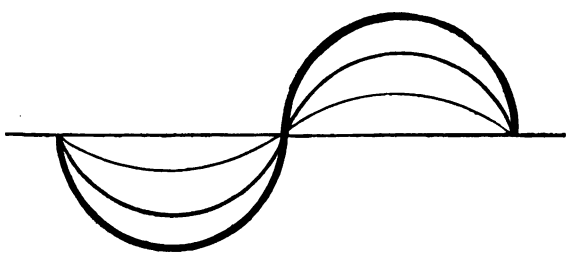


Fig. 67.

be a combination of what each would have been separately.

The student will remember that the vibrations of a simple tone—a tone, that is, which is entirely free from upper partials—are always pendular vibrations; and, as has been said, no matter how complicated be the form of the wave which represents any given quality of sound, that form can be traced to a combination of two, three, four, or more, as the case may be, simple pendular vibrations. The rule by which this fact is proved is known as *Fourier's Theorem*, and this law furnishes a theory, not only for the analysis

of compound sounds, but also for the synthetical construction of any required tone. The student is referred to the extract from Helmholtz on this subject given above.

Helmholtz proved his theory by the most certain of all means, and starting with the theory that a compound tone is made up of a certain number of simple tones, he succeeded in producing sounds of different qualities by the aid of tuning-forks so made as to give the prime and the upper partials of the tone required. So long as it was merely *stated* that different qualities of tone depended upon the presence or absence of upper partials, such statement was open to doubt; but when given qualities of tone were built up, and, in a few cases at least, the quality of tone given by different instruments was imitated by an arrangement of the simple tones of tuning-forks, the theory itself received a most triumphant verification. One simple tone is thus superposed upon another, and the result is, that to the eye the form of the wave is changed, and to the ear the quality of tone (with which, of course, the form always corresponds) is likewise changed.

If, in the case of the wave given above, the trough of A B is simultaneous with the crest of A C, the result will be that A D will be less to that extent than it would have been. The next figure will illustrate this, and the figures which are now given, in which the effects of two, three, and four waves are united, and their resultant form shown by a thick

line, will render the principle of the superposition of waves easy to be understood.

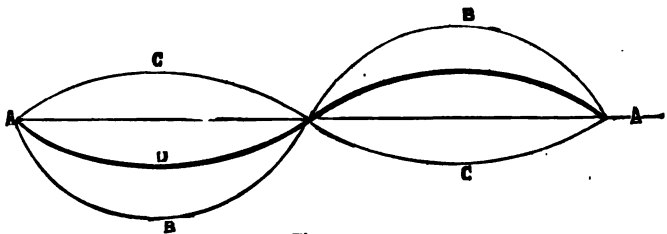


Fig. 68.

Two equal waves in opposition; the one nullifies the effect of the other, and the resultant is a straight line:—

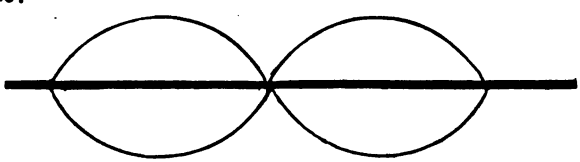


Fig. 69.

Two waves of different phase, starting, *i.e.*, at different points:—

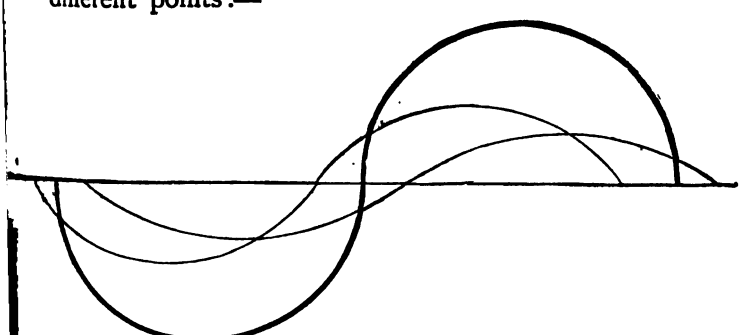


Fig. 70.

Two waves opposed to one:—

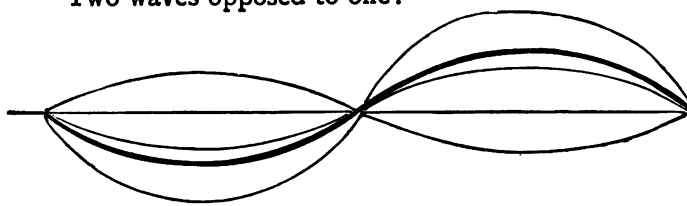


Fig. 71.

Two waves opposed to two:—

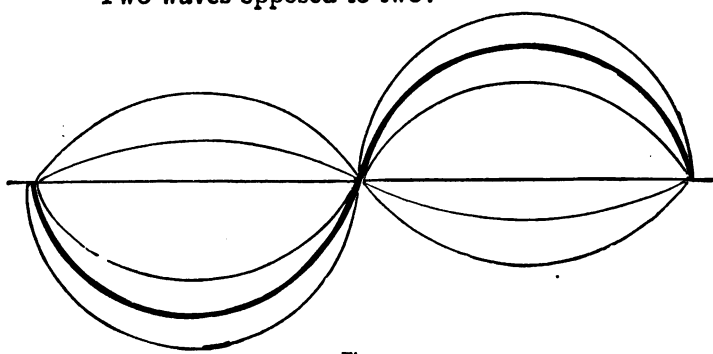


Fig. 72.

Three waves and their resultants:—

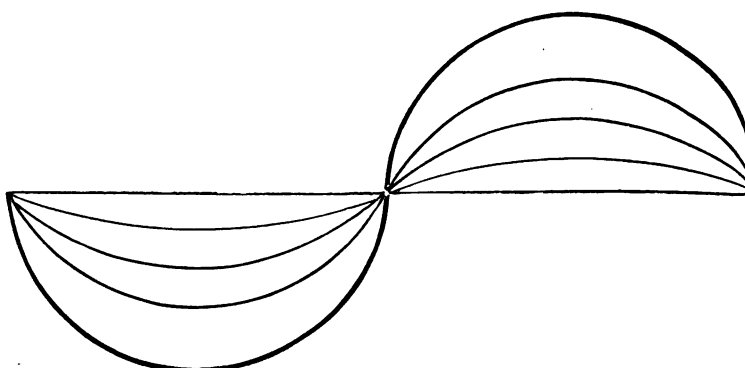


Fig. 73.

Two waves and their resultant: the thin and thick waves added, result in the largest wave of the three:—

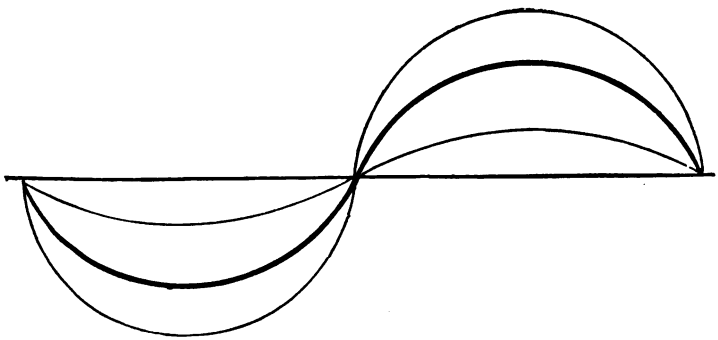


Fig. 74.

These forms will enable the student to understand the principle that, just as quality is the result of the combinations of various simple tones, so the wave-forms which represent certain qualities are a compound of the simple pendular vibration forms of which the tone is made up. If we take, *e.g.*, a tone composed of one prime and the first two upper partials, the latter falling off in intensity in inverse proportion to their rates of vibration, we shall have the following figures (the thick line representing the prime, the thin line the first upper partial, and the dotted line the second upper partial):—

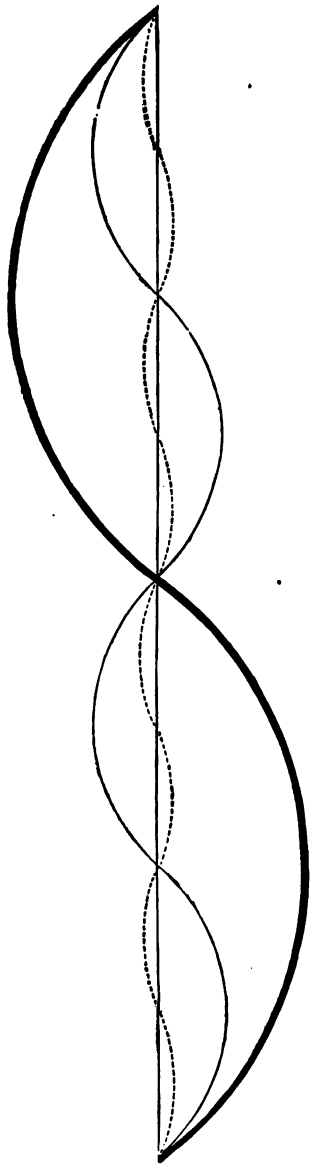


Fig. 75.

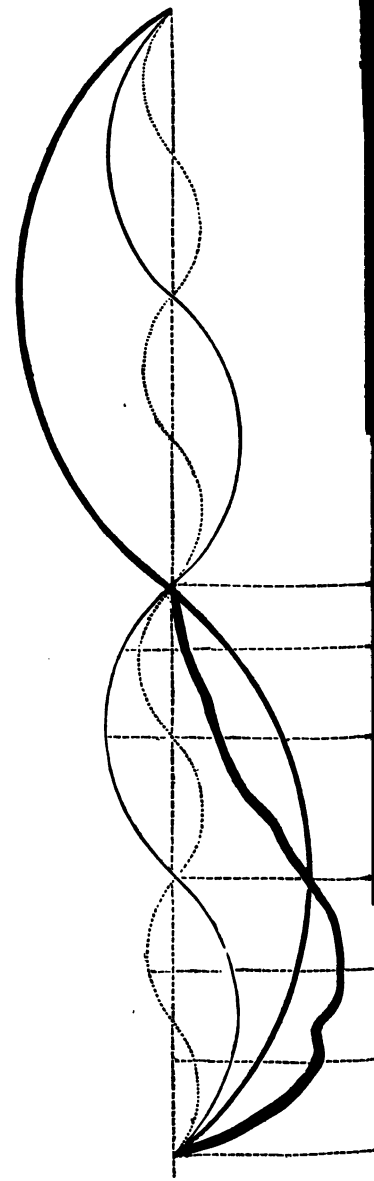


Fig. 76.

1. At starting, all three waves are moving in the same direction, and the resultant wave suddenly dips. 2. The second upper partial soon rises, and causes a temporary elevation. 3. The first upper partial then rises, the second dips, and the one partially neutralises the other, the elevation of the first having more influence on the form of the curve than the depression of the second. 4. At this point the prime also begins to rise, and elevation prevails, the slight effect of the falling of the second upper partial doing little to prevent the rise. 5. All three waves rise together. 6. First upper partial begins to fall, and stops the suddenness of the rise. 7. Second upper partial begins to dip, and still further arrests the rise, though not to the same extent as when (at 6) the first began to dip.

CHAPTER IX.

THE MOTIONS OF SOUNDING STRINGS.

ALTHOUGH the vibrations of a string which emits a musical tone cannot be seen beyond the fact that it is moving, it is yet possible by a very simple experiment to demonstrate the mode in which the vibration of a string takes place. The easiest way to do this is to take a piece of flexible tubing or rope, say twenty feet long, and fasten it loosely between two points. It can be then set in motion by the hand, and according to the rapidity with which the hand moves, the tube or rope will be seen to adopt different forms. The first of these is that in which the whole cord sways from side to side. (See fig. 77.)

It may also vibrate in any of the following ways, according to the rapidity of the motion imparted to it. (See figs. 78, 79, 80).

These are called ventral segments, and the points where they meet are called nodes. If now one end of the cord or string be unfastened, and a slight jerky

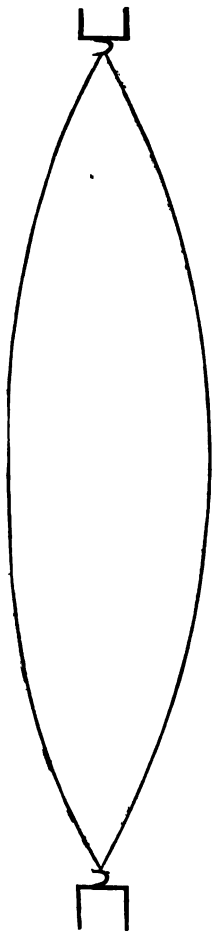


Fig. 77.

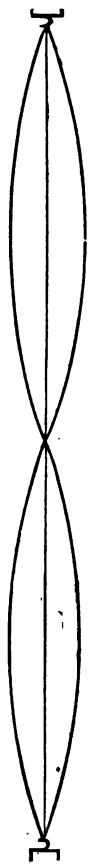


Fig. 78.

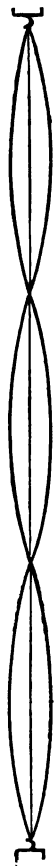


Fig. 79.

Fig. 80.

motion be given to it with the hand, a little wave will be made in the tube, thus—

Fig. 81.

This wave will pass along the tube until it reaches the other end, when it will return on the opposite side of the tube to that on which it came, thus—

Fig. 82.

and however many times this wave may pass along the tube, it will return on the opposite side to that on which it started; that is to say, when each wave reaches the fastened end, it will be reflected and pass back again, reversed.

Now fasten both ends of the tube, and with the thumb and finger jerk the middle of the tube. This will send a wave to each end (see fig. 83), and each wave will be reflected from the fixed end of the tube, and return in an inverted form. (See fig. 84).

Now remove the hand, and two waves are travelling along the tube in opposite directions. Let us see what may now happen.

It is easy to perceive that if two crests coincide, their joint effect will be larger than that of each one alone. If, therefore, the following figure represents two crests, the upper line will represent the joint effect if

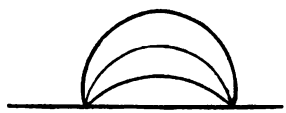


Fig. 85.

two such crests coincide. If, however, a crest and a trough meet, the one will nullify the other, and the result will be a straight line, as in the following figure, the joint effect

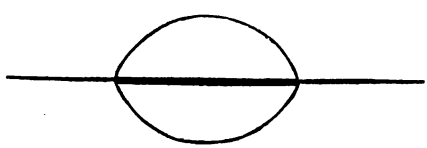


Fig. 86.

being that the crest destroys the wave and results in the thick line. It is therefore clear that crest superimposed on crest will make a crest of the height of both; and that trough superimposed on trough will make a trough of the depth of both; but that a trough met by a crest equal to it will result in the destruction of both. It is also equally clear that when a crest meets a trough, say at

Fig. 83. Fig. 84.

R in the next figure, that point R, being between two opposed equal motions, will remain at rest—

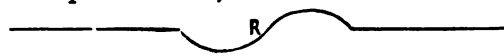


Fig. 87.

A *node*, or point of rest, is therefore formed whenever a crest meets a trough (an inverted crest), equal to it in extent and motive power.

Fig. 88.

It will be seen that, however many crests and troughs (ventral segments) are travelling along a string, the points of rest (nodes) must be one less than the number of segments. Thus, not counting the fixed ends of the string (which only serve to reflect the motion of the segments), there are in fig. 88 five segments and four nodes.

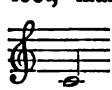
The same principle holds good if instead of a string, say, vibrating in two segments, we take a string of half the length vibrating in one segment; or, in mathematical language, the rate at which a string vibrates is in inverse proportion to its length. A string that is four feet long will vibrate twice as fast as one eight feet long; and a string two feet long will vibrate twice as fast as one four feet long. This is, of course, on the assumption that the material, tension, and thickness of each string are precisely the same.

"The most important inquiry as to a stretched string is, What is the *pitch* of the note it will sound?

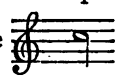
"This is influenced by three qualities which the string possesses, namely—

1. The weight.
2. The tension,
3. The length.*

"First as to the *weight* of the string. In proportion as the string is thicker, or *heavier*, it will, on mechanical principles, vibrate more slowly, and consequently it will speak a *lower note*. A glance inside a pianoforte will show that the strings increase in thickness and weight as they descend in pitch; and the lowest of all are usually lapped with wire twisted spirally round them, to give them sufficient weight to produce the grave tone. The lower strings of the violin and violoncello are also lapped with silver wire for the same reason.

"The rapidity of the vibrations is proportionate, *inversely*, to the *square root of the weight* of the string:—for example, if it is found that a pianoforte wire of a certain length and tension, and weighing 40 grains per foot, makes 256 vibrations per second, sounding the note  by substituting a wire of one-fourth the weight or 10 grains per foot, keeping the tension the same, we shall find it makes

* The principles of stretched strings may be well and easily studied practically by means of a very useful instrument called a *monochord*. It is simply an arrangement for stretching a wire so that its various elements of length, tension, &c., can be easily adjusted at pleasure, and their effect on the note sounded thereby demonstrated. Every student of the subject should make himself practically familiar with this excellent contrivance; it is of great use in many other ways for the study of musical theory, on account of the facility with which, by adjusting the length of the string, definite and very small differences of pitch may be obtained.

512 vibrations per second, and will consequently sound the note 

"The second element is the *tension* of the string. The *tighter* the string is stretched the more rapidly will it vibrate, and the *higher* or more acute will be the sound. This is the foundation of the familiar principle of *tuning* all stringed instruments; which consists merely of altering the tension of the strings, in order to bring them to sound the note desired. The tension on a pianoforte string varies from 100 to 260 lbs.; on the whole of a large grand piano it reaches as much as 16 or 17 tons.

"The rapidity of vibration varies *directly as the square root* of the tension. Thus if a wire be stretched with a tensile force of 25 lbs., by increasing this to 100 lbs. the vibration-number will be doubled, and the string made to speak an octave higher.

"The third element is the *length* of the string. With the same string, stretched under the same tension, the rapidity of vibration is exactly *in inverse proportion to the vibrating length* of the string. Doubling the length will give half the number of vibrations per second, and will produce a note an octave lower. Halving the length of the string will give twice the number of vibrations, and will produce a note an octave higher.

"Hence all gradations of pitch can be produced by stopping off the string to such a length as is suitable to the note required. This is the principle of violin-playing, and it is this that gives to that class of instruments their exquisite perfection of intonation; seeing that the performer can adjust the length with the greatest accuracy.

"This fact, namely, that the note sounded by a string may be defined by its proportionate length, was the great discovery of Pythagoras; and it will be seen, in a future

chapter, the important use he made of it in settling the music of the Greeks on a basis intelligible for all future time.

"Taking the three elements of a stretched string all into calculation together, it is possible to determine, by mathematical reasoning, what note any string ought to sound. Or in other words, having a string of a given weight and length, and stretched with a given tension, we may calculate, on unerring mathematical principles, what number of vibrations it ought to make in a second of time.* *

"Hence, since we can try by experiment what note such a string will sound, we have a means of determining what musical note corresponds to a certain given *vibration number*, and conversely, what number of vibrations corresponds to any given note."—*Pole.*

When a string is being used, not for experimental purposes, but to produce musical sounds, it obeys of

* The formula for doing this was first worked out in 1715 by Dr. Brook Taylor, the learned mathematician and author of the celebrated "Methodus Incrementorum," and it is as follows :—

Let W = whole weight of string.
T = tension.
l = length of string.
L = length of a pendulum vibrating seconds:—

$$\text{Double Vibrations per Second.} \Big\} = \frac{\pi}{2}\sqrt{\frac{T}{W} \times \frac{L}{l}}$$

Or it may be put in a more practical form thus :—

Assuming L, in the latitude of London, = 39.126 inches; make w = weight of the string per inch of its length; and l = length in inches, T and w being both taken in the same unit.

$$\text{Double Vibrations per Second.} \Big\} = \frac{9.825}{l}\sqrt{\frac{T}{w}}$$

Or, for arithmetical calculation, multiply the tension in lbs. by 84,000, divide by the weight in grains of a foot-length of the string, and take the square root of the quotient. Multiply this by 9.825, and divide by the length of the string in inches. The result will be the number of double vibrations per second.

course the same laws. It is known, for instance, and can be proved by experiment, that if a string of four feet in length be divided at the centre, the two halves will give each a note exactly an octave above that of the prime tone; that is, each half vibrates twice as fast as does the whole string.

So also it is easy to prove, by stopping a string in the middle, that each half sounds an octave higher than the whole length; and it is also easy to show, by experiments with the syren, that the half vibrates twice as rapidly as the whole. Mr. Sedley Taylor ("Sound and Music," page 103) says:—"Thus" (referring to a diagram and explanation just given) "the tube executes one complete vibration in the time occupied by a pulse in passing along a length of the tube equal to *twice one of its own ventral segments.* In other words, *the tube's rate of vibration varies as the number of segments into which it is divided.*" And, a little farther on:—"It is easy to confirm this by direct experiment, the swaying movement of the hand on the tube needing to be twice as rapid for a form of vibration with two segments, as for a form with one, and so on." It is hardly correct to say that the *tube's* rate varies as the number of segments. The correct way of putting it is where Mr. Taylor says (page 105), "that the rapidity of vibration *in any one of these forms* is, as we have seen, proportional to the number of segments formed."

We saw in the last chapter how the qualities of strings are affected by the force with which they are

plucked, and by the nature of the material with which the blow is given, or by which the string is moved, whether it be by stroking it with a resined bow as in the violin, or striking it with a hammer as in the pianoforte. The waves thus created travel to the end of the string, are there reflected, and then return—following, in fact, exactly the same laws as in the case of the tube the vibrations of which we have just been considering.

If a string vibrates in a single segment, its tone is a simple tone, and will be free (other things being equal) from upper partials. The tone of such a string would be dull and characterless. If it vibrates in two segments, its first upper partial will be present, and the quality of the tone produced will be materially altered; vibrating in three segments, with two nodes, the first and the second upper partials, as well as the prime tone, will be present; and so on. The notes which are produced by the segments of a string are its upper partials.

If the student will stretch a string between two points of rest, and mark accurately the half, third, fourth, fifth, sixth, seventh, eighth, &c., of its length, he will be able to determine for himself, in a very pleasing and interesting manner, the laws that regulate the vibrations of strings. For instance, it has been stated that a string does not vibrate at the point where nodes are formed; and this can be proved by folding up a bit of paper and placing it on the string at those points at which theory determines nodes ought to be found.

If the string be made to vibrate, it will be found that the piece of paper remains at rest; whereas if it be moved, however little, to the right or left, it will immediately be thrown off. If, for instance, it be desired to show that a string can vibrate in three segments with two nodes, place a paper rider across the string at exactly one-third of its length; then while the string is moved with a violin bow, place the point of the finger at one-third the length of the string from the other end, and the finger, making a node at one-third the length, the string will divide itself into three segments and the paper will remain in its place; whereas, if the finger be held at one-third the length of the string, and the paper be placed at one half the length, it will immediately fly off, because it is just the centre of a segment, and consequently at a point of maximum vibration.

After what has been said about the upper partials of any compound tone and their connection with the quality, it will be understood that the quality of tone produced by any vibrating string depends upon the number of sections or segments into which it divides and subdivides itself in addition to its full length; and the varying tones of different stringed instruments are determined not only by the material of which the body of the instrument is made, but also by the influence which that material, in conjunction with the quality of the material of which the string itself is made, brings to bear on the division and subdivision of the string into segments.

"For experiments on musical strings, and for the most convenient apparatus and manipulation, we would refer to Professor Tyndall's Lectures on Sound. The apparatus which we would recommend, as sufficient for the repetition and variation of these, is very simple and inexpensive. The basis may be a stiff piece of wood 3 or 4 inches broad and 3 or 4 feet long; to one end of this is fastened a common violin string, passing over a bridge near that end, and passing over another bridge near the other end, and then passing over a pulley and sustaining a scale-pan, in which various weights may be placed. For the experiments on harmony there should be a second string mounted in a similar way parallel to the first: one of the strings should be provided with a movable bridge, which can be planted at any arbitrary point under it. For producing sound, a common violin-bow is to be used. Confining ourselves for the present to the single string, we may point out as the experiments most worthy of attention that, by damping the motion of the string by a touch of the finger at the middle, at $\frac{1}{3}$ length, at $\frac{1}{4}$ length, &c., still exciting the movements by the bow, pure notes will be produced which the musical ear will recognise as the harmonics of the fundamental note; and that, by putting small pieces of paper on the string, when the damping is at $\frac{1}{3}$ length or $\frac{1}{4}$ length, those which are (in the former) at $\frac{2}{3}$ length, or those which are (in the latter) at $\frac{2}{4}$ length and $\frac{3}{4}$ length will remain, showing that those points of the string are quiescent, while all others are thrown off by the vibrations. Also, by weighing the string, and ascertaining the weight in the pan, the number of vibrations per second can be found."—*Airy.*

"It has been seen in Article 48 that in a divergent oscillating wave of air, such as we may suppose to be caused by the vibrations of a string, the motion of the particles is

of the order of R, whose first term varies as the distance raised to the power $-\frac{1}{2}$. Moreover, the smallness of dimension of a wire makes it impossible that it can communicate great motion even to the air it touches. Hence, it is impossible that a wire can, by immediate action on the air, produce a sound easily audible to a considerable or convenient distance. To make it audible, the wire must be connected with an intermediate substance whose vibrations can produce a stronger effect on the air, and those vibrations must be excited by the vibrations of the wire. The intermediate substance used for this purpose is the sounding board. In the violin, the wires pass over a bridge which rests by two feet upon the upper board; and under that board, at the place where one foot of the bridge presses, is a little post (known by the name of the 'sound-post' or the 'soul') connecting the upper board with the lower board. Every tremulous motion of a wire of the violin acts directly upon the bridge and upon the upper and lower boards; and the tremors of these produce effective vibrations of the air, and diffuse the sound. We know that everything depends on the elastic properties of these boards; but we know nothing of their precise laws of vibration. In the pianoforte, the general construction is simpler, but the sounding board is so connected with the supports of the wires that it is made to vibrate by the vibrations of the wires."—*Airy.*

"Very rigid strings will not form any very high upper partials, because they cannot readily assume inflections in opposite directions within very short sections. This is easily observed by stretching two strings of different thicknesses on a monochord and endeavouring to produce their high upper partial tones. We always succeed much better with the thinner than with the thicker string. To produce

very high upper partial tones, it is preferable to use strings of extremely fine wire, such as gold lace makers employ, and when they are excited in a suitable manner, as, for example, by plucking or striking with a metal point, these high upper partials may be heard in the compound itself. The numerous high upper partials which lie close to each other in the scale, give that peculiar high inharmonious noise which we are accustomed to call 'tinkling.' From the eighth partial tone upwards these simple tones are less than a whole tone apart, and from the fifteenth upwards less than a semitone. They consequently form a series of dissonant tones. On a string of the finest iron wire, such as is used in the manufacture of artificial flowers, 700 centimetres (22·97 feet) long, I was able to isolate the eighteenth partial tone. The peculiarity of the tones of the zither depends on the presence of these tinkling upper partials, but the series does not extend so far as that just mentioned, because the strings are shorter.

"Strings of gut are much lighter than metal strings of the same compactness, and hence produce higher partial tones. The difference of their musical quality depends partly on this circumstance and partly on the inferior elasticity of the gut, which damps their partials, especially their higher partials, much more rapidly. The tone of plucked cat-gut strings (*guitar, harp*) is consequently much less tinkling than that of metal strings."—*Helmholtz.*

The following summary of part of Tyndall's chapter on Strings will interest the student:—

"The following four laws regulate the vibrations of strings:—The rate of vibration is inversely proportional to the length; it is inversely proportional to the diameter; it is directly proportional to the square root of the stretch-

ing weight or tension; and it is inversely proportional to the square root of the density of the string.

"When strings of different diameters and densities are compared, the law is, that the rate of vibration is inversely proportional to the square root of the weight of the string.

"When a stretched rope, or an indiarubber tube filled with sand, with one of its ends attached to a fixed object, receives a jerk at the other end, the protuberance raised upon the tube runs along it as a pulse to its fixed end, and, being there reflected, returns to the hand by which the jerk was imparted.

"The time required for the pulse to travel from the hand to the fixed end of the tube and back is that required by the whole tube to execute a complete vibration.

"When a series of pulses are sent in succession along the tube, the direct and reflected pulses meet, and by their coalescence divide the tube into a series of vibrating parts, called *ventral segments*, which are separated from each other by points of apparent rest called *nodes*.

"The number of ventral segments is directly proportional to the rate of vibration at the free end of the tube.

"The hand which produces these vibrations may move through less than an inch of space; while by the accumulation of its impulses the amplitude of the ventral segments may amount to several inches, or even to several feet.

"If an indiarubber tube, fixed at both ends, be encircled at its centre by the finger and thumb, when either of its halves are pulled aside and liberated, both halves are thrown into a state of vibration.

"If the tube be encircled at a point one-third, one-fourth, or one-fifth of its length from one of its ends, on pulling the shorter segment aside and liberating it, the longer segment divides itself into two, three, or four vibrating parts, separated from each other by nodes.

"The number of vibrating segments depends upon the rate of vibration at the point encircled by the finger and thumb.

"Here also the amplitude of vibration at the place encircled by the finger and thumb may not be more than a fraction of an inch, while the amplitude of the ventral segments may amount to several inches.

"A musical string damped by a feather at a point one-half, one-third, one-fourth, one-fifth, &c., of its length from one of its ends, and having its shorter segment agitated, divides itself exactly like the indiarubber tube. Its division may be rendered apparent by placing little paper riders across it. Those placed at the ventral segments are thrown off, while those placed at the nodes retain their places.

"The notes corresponding to the division of a string into its aliquot parts are called the *harmonics* of the strings.

"When a string vibrates as a whole, it usually divides at the same time into its aliquot parts. Smaller vibrations are superposed upon the larger, the tones corresponding to those smaller vibrations, and which we have agreed to call overtones, mingling at the same time with the fundamental tone of the string.

"The addition of these overtones to the fundamental tone determines the *timbre* or *quality* of the sound, or, as we have agreed to call it, the *clang-tint.*

"It is the addition of such overtones to fundamental tones of the same pitch which enables us to distinguish the sound of a clarionet from that of a flute, and the sound of a violin from both. Could the pure fundamental tones of these instruments be detached, they would be undistinguishable from each other; but the different admixture of overtones in the different instruments renders their clang-tints diverse, and therefore distinguishable.

"Instead of the heavy indiarubber tube in the experi-

ment above referred to, we may employ light silk strings, and, instead of the vibrating hand, we may employ vibrating tuning-forks, and cause the strings to swing as a whole, or to divide themselves into any number of ventral segments. Effects of great beauty are thus obtained, and by experiments of this character all the laws of vibrating strings may be demonstrated.

"When a stretched string is plucked aside or agitated by a bow, all the overtones which require the agitated point for a node vanish from the clang of the string.

"The point struck by the hammer of a piano is from one-seventh to one-ninth of the length of the string from its end: by striking this point, the notes which require it as a node cannot be produced, a source of dissonance being thus avoided."—*Tyndall.*

"We may extend the experiments of M. Melde to the establishment of all the laws of vibrating strings. Here are four tuning-forks, which we may call *a*, *b*, *c*, *d*, whose rates of vibration are to each other as the numbers 1, 2, 4, 8. To the largest fork is attached a string, *a*, stretched by a weight, which causes it to vibrate as a whole. Keeping the stretching weight the same, I determine the lengths of the same string, which, when attached to the other three forks, *b*, *c*, *d*, swing as a whole. The lengths in the four respective cases are as the numbers 8, 4, 2, 1.

"From this follows the first law of vibration, already established by another method, viz.:—*the length of the string is inversely proportional to the rapidity of vibration.*

"In this case the longest string vibrates as a whole when attached to the fork *a*. I now transfer the string to *b*, still keeping it stretched by the same weight. It vibrates when *b* vibrates; but how? By dividing into two equal ventral segments. In this way alone can it accommodate itself to the swifter vibrating period of *b*. Attached to *c*, the same

string separates into four, while when attached to *d*, it divides into eight ventral segments. The number of the ventral segments is proportional to the rapidity of vibration. It is evident that we have here, in a more delicate form, a result which we have already established in the case of our india-rubber tube set in motion by the hand. It is also plain that this result might be deduced theoretically from our first law.

"We may extend the experiment. Here are two tuning-forks separated from each other by the musical interval called a fifth. Attaching a string to one of the forks, I stretch the string until it divides into two ventral segments: attached to the other fork, and stretched by the same weight, it divides instantly into three segments when the fork is set in vibration. Now, to form the interval of a fifth, the vibrations of the one fork must be to those of the other in the ratio of 2 : 3. The division of the string, therefore, declares the interval. In the same way the division of the string in relation to all other musical intervals may be illustrated.

"Again. Here are two tuning-forks, *a* and *b*, one of which (*a*) vibrates twice as rapidly as the other. A string of silk is attached to *a*, and stretched until it synchronises with the fork, and vibrates as a whole. Here is a second string of the same length, formed by laying four strands of the first one side by side. I attach this compound thread to *b*, and keeping the tension the same as in the last experiment, set *b* in vibration. The compound thread synchronises with *b*, and swings as a whole. Hence, as the fork *b* vibrates with half the rapidity of *a*, by quadrupling the weight of the string we halved its rapidity of vibration. In the same simple way it might be proved that by augmenting the weight of the string nine times we reduce the number of its vibration to one-third. We thus demonstrate the law:—

"*The rapidity of vibration is inversely proportional to the square root of the weight of the string.*

"An instructive confirmation of this result is thus obtained:—Attached to this tuning-fork is a silk string six feet long. Two feet of the string are composed of four strands of the single thread, placed side by side, the remaining four feet are a single thread. A stretching force is applied, which causes the string to divide into two ventral segments. But how does it divide? Not at its centre, as is the case when the string is of uniform thickness throughout, but at the precise point where the thick string terminates. This thick segment, two feet long, is now vibrating at the same rate as the thin segment four feet long, a result which follows by direct deduction from the two laws already established.

"Here again are two strings of the same length and thickness. One of them is attached to the fork *a*, the other to the fork *b*, which vibrates with twice the rapidity of *a*. Stretched by a weight of 20 grains, the string attached to *b* vibrates as a whole. Substituting *b* for *a*, a weight of 80 grains causes the string to vibrate as a whole. Hence, to double the rapidity of vibration, we must quadruple the stretching weight. In the same way it might be proved, that to treble the rapidity of vibration we should have to make the stretching weight nine-fold. Hence our third law:—

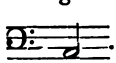
"*The rapidity of vibration is proportional to the square root of the tension.*

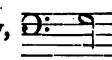
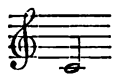
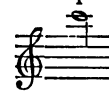
"Let us vary this experiment. This silk cord is carried from the tuning-fork over the pulley, and stretched by a weight of 80 grains. The string vibrates as a whole as at A, fig. 50.* By diminishing the weight the string is relaxed,

* The figures at the beginning of this chapter are similar to those referred to in this extract from Tyndall.

and finally divides sharply into two ventral segments, as at B, fig. 50. What is now the stretching weight? 20 grains, or one-fourth of the first. With a stretching weight of almost exactly 9 grains it divides into three segments, as at C; while with a stretching weight of 5 grains it divides into four segments, as at D. Thus, then, a tension of one-fourth doubles, a tension of one-ninth trebles, and a tension of one-sixteenth quadruples the number of ventral segments. In general terms, the number of segments is inversely proportional to the square root of the tension. This result may be deduced by reasoning from our first and third laws, and its realisation here confirms their correctness.

"Thus, by a series of reasonings and experiments totally different from those formerly employed, we arrive at the self-same laws. In science, different lines of reasoning often converge upon the same truth; and if we only follow them faithfully, we are sure to reach that truth at last. We may emerge, and often do emerge, from our reasoning with a contradiction in our hands; but on retracing our steps, we infallibly find the cause of the contradiction to be due, not to any lack of constancy in nature, but of accuracy in man. It is the millions of experiences of this kind which science furnishes that give us our present faith in the stability of nature."—*Tyndall.*

"By certain means well known, it is easy to alter the mode of vibration, dividing the string or column of air into two parts, each of which will vibrate at double the former rate, viz., 128 vibrations, which will give a note an octave higher than before, namely,  . This is called a *natural harmonic note* of the fundamental lower C. Again, similarly, the string or column of air may easily be made to divide itself into three equal parts, each of which

will make $3 \times 64 = 192$ vibrations per second, and will produce a second natural harmonic, namely,  It may further divide itself into four parts, each vibrating $4 \times 64 = 256$, and giving a third natural harmonic,  The division might be carried further, at pleasure, each harmonic note becoming higher in the scale. For example, the division into eight parts would give 512 vibrations, producing a seventh natural harmonic, and the division into sixteen parts would produce a fifteenth natural harmonic 

"It was shown by Delezenne in 1842 that it is impossible to make a string sound if it be excited in the centre by a bow. Duhamel was of opinion that in a string which is giving its foundation tone the first partial is vibrating also, and that since the bow prevents this form of motion, sound cannot be produced. To verify this hypothesis, he endeavoured to sound a string by means of two bows moving in the same direction, to the right and left respectively of the middle point of the string. Still no sound was produced. But on the other hand, if the position of the bows were retained unchanged, and an opposite direction of motion with equal velocity were given them, the foundation tone came out instantly, accompanied by the first upper partial. If the string be attacked successively close to each of the consecutive harmonic points, so as to produce the fundamental tone, the corresponding upper partial is reinforced. At one-third of the length the twelfth has about equal intensity with the fundamental, at one-fourth the

double octave, at one-fifth the major seventeenth. The harmonic always slightly precedes the fundamental tone. Speaking generally, a string vibrating transversally can only sound on the condition that it gives two transversal tones, the sharper of which depends on the point of attack, or the mode of excitement."—*Stone.*

"*Effects of Heat on Strings.*—A vibrating string of metal, stretched between two fixed supports, materially alters its tension with heat from the expansion and contraction of the metal itself. There is also a modification of elasticity due to the same cause.

"An experiment proving the latter proposition may be performed by extending a string of steel or platinum over a long trough of lighted alcohol. The string, fixed at one extremity, passes over a pulley, or on to the short arm of a bent lever weighted at the other end; and though the tension thus remains the same, the note sinks in pitch from the cause above-named, and at last becomes extinguished.

"The former influence is very perceptible in the pianoforte, which is materially sharper in frosty weather, the strings themselves not unfrequently snapping from this cause. The frame of the instrument, especially if made of wood, being less susceptible to variation of bulk from increments of heat, remains fixed, while the string shortens.

"The writer has made experiments by passing an electric current through a steel and brass string strained on a sonometer. He found that the heat thus developed was competent to lower the pitch through the interval of an octave.* Even when the tension was produced by means of a weight, and the string thus allowed to lengthen, there was still a

* *Proceedings of Physical Society* and of *College of Organists.*

notable fall of pitch, due, in all probability, to alteration of its elasticity.

"Strings of catgut, being very hygrometric, are also materially affected by moisture, which swells the material laterally and tends to shorten it. In a hot damp concert-room violins vary rapidly and somewhat irregularly from this cause."—*Stone.*

CHAPTER X.

THE MOTION OF SOUNDING AIR-COLUMNS.

If a tube, four feet long and open at both ends, be blown across at one end, the fundamental tone of the tube will be sounded; but if the hand be placed at one end of the tube, so as to effectually close it, and the open end be blown across as before, a sound will be heard exactly one octave below that which was heard when both ends of the tube were open. One of these pipes was an open pipe, the other a stopped pipe; and the difference between the two is that which constitutes the two great classes into which the flue pipes of organs are divided.

It will be remembered that vibrating strings divide into segments, having nodes between, and the same general principle applies to the vibrations of air in pipes. It is important to remember that *there is always a node at the stopped end of the pipe, and a segment at the open end*, and all the nodes which are formed in stopped pipes will, therefore, be at stated distances from the stopped end of the pipe. What those distances are, as well as the law which determines them, will be considered presently. Just

now the student, bearing in mind the law which governs the vibrations of a string, has to remember this one important fact, that in any pipe, whether stopped or open, there is always a segment at the open end; and that in a stopped pipe there is always a node at the closed end. The connection between this fact, and that of the stopped pipe giving an octave below the open pipe, will be dealt with in this chapter.

Let us take, first of all, the case of an open pipe, which will always have a segment at each end. The simplest form of vibration in this pipe will be that with one node, which gives one vibration at one end of the pipe, the other half vibrating at the other end, thus making one complete vibration of the whole body of air contained in it. This will give the fundamental note of the pipe. The form of vibration in an open pipe giving its fundamental tone is, therefore, as follows:—

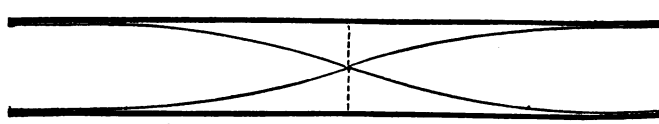


Fig. 89.

If increased pressure of wind takes place, two nodes will be formed within the pipe; but as there must be a segment at each open end, there will be necessarily two nodes, one at one-fourth the length from one end, the other at an equal length from the

other end, there being a segment exactly in the middle of the pipe, thus—

Fig. 90.

This form of vibration gives the octave of the fundamental tone.

Further pressure will produce sounds having three nodes, and giving a note a twelfth above the prime, thus—

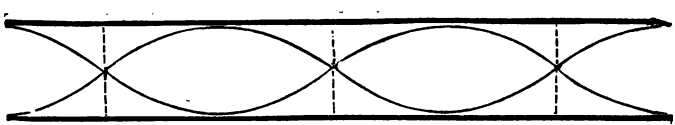


Fig. 91.

and four nodes, which is the double octave of the fundamental note.

The lengths of these waves will be seen, from the figures, to be respectively one-half, one-fourth, one-sixth, one-eighth, &c., the length of the pipe; therefore, the times of vibrations will be as one to two, one to three, one to four, &c.; thus the harmonics of an open pipe will correspond with the tones of the regular harmonic series.

"The condition of the air within an open organ-pipe when its fundamental note is sounded is that of a rod free at both ends, held at its centre, and caused to vibrate longitudinally. The two ends are places of vibration, the

centre is a node. Is there any way of *feeling* the vibrating air-column so as to determine its nodes and its places of vibration? The late excellent William Hopkins has taught us the following mode of solving this problem. Over a little hoop is stretched a thin membrane, forming a little tambourine. The front of this organ-pipe is of glass, through which you can see the position of the body within it. By means of a string, the little tambourine can be raised or lowered at pleasure through the entire length of the pipe. When held above the upper end of the pipe you hear the loud buzzing of the membrane. When lowered into the pipe it continues to buzz for a time; the sound becoming gradually feebler, and finally ceasing totally. It is now in the middle of the pipe, where it cannot vibrate, because the air around it is at rest. On lowering it still further, the buzzing sound instantly recommences, and continues down to the bottom of the pipe. Thus, as the membrane is raised and lowered in quick succession, during every descent and ascent, we have two periods of sound separated from each other by one of silence. If sand were strewn upon the membrane, it would dance above and below, but it would be quiescent at the centre. We thus prove experimentally that when an organ-pipe sounds its fundamental note it divides itself into two semi-ventral segments separated by a node.

"What is the condition of the air at this node? Again that of the middle of a rod, free at both ends, and yielding the fundamental note of its longitudinal vibration. The pulses reflected from both ends of the rod, or of the column of air, meet in the middle, and produce compression; they then retreat and produce rarefaction. Thus, while there is no vibration in the centre of an organ-pipe, the air there undergoes the greatest changes of density. At the two ends of the pipe, on the other hand, the air-particles

merely swing up and down without sensible compression or refraction.

"If the sounding pipe were pierced at the centre, and the orifice stopped by a membrane, the air, when condensed, would press the membrane outwards, and, when rarefied, the external air would press the membrane inwards. The membrane would therefore vibrate in unison with the column of air. An organ-pipe is so arranged that a small jet of gas can be lighted opposite the centre of the pipe, and there acted upon by the vibrations of a membrane. Two other gas jets are placed nearly midway between the centre and the two ends of the pipe. The three burners are fed through the tube, the gas enters the hollow chamber, from which issue three little bent tubes, each communicating with a capsule closed underneath by the membrane. This is in direct contact with the air of the organ-pipe. From the three capsules issue the three little burners, with their flames.

"Blowing into the pipe so as to sound its fundamental note, the three flames are agitated, but the central one is most so. Lowering the flames so as to render them very small, and, blowing again, the central flame is extinguished, while the others remain lighted. The experiment may be performed half-a-dozen times in succession; the sounding of the fundamental note always quenches the middle flame.

"By blowing more sharply into the pipe, it is caused to yield its first overtone. The middle node no longer exists. The centre of the pipe is now a place of maximum vibration, while two nodes are formed midway between the centre and the two ends. But if this be the case, and if the flame opposite a node be always blown out, then, when the first overtone of this pipe is sounded, the two flames ought to be extinguished, while the central flame remains lighted. This is the case. When the first harmonic is

sounded the two nodal flames are infallibly extinguished, while the flame in the middle of the ventral segment is not sensibly disturbed.

"There is no theoretic limit to the subdivision of an organ-pipe, either stopped or open. In stopped pipes we begin with one semi-ventral segment, and pass on to 3, 5, 7, &c., semi-ventral segments; the number of vibrations of the successive notes augmenting in the same ratio. In open pipes we begin with two semi-ventral segments, and pass on to 4, 6, 8, 10, &c., the number of vibrations of the successive notes augmenting in the same ratio; that is to say, in the ratio 1 : 2 : 3 : 4 : 5, &c. When, therefore, we pass from the fundamental tone to the first overtone in an open pipe, we obtain the octave of the fundamental. When we make the same passage in a stopped pipe, we obtain a note a fifth above the octave. No intermediate notes of vibration are in either case possible. If the fundamental tone of a stopped pipe be produced by 100 vibrations a second, the first overtone will be produced by 300 vibrations, the second by 500, and so on. Such a pipe, for example, cannot execute 200 or 400 vibrations in a second. In like manner the open pipe whose fundamental note is produced by 100 vibrations a second, cannot vibrate 150 times in a second, but passes, at a jump, to 200, 300, 400, and so on.

"In open pipes, as in stopped ones, the number of vibrations executed in the unit of time is inversely proportional to the length of the pipe. This follows from the fact, already dwelt upon so often, that the time of a vibration is determined by the distance which the sonorous pulse has to travel to complete a vibration."—*Tyndall.*

With stopped pipes the case is different, as a different law prevails. The waves which enter at

one end of the pipe cannot escape at the other, which is stopped, and it follows, therefore, that whatever be the condition of the air in the pipe when it reaches the stopped end, it will be turned back in exactly the same condition; that is to say, if the air reaches the stopped end of the pipe as a condensation, it will so return; if it reaches it as a rarefaction, it will be returned so likewise. Remembering that a segment is always at the open end, and a node at the stopped end, we shall see that the simplest form of vibration in a stopped pipe is that shown in the following figure:—

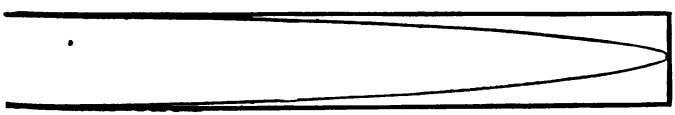


Fig. 92.

having one node and one segment. Increased pressure will in this case, just as in the other, divide the wave into two parts, and there will be consequently a node at one-third the length of the pipe from the open end as well as at the closed end, the segments being one at the open end, the other at one-third the length of the pipe from the closed end, thus—

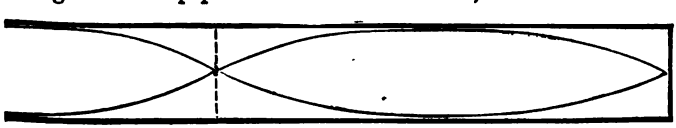


Fig. 93.

so that we have here a wave and a half, producing a note a twelfth above the prime. If the pressure is

again increased, the waves will be more divided, and we shall have two waves and a half, three waves and a half, &c.; and these waves bear the same pro-

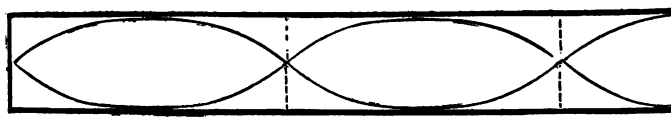


Fig. 94.

portion to each other as the numbers one to three, three to five, &c., so that a stopped pipe makes only the *odd* numbers of the harmonic series.

"But that the current of air should be thus able to accommodate itself to the requirements of the tube, it must enjoy a certain amount of *flexibility*. A little reflection will show you that the power of the reflected pulse over the current must depend to some extent on the force of the current. A stronger current, like a more powerfully stretched string, requires a greater force to deflect it, and when deflected vibrates more quickly. Accordingly, to obtain the fundamental note of this 24-inch tube, we must blow very gently across its open end; a rich, full, and forcible musical tone is then produced. With a little stronger blast the sound approaches a mere rustle; blowing stronger still, a tone is obtained of much higher pitch than the fundamental one. This is the first overtone of the tube, to produce which the column of air within it has divided itself into two vibrating parts with a node between them. With a still stronger blast a still higher note is obtained. The tube is now divided into three vibrating parts, separated from each other by two nodes. Once more I blow with sudden strength; a higher note than any before obtained is the consequence. We have now to

inquire into the relation of these successive notes to each other. The space from node to node has been called all through 'a ventral segment;' hence the space between the middle of a ventral segment and a node is a semi-ventral segment. You will readily bear in mind the law, that *the number of vibrations is directly proportional to the number of semi-ventral segments* into which the column of air within the tube is divided. Thus, when the fundamental note is sounded, we have but a single semi-ventral segment. The bottom here is a node, and the open end of the tube, where the air is agitated, is the middle of a ventral segment. The mode of division represented in *c* and *d* yields three semi-ventral segments; in *e* and *f** we have five. The vibrations, therefore, corresponding to this series of notes, augment in the proportion of the series of odd numbers, 1 : 3 : 5. And could we obtain still higher notes, their relative rates of vibration would continue to be represented by the odd numbers, 7, 9, 11, 13, &c., &c. It is evident that this *must* be the law of succession. For the time of vibration in *c* or *d* is that of a stopped tube of the length of xy; but this length is one-third of the length of the whole tube, consequently its vibrations must be three times as rapid. The time of vibration in *e* or *f* is that of a stopped tube of the length $x'y'$, and inasmuch as this length is one-fifth that of the whole tube, its vibrations must be five times as rapid. We thus obtain the succession 1, 3, 5, and if we pushed matters further we should obtain the continuation of the series of odd numbers. And here it is once more in your power to subject my statements to an experimental test. Here are two tubes, one of which is three times the length of the other. I sound the fundamental note of the longest tube, and then the next note above the fundamental.

* These refer to figures representing respectively three and five segments, on the same principle as the illustrations given above.

The vibrations of these two notes are stated to be in the ratio of 1 : 3. This latter note, therefore, ought to be of precisely the same pitch as the fundamental note of the shorter of the two tubes. When both tubes are sounded their notes are identical. It is only necessary to place a series of such tubes of different lengths thus together to form that ancient instrument, Pan's pipes, with which we are so well acquainted."—*Tyndall.*

Let us now see how to determine the pitch of the lowest note which a pipe can give. The rule is this:—With a stopped pipe, the length of the tube must be multiplied by four, because the wave length of the note is four times the length of the tube; and if the velocity of sound in air be divided by the product which results from multiplying the length of the pipe in feet by four, it will give the vibration number of the lowest tone of the pipe; so also the length of a stopped pipe giving any tone can be found by dividing the velocity of sound in air per second per foot, by four times the vibration number of the note. The open pipe being always twice the length of the stopped pipe, the multiplication or division must be by two instead of four.

The length of an *open* pipe being 8 feet, what is the vibration number of the note produced?

Length of pipe 8 feet $\times$ 2 = 16; and taking the velocity of sound in air, at ordinary temperatures, as 1130 feet, we have

$$\frac{1130}{16} = 70\tfrac{5}{8} \text{ vibrations per second.}$$

The note known as "eight-foot C," usually vibrates

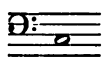
less than this number (about 64), the length of the pipe being actually rather less than 8 feet.

The length of a stopped pipe being 4 feet, what is the vibration-number of the note produced?

Length of pipe 4 feet $\times$ 4 = 16, and

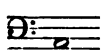
$$\frac{1130}{16} = 70\tfrac{5}{8} \text{ as before,}$$

showing that the stopped pipe of 4 feet gives the same note as the open pipe of 8 feet.

The vibration-number of the note C  being 132 per second, what will be the length of the stopped pipe producing it?

Velocity of sound in air = 1130; 132 $\times$ 4 = 528;

$$\frac{1130}{528} = 2{\cdot}13 = 2 \text{ feet } 1\tfrac{3}{4} \text{ inches (about).}$$

What will be the length of the open pipe which produces the same note C  ?

$$132 \times 2 = 264;$$

$$\frac{1130}{264} = 4{\cdot}28 = 4 \text{ feet } 3\tfrac{1}{2} \text{ inches (about).}$$

Upon these models the student should make calculations of his own, until he can find without difficulty either the length of a pipe from the vibration number of its note, or *vice versâ*.

"Wide stopped pipes, on account of peculiar relations of the mass of air in vibration, give their fundamental note almost pure, with but little accompaniment of overtones; but narrower ones have the addition of the third partial

tone or twelfth, an effect well known in what is called the 'stopped diapason' register of the organ. In modified forms the fifth partial tone may also be heard; but the partial tones with even numbers, representing the octave, double octave, &c., are absent.

"This peculiarity of stopped pipes, and their general deficiency in overtones, gives them a dull, weak, hollow quality of tone, especially in the bass, which contrasts remarkably with that of the open pipes, the latter being much clearer and more brilliant from their greater richness in overtones.

"In narrow, open pipes, such as those of the organ stops called the violone, viola di gamba, &c., the fundamental notes are weak, and are accompanied by a series of strong overtones (often audible up to the fifth) which give a thin tone, somewhat imitating the stringed instruments from which they take their name. In the larger open pipes the fundamental tone is stronger, and the overtones are less prominent; and these are consequently the most useful pipes in the organ. In open wood-pipes Helmholtz found only the second partial tone prominent, the third weak, and the higher ones inappreciable. In metal ones the fourth was also audible. The well-known difference in quality between pipes of wood and metal is partly owing to this, and partly to the fact that the wood surfaces do not resist the vibrations so well as the metal ones, whereby the higher tones appear to be more readily extinguished by friction.

"The peculiar qualities due to special forms of organ pipes are due to exceptional arrangements of the harmonics produced thereby; for example, in pipes with a conically diminished top, the fifth to seventh partials are especially prominent, giving a thin but clear and characteristic tone."—*Pole.*

It will be seen that the length of the pipe varies inversely as the vibration number, just as was shown to be the case with the length of a vibrating string. When, therefore, an organ-builder speaks of "two-foot C," "four-foot C," "eight-foot C," he means the note produced by an open pipe of that length. In each of these cases, a stopped pipe of the same length would produce a note an octave below, and therefore the pipe which is opened when the middle C is played with the stopped diapason drawn out, is exactly half the length of that used when the open diapason is drawn out. The stop known as Bourdon sixteen-feet *tone*, is produced by a series the lowest of which is eight feet in length.

The stops of the open diapason class, or, as they are called, flute or flue stops, constitute the main body of tone in an organ. The principal is a stop of four feet in length, and is therefore an octave higher than that of the open diapason. The twelfth is an octave and a fifth higher, and the fifteenth, as its name implies, is two octaves higher; that is to say, when the lowest note on an organ is pressed down with the open diapason, principal, twelfth, and fifteenth stops drawn, flue pipes of eight feet, four feet, three feet, and two feet in length respectively, are sounded.

"Columns of air of definite length resound to tuning-forks of definite rates of vibration.

"The length of a tube filled with air, and closed at one end, which resounds to a fork, is one-fourth of the length of the sonorous wave produced by the fork.

"This resonance is due to the synchronism which exists between the vibrating period of the fork and that of the column of air.

"By blowing across the mouth of a tube closed at one end, we produce a flutter of the air, and some pulse of this flutter may be raised by the resonance of the tube to a musical sound.

"The sound is the same as that obtained when a tuning-fork, whose rate of vibration is that of the tube, is placed over the mouth of the tube.

"When a tube closed at one end—a stopped organ-pipe for example—sounds its lowest note, the column of air within it is undivided by a node. The overtones of such a column correspond to its division into parts, like those of a rod fixed at one end and vibrating longitudinally. The order of its tones is that of the odd numbers, 1, 3, 5, 7, &c. That this must be the order follows from the manner in which the column is divided.

"In organ-pipes the air is agitated by causing it to issue from a narrow slit, and to strike upon a cutting edge. Some pulse of the flutter thus produced is raised by the resonance of the pipe to a musical sound.

"When, instead of the aerial flutter, a tuning-fork of the proper rate of vibration is placed at the embouchure of an organ-pipe, the pipe *speaks* in response to the fork. In practice, the organ-pipe virtually creates its own tuning-fork, by compelling the sheet of air at its embouchure to vibrate in periods synchronous with its own.

"An open organ-pipe yields a note an octave higher than that of a closed pipe of the same length. This relation is a necessary consequence of the respective modes of vibration.

"When, for example, a stopped organ-pipe sounds its deepest note, the column of air, as already explained, is

undivided. When an open pipe sounds its deepest note, the column is divided by a node at its centre. The open pipe in this case virtually consists of two stopped pipes with a common base. Hence it is plain that the fundamental note of an open pipe must be the same as that of a stopped pipe of half its length.

"The length of a stopped pipe is one-fourth that of the sonorous wave which it produces, while the length of an open pipe is one-half that of its sonorous wave.

"The order of the tones of an open pipe is that of the even numbers 2, 4, 6, 8, &c., or of the natural numbers 1, 2, 3, 4, &c.

"In both stopped and open pipes the number of vibrations executed in a given time is inversely proportional to the length of the pipe.

"The places of maximum vibration in organ-pipes are places of minimum changes of density; while at the places of minimum vibration the changes of density reach a maximum.

"The velocities of sound in gases, liquids, and solids may be inferred from the tones which equal lengths of them produce; or they may be inferred from the lengths of these substances which yield equal tones.

"Reeds, or vibrating tongues, are often associated with vibrating columns of air. They consist of flexible laminæ which vibrate to and fro in a rectangular orifice, thus rendering intermittent the air-current passing through the orifice.

"The action of the reed is the same as that of the syren.

"The flexible wooden reeds sometimes associated with organ-pipes are compelled to vibrate in unison with the column of air in the pipe; other reeds are too stiff to be thus controlled by the vibrating air. In this latter case the column of air is taken of such a length that its vibrations synchronise with those of the reed."—*Tyndall.*

"In these instruments the tone is produced by driving a stream of air against an opening, generally furnished with sharp edges, in some hollow space filled with air. To this class belong the bottles described in the last chapter, . . . and especially flutes and the majority of organ pipes. For flutes, the resonant body of air is included in its own cylindrical bore. It is blown with the mouth, which directs the breath against the somewhat sharpened edges of its mouth hole. The construction of organ pipes will be seen from the two adjacent figures.*

"Any air chamber can be made to give a musical tone, just like organ pipes, flutes, the bottles previously described, the wind-chests of violins, &c., provided they have a sufficiently narrow opening, furnished with somewhat projecting sharp edges, by directing a thin flat stream of air across the opening and against its edges.

"These edges are the source of the musical tone of all such instruments. The directed stream of air breaking against the edges, generates a peculiar hissing or rushing noise, which is all we hear when the pipe does not speak, or when we blow against the edges of a hole in a flat plate instead of a pipe. The narrower the opening and the stronger the blast, the higher will be this noise of the wind. Such a noise, as we have already found, may be considered as a mixture of several inharmonic tones of nearly the same pitch. When the air chamber of the pipe is brought to bear upon these tones its resonance strengthens, such as correspond with the proper tones of that chamber, and makes them predominate over the rest, which this predominance conceals. Hence in all such pipes we always hear the tone accompanied more or less distinctly by a rush of wind, and this gives a peculiar character to its quality. Precisely in the same way as the tones of the

* See figures 61 and 62, page 170.

noise created by the wind are strengthened by resonance, the tone of a tuning fork can be also reinforced by bringing it near the mouth of the pipe, when the pitch of the fork corresponds to one of the proper tones of the enclosed mass of air. By a series of different tuning forks, then, we are enabled to find and determine the proper tones of the air chamber with ease and certainty. The character of the musical quality of tone in such pipes, of course, essentially depends upon whether or not the harmonic upper partials of the tone created by blowing correspond to the proper tones of the pipe with sufficient accuracy to admit of being reinforced at the same time as the prime tone. It is only in narrow cylindrical open pipes, as flutes, and the fiddle stop of organs, that the higher upper partials of the tube exactly correspond with the harmonic upper partials of the prime tone. By blowing more strongly, and thus raising the pitch of the exciting wind-rush itself, the higher proper tones of the tube can be made to speak without the lower. A flute, which when gently blown, with all its holes stopped, gives d', will, on stronger blowing, give d'', and on still stronger d'' and d''', that is, the first, second, and third upper partial tone of d'. Hence for narrow cylindrical pipes, not only the prime tone, but also a series of its harmonic upper partial tones are reinforced by the resonance of the tube, when it is blown with sufficient force for the wind-rush itself to contain several higher partial tones. And in reality on forcibly blowing the narrow cylindrical pipes of an organ (in the *geigenprincipal*, *violoncell*, *violon-bass*, *viola di gamba* stops) we hear a series of upper partials distinctly and powerfully accompany the prime tone, giving them a more cutting quality, resembling a violin. By using resonators I find that on narrow pipes of this kind, the partial tones may be clearly heard up to the sixth. For wide open pipes, on the other hand, the

adjacent proper tones of the tube are all somewhat sharper than the corresponding harmonic tones of the prime, and hence these tones will be much less reinforced by the resonance of the tube. Wide pipes, having larger masses of vibrating air, and admitting of being much more strongly blown without jumping up into a harmonic, are used for the great body of sound on the organ, and are hence called *principalstimmen.* For the above reasons they produce the prime tone alone, strongly and fully, with a much weaker retinue of secondary tones. For wooden 'principal' pipes, I find the prime tone and its octave, or first upper partial, very distinct; the twelfth, or second upper partial, is but weak, and the higher upper partials no longer distinctly perceptible. The quality of tone in these pipes is fuller and softer than that of the *geigenprincipal.* When flute or flue stops of the organ, and the German flutes are blown softly, the upper partials lose strength at a greater rate than the prime tone, and hence the musical quality becomes weak and soft.

"Another variety is observed on the pipes which are conically narrowed at their upper end, in the *salicional, gemshorn,* and *spitzflöte* stops. Their upper opening has generally half the diameter of the lower section; for the same length the *salicional* pipe has the narrowest, and the *spitzflöte* the widest section. These pipes have, I find, the property of rendering some higher partial tones, from the fifth to the seventh, comparatively stronger than the lower. The quality of tone is consequently poor, but peculiarly bright.

"The *narrower stopped cylindrical pipes* have proper tones corresponding to the uneven partials of the prime, that is, the third partial or twelfth, the fifth partial or higher major third, and so on. For the *wider* stopped pipes, as for the wide open pipes, the next adjacent proper

tones of the mass of air are distinctly higher than the corresponding upper partials of the prime, and consequently these upper partials are very slightly, if at all, reinforced. Hence wide stopped pipes, especially when gently blown, give the prime tone almost alone, and they were therefore previously adduced as examples of simple tones. Narrow stopped pipes, on the other hand, let the twelfth be very distinctly heard at the same time with the prime tone; and have hence been called *quintaten* (*quintam tenentes*). When these pipes are strongly blown, they also give the fifth upper partial, or higher major third, very distinctly. Another variety of quality is produced by the *rohrflöte*. Here a tube, open at both ends, is inserted in the cover of a stopped pipe, and in the examples I examined, its length was that of an open pipe giving the fifth partial tone of the prime tone of the stopped pipe. The fifth partial tone is thus proportionably stronger than the rather weak third partial on these pipes, and the quality of tone becomes peculiarly bright. Compared with open pipes, the quality of tone in stopped pipes, where the even partial tones are absent, is somewhat hollow; the wider stopped pipes have a dull quality of tone, especially when deep, and are soft and powerless. But their softness offers a very effective contrast to the more cutting qualities of the narrower open pipes and the noisy *compound stops*, of which I have already spoken, and which, as is well known, form a compound tone by uniting many pipes corresponding to a prime and its upper partial tones.

"Wooden pipes do not produce such a cutting windrush as metal pipes. Wooden sides also do not resist the agitation of the waves of sound so well as metal ones, and hence the vibrations of higher pitch seem to be destroyed by friction. For these reasons wood gives a softer, but duller, less penetrating quality of tone than metal.

"It is characteristic of all pipes of this kind that they speak readily, and hence admit of great rapidity in musical divisions and figures, but, as a little increase of force in blowing distinctly alters the pitch, their loudness of tone can scarcely be changed. Hence on the organ *forte* and *piano* have to be produced by stops, which regulate the introduction of pipes with various qualities of tone, sometimes more, sometimes fewer, now the loud and cutting, now the weak and soft. The means of expression on this instrument are therefore somewhat limited, but, on the other hand, it clearly owes part of its magnificent properties to its power of sustaining tones with unaltered force, undisturbed by subjective excitement.

"The mode of producing the tones on reed pipes resembles that used for the syren: the passage for the air being alternately closed and opened, its stream is separated into a series of individual pulses. This is effected on the syren, as we have already seen, by means of a rotating disc pierced with holes. In reed instruments, elastic plates, or tongues, are employed, which are set in vibration, and thus alternately close and open the aperture in which they are fastened."—*Helmholtz.*

"The construction of the syren and our experiments with that instrument are, no doubt, fresh in your recollection. Its musical sounds are produced by the cutting up into puffs of a series of air-currents. The same purpose is effected by a vibrating reed, as employed in the accordion, the concertina, and the harmonica. In these instruments it is not the vibrations of the reed itself which, imparted to the air, and transmittted through it to our organs of hearing, produce the music; the function of the reed is *constructive*, not *generative;* it moulds into a series of discontinuous puffs that which without it would be a continuous current of air."—*Tyndall.*

"We now proceed to investigate the *quality of tone* produced on reed pipes, which is our proper subject. The sound in these pipes is excited by intermittent pulses of air, which at each swing break through the opening that is closed by the tongue. A freely-vibrating tongue has far too small a surface to communicate any appreciable quantity of resonant motion to the surrounding air; and it is as little able to excite the air enclosed in pipes. The sound seems to be really produced by pulses of air, as in the syren, where the metal plate that opens and closes the orifice does not vibrate at all. By the alternate opening and closing of a passage, a continuous influx of air is changed into a periodic motion, capable of affecting the air. Like any other periodic motion of the air, the one thus produced can also be resolved into a series of simple vibrations. We have already remarked that the number of terms in such a series will increase with the discontinuity of the motion to be thus resolved. Now the motion of the air which passes through a syren, or past a vibrating tongue, is discontinuous in a very high degree, since the individual pulses of air must be generally separated by complete pauses during the closures of the opening. Free tongues without a resonance tube, in which all the individual simple tones of the vibration which they excite in the air are given off freely to the surrounding atmosphere, have consequently always a very sharp, cutting, jarring quality of tone, and we can really hear, with either armed or unarmed ears, a long series of strong and clear partial tones up to the 16th or 20th, and there are evidently still higher partials present, although it is difficult or impossible to distinguish them from each other, because they do not lie so much as a semitone apart. This whirring of dissonant partial tones makes the musical quality of free tongues very disagreeable. A tone thus produced also shows that it is really due to puffs of

air. I have examined the vibrating tongue of a reed pipe, like that in fig. 28,* when in action with the vibration microscope of Lissajous, in order to determine the vibrational form of the tongue, and I found that the tongue performed perfectly regular simple vibrations. Hence it would communicate to the air merely a simple tone, and not a compound tone, if the sound were directly produced by its own vibrations.

"The intensity of the upper partial tones of a free tongue, unconnected with a resonance tube, and their relation to the prime, are greatly dependent on the nature of the tongue, its position with respect to its frame, the tightness with which it closes, &c. Striking tongues which produce the most discontinuous pulses of air, also produce the most cutting quality of tone. The shorter the puff of air, the more sudden its action, the greater number of high upper partials should we expect, exactly as we find in the syren, according to Seebeck's investigations. Hard, unyielding material, such as that used for brass tongues, will produce pulses of air which are much more disconnected than those formed by soft and yielding substances."—*Helmholtz.*

Various qualities of tone are given to reed pipes by making them of different shapes, as pointed at the top, pointed at the bottom, covered with a cap, &c.

The broad characteristics of the principal orchestral instruments which produce their tones by means of sounding air-columns are admirably described by Dr. W. H. Stone in his little work "On Sound," and we now quote from him:—

"*Orchestral Wind Instruments* have already been de-

* See illustrations of reed pipes, *ante.*

scribed as regards their principle. Those in actual musical use are of three kinds.

1. Flutes.
2. Reeds.
3. Instruments with cupped mouthpieces.

"They all require two essential parts: I. the windchest; II. the embouchure.

"The windchest in this case is invariably the human thorax. The writer made a series of experiments some years ago for the purpose of determining what the pressures within the thorax actually were. A water gauge was connected with a small curved piece of tube by means of a long flexible indiarubber pipe. The curved tube being inserted in the angle of the mouth, did not, after a little practice, interfere with the ordinary playing of the instrument. The various notes were then sounded successively, and the height at which the column stood was noted. The following table of pressures was obtained as an average of many experiments:—

Table of Pressures.

Oboe	9 inches	to	17
Clarinet	15 ,,	,,	18
Bassoon	12 ,,	,,	24
Horn	5 ,,	,,	27
Cornet	10 ,,	,,	34
Trumpet	12 ,,	,,	33
Euphonium	3 ,,	,,	40
Bombardon	3 ,,	,,	36

"The *Flute* is an instrument of great antiquity, but not in the form in which it is now played. It acts like the ordinary open organ pipe, by driving a current of air from the lips against a thin edge. This edge in the modern form is fashioned in the side of a large lateral hole near the upper extremity. In the olden form, that of the flute *à bec*,

or flageolet, there is a fixed contrivance like the mouth of an organ pipe for producing the tone, and the wind is simply blown into the mouthpiece. There is, however, no reason why either of these systems should be adopted. The writer has particular pleasure in drawing attention to a reed flute brought from Egypt by his friend Mr. Girdlestone, of the Charterhouse, which exactly illustrates the stage of development of this instrument hitherto wanting. It is about fourteen inches long, possessing the usual six finger-holes, but the upper extremity or head is continuous. The top end is not stopped with a cork, as in the ordinary flute, but is thinned off to a feather edge, leaving a sharp circular ring at right angles to the axis of the bore. If this flute be held obliquely towards the right hand of the player, and the stream of wind from the all but closed lips directed against the opposite edge of the ring, a fair but somewhat feeble flute tone can be elicited.

"Here the mechanism is reduced to its very simplest form. It is, moreover, interesting to observe that the flute still played on by the peasants about the Nile is the counterpart of that to be seen distinctly on the Egyptian hieroglyphics of many thousand years ago.

"The *Oboe* or *Hautbois* is one of the very earliest instruments known; Mr. Chappell has succeeded in reproducing exact copies of real specimens found in the Egyptian tombs, and the writer has fitted reeds to them, by means of which a fair musical scale can be elicited. Beside the originals in the tomb lies usually a small piece of grass or reed, obviously intended to furnish the means of playing. In some cases this has been actually within the bore.

"The 'reed' or 'cane' now used for all reed instruments is formed of the outer siliceous layer of a tall grass, the *Arundo donax* or *sativa*, which grows in the south of Europe. This is fashioned in the oboe and bassoon into

a broad spatula-like form, with two thinned plates of the cane in close approximation to one another.

"It is therefore termed a double reed, in opposition to that of the clarinet and some other instruments where the vibrating plate of cane is single. It has been materially reduced in size of late years with a corresponding improvement in the tone of the instrument. Even as late as the visit of the composer Rossini to this country a reed resembling that of the bassoon was in use for the oboe.

"The bore of the oboe is conical, enlarging at the lower extremity into an expanded bell. Its scale is founded on the interval of the octave, beginning at the *B*♭ or *B*♮ of the four foot or small octave, and extending to *F* in alto in the twice accented, or one foot, or 9th octave.

"The *Clarinet* is an instrument of four-foot tone, with a single reed and smooth quality, commonly said to have been invented in 1690 at Nuremberg. It is probable, however, that in one form or another it existed long before. Its name is evidently a diminutive of *Clarino*, the Italian name of the trumpet, to which its tone has some similarity.

"The clarinet consists of a peculiar mouthpiece furnished with a single beating reed, a cylindrical tube terminating in a bell, with eighteen openings in the side, half of which are closed by the fingers, and half by keys. The lower scale comprises nineteen semitones, from *F* in the bass stave to B♭ in the octave above. The lowest note is emitted through the bell, the *G* of the two-foot octave through a hole at the back of the tube, peculiar to this instrument. This register is termed *Chalumeau*. By opening a lever above the back hole named above, the pitch is raised a twelfth, so that the *E* of the small or four-foot octave becomes the B♮ of that above. By the successive removal of fingers, fifteen more semitones are obtained,

reaching to high $C\sharp$, and above this note is another octave obtained by cross-fingering.

"The mouthpiece is a conical stopper flattened at one side to form the table for the reed, and thinned to a chisel edge on the other for the convenience of the lips. From the bore a lateral orifice is cut into the table which is closed in playing by the thin end of the reed. The table on which the reed lies, instead of being flat, is curved backwards towards the point, so as to leave a gap or slit about the thickness of a sixpence between the end of the mouthpiece and the point of the reed.

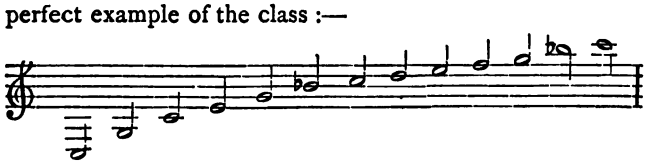
"Helmholtz has analysed the tone and musical character of the clarinet, as has been stated above. It stands apart from all other instruments, both in its quality, in its scale, founded on the twelfth, and according to the writer's experiments, in the wind-pressure required for its various registers. The clarinet is made in many keys, to meet difficulties of execution. This fact enables a very close approach to true intonation to be obtained on it, as described in the chapter on Temperament.

"The *Bassoon* is a double-reed instrument of eight-foot tone, as implied in its name; it being the natural bass of the oboe. In one form or another it is probably of great antiquity; though it is said to have been invented in 1539 by Afranio, a canon of Ferrara. A class of instruments named *bombards*, *pommers*, or *brummers*, seem to have been the immediate predecessors of the bassoon. It is a contrivance which has evidently originated in a fortuitous manner, developed by successive improvements of an empirical character. Various attempts have been made to give greater accuracy and completeness to its singularly capricious scale, but with only partial success. Its compass is from $B\flat$ in the contra or sixteen-foot, to $A\flat$ in the once accented or two-foot octave, but additional mechanism

has greatly raised the upper limit, so that the *C* or even the *F* above that note can be obtained.

"Like the oboe, it gives the consecutive harmonics of an open pipe.

"*Instruments with cupped mouthpieces* may be cited as the simplest musical instances of consonant tubes. They all consist essentially of an open conical tube, often of great length, in the French horn about 17 feet. The fundamental note of such a tube is consequently very deep. At the smaller extremity is the cup, forming an expansion of the bore, carrying a rounded edge against which the tense lips of the player are steadily pressed. The reed thus constituted is of the membranous kind, not dissimilar to the vocal cords of the human larynx. The method of its vibration is totally different from the reed of the oboe or clarinet: for whereas in these the lower harmonic notes are damped by the appended tube, and one of the higher and sweeter partials is reinforced; in the cupped instruments every successive harmonic from the very lowest is practicable, and all but the extreme bass sounds are actually used successively in producing the scale. The sequence of sounds is the harmonic series already given, modified slightly according to the particular instrument; it depends for its production entirely on the varied tension of the lips, and is commonly termed the scale of Open Notes. It is to bridge over the long gaps and intervals between these open notes that all systems of valves, slides, and keys are intended. The natural or open notes are as follows in the French Horn, which furnishes the most perfect example of the class :—



"It will be seen that in the lower part of the series the intervals between the sounds are large, but that the upper harmonics approach closer and closer together, so that from the middle B♭ a nearly perfect octave scale of continuous notes can be obtained. It has long been the custom to interpolate the missing semitones on the French horn by thrusting the hand into the bell, and so lowering the pitch by a variable quantity. The instrument is hence named the 'hand' horn, and the notes so modified hand-notes. Of late years, however, valves have been applied, as will be described in a subsequent paragraph.

"The *Trumpet*, speaking in a higher octave, possesses the first eleven open notes of the French horn. In this instrument, and in the trombone, its natural bass, a totally different and far more perfect system has been adopted for completing the scale. An U-shaped portion of the tube is made to slide with gentle friction upon the body of the instrument, so that the length of the bore can be increased and diminished by any given quantity within certain limits, at the will of the player. The note emitted can thus be lowered insensibly, and without abrupt changes through a variable interval. The absence of fixed notes enables the intonation to be guided at the will of the player, by accurate ear, exactly as is the case on the violin family."—*Stone.*

The leading points to be remembered by the student in connection with the vibration of air-columns are these:—that a column of air vibrates in segments corresponding with the segments of a vibrating string; that increased pressure has the same effect upon the waves as that of dividing and sub-dividing the string; that an open pipe has invariably a segment at each end, and produces the odd *and* even

tones of the harmonic series; that a stopped pipe invariably has a node at the closed and a segment at the open end, and makes only the odd vibrations of the series.

"The investigation of the reaction on the reed seems to throw some light upon that obscure subject, the production of musical vibrations in a pipe by a simple blast of air. In the ordinary mouth-piece of an organ-pipe a strong blast is forced through a very narrow slit, and is received upon a sharp edge, after which it partly enters the body of the pipe, partly passes into the external air. In the flute, a blast of air is directed by the lips of the flute-player upon the sharp edges of a hole in the tube, and then partly enters the tube. In both cases it appears necessary to suppose that the air, which enters the tube, bears a vibration or many kinds of simultaneous vibrations (which as mixed could not be distinguished from ordinary noise). The same supposition appears necessary to explain the sounding of a stretched wire opposite a chink of a door, or the singing of telegraph-wires, or the whistle of a locomotive (in which an annular jet of steam is thrown upon the circular edge of a bell, and excites the note peculiar to the bell). Every one of these vibrations may be considered as the vibration of a reed-tongue; and the reaction of the air in the pipe will modify these in the way which we have described for the reed. There appears to be only this possible difference; that these external air-vibrations have not that stubborn attachment to arbitrary times of vibration which the reed-tongue has, and therefore every one of them will be so changed as to correspond exactly to the vibration natural to the organ-pipe.

"A skilful flute-player, making no alteration in the fingering of the holes, but altering the character of his blast, can produce not only the first note but any one of several of its

harmonics. Here it appears to be necessary that the external vibrations should have an approximate similarity to those in the note which is to be produced; since the same mode of blowing which produces one note will not produce another.

"The matter, however, demands more complete explanation.

"It is scarcely necessary to say that the energy of the vibrations of the air in the organ-pipes is consumed in producing vibrations in the external air, either directly or indirectly through the vibrations in the sides of the pipes; and that, for the maintenance of the vibrations, a continued application of energy to the reed or the mouth-piece is necessary."—*Airy.*

"For experiments on pipes, we would recommend the student, having furnished himself with a tuning fork as above mentioned, to procure some pipes which admit of alteration of length, and which can also be stopped at pleasure. A light wooden pipe 1½ inch square and 18 inches long, open at both ends, with a tin pipe 12 inches long that will just slide in it, and with plugs for the two parts of the pipe, each plug managed by a long wire-stalk, will be found convenient. The student will be struck with the effect of a stopped pipe 6 inches or 18 inches long, or an open pipe 12 inches or 24 inches long, in resounding to a tuning-fork whose note without the pipe could scarcely be heard at all. Various openings in the side of the pipe will suggest themselves. This combination also facilitates the observation of the different intensities of sound produced by the tuning-fork as it is turned into different positions, and of the vanishing of the sound in some positions.

Among the special experiments on the vibrations of air in musical pipes, we know none so important as those by

the late Mr. Hopkins, published in the *Transactions of the Cambridge Philosophical Society*, vol. v. The vibrations of air at the mouth of the pipe were produced by the vibrations of a plate of glass, vibrating (apparently) in a known time: it was found that a small distance of the glass-plate from the pipe made the phenomena the same as if the pipe had been quite open to the external air (as the experimenter who uses a tuning-fork will also find), but with particular cautions in some instances the phenomena were made identical with those which belong to a closed pipe. The examination, however, in which Mr. Hopkins was most successful was the determination of nodal points; which was effected by gradually lowering into the tube a stretched membrane carrying a very small quantity of sand, and noting its place when the sand was not shaken by the air. It was thus found that the node next to the open mouth of the pipe was somewhat less distant from it than that given by theory; or, which amounts to the same thing, that the place where the air has always the same density as the external air is not exactly at the pipe's mouth, but somewhat exterior to it. The experiments, generally, were experiments on resonance; and in one of these, Mr. Hopkins appears to have fallen precisely on the case described in Article 81, where a very loud sound is produced. The whole of this paper, theory and experiment, is well worthy of the reader's attention."—*Airy*.

CHAPTER XI.

THE HUMAN VOICE.

THE human voice is essentially a reed instrument, for although it may be called a wind instrument for the reason that every sound-producing instrument sounds by reason of the air through which it makes its vibrations known, there is yet no other mode in which the human voice is produced than by the vibrations of what are, in reality, reeds suspended from the front to the back of the larynx. Let us consider—

I. The organs which essentially produce sound, and—

II. Those which only modify it.

I. The organ which actually produces voice is called the larynx, a mass of muscles and cartilages situated at the top of the windpipe. The length of the windpipe is between four and five inches, and it is connected at its lower extremity with the two lungs, with each of which it is joined by a branch. The following illustration (p. 239) will furnish a general view of the cartilages of the larynx as seen from behind.

The larynx is that prominence seen in the front of the throat under the chin, and which is known as "Adam's apple," that name having been given to it

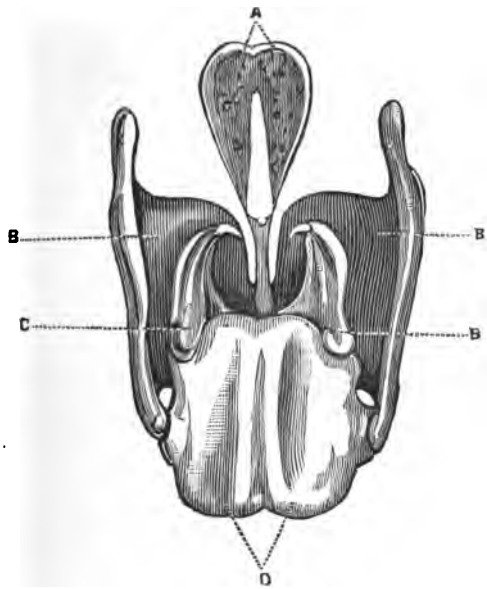


Fig. 95.

on account of the tradition which says that when our first progenitor partook of that unfortunate fruit it stuck in his throat.

The larynx consists of five principal cartilages: the epiglottis, the thyroid cartilage, the cricoid cartilage, and two arytenoid cartilages. The next figure (p. 240) gives a view of each of these cartilages separately.

The cartilages just enumerated, and called five, are

in reality nine. There are, for instance, two arytenoid cartilages, two "small horns of the larynx" (*cornicula*

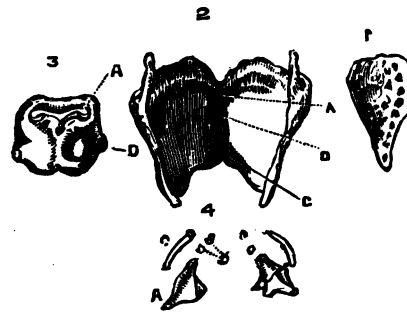


Fig. 96.

laryngis) or Santorini cartilages, and the cuneiform cartilage of Wristberg. The two cornicular and cuneiform cartilages are, as will be seen from the figure, very much smaller than the five principal cartilages above referred to. The epiglottis, fig. 95, stands highest of all, and occupies an erect position against the back of the tongue, in front of and above the rest of the larynx. It is in shape something like a leaf, as the engraving shows.

The thyroid cartilage (*thyreos*, a shield) is the largest of all, and constitutes really the front and sides of the larynx. Its two extremities widen, and are both above and below continued as horns, four in number.

In front it is, so to speak, jointed together for about half its length, and contains that elastic material to which on the inside the vocal cords are attached.

The cricoid cartilage (*krikos*, a ring), as its name will lead us to infer, is shaped like a ring, and lies at the bottom of the larynx, and forms a support for the whole structure. It is held by the lower horns of the thyroid cartilage, by two short fibres, and a sort of hinge is thus formed. The back part of the cricoid cartilage projects upwards into the open portion of the thyroid.

The arytenoid cartilages (*arutaine*, a pitcher) are fixed on the two smooth projections behind the upper edge of the cricoid. They are something like a pyramid in shape, but the sides which form their bases are at right angles to each other. These are called the *processus vocales*, or vocal processes, and to these are attached the vocal cords.

The vocal cords, or bands, are of two kinds: the true, which are usually called the vocal cords, and the false, called the superior vocal cords.

The true vocal cords are two membranous folds which project on each side toward the middle line, and which are in the rear attached to the *processus vocales* which belong to the arytenoid cartilage.

The false vocal cords are two folds of mucous membrane of the same general shape as the true vocal cords; behind they are attached to the arytenoid cartilage, and above and on the front to the thyroid, just over the place where the true vocal cords are inserted. The near edges of the false vocal cords do not so nearly approach the central line as do the true. Between the cords, on each side, is an open-

ing to the ventricle of the larynx—a small cavity for the secretion of mucous membrane.

The muscles which are engaged in the motions of the larynx—at any rate, those of them which assist in the formation of voice—will be best recognised by the following illustration :—

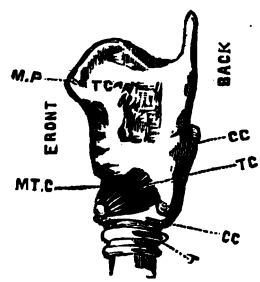


Fig. 97.

This is a side view of the larynx, from the right side. T C is the thyroid cartilage; M P, Adam's apple; C C, cricoid cartilage; T C, crico-thyroid muscle; M T C, the crico-thyroid membrane; T, the trachea or windpipe.

The nerves of the larynx are the superior laryngeal nerve, and the inferior or recurrent laryngeal nerve. Each of these is a branch of the pneumogastric nerve, from which the superior laryngeal nerve arises, just below the point where the pneumogastric leaves the skull, and divides into the external and internal laryngeal; the first sending branches to control the act of swallowing, and the second supplying the arytenoid muscle and the lining of the larynx. The internal laryngeal nerve branches off from the pneu-

mogastric, just after the latter enters the cavity of the chest. The inferior laryngeal nerve bends beneath the aorta and runs upwards to the larynx, every movement of the muscles of which is directed from the medulla oblongata, along the inferior laryngeal down into the chest and then upwards to the larynx. There are abundant reasons for this apparently objectless excursion of a nerve from the back of the neck, down as low as the heart, and back to the throat again; but as these reasons are connected with the science of physiology more than with that of acoustics, we shall leave those of our readers, who may feel curious on the point, to study for themselves works which are specially written with physiological ends in view. The inferior laryngeal nerve supplies all the muscles of the larynx, except the crico-thyroid.

We will now explain the movements of the various muscles of the larynx.

The crico-thyroid turns the front of the thyroid and the cricoid both downwards and forwards, and this forward and downward movement of the thyroid cartilage lengthens the vocal cords. The higher notes of the voice are produced, of course, by increased tension of the vocal cords, and the act of singing a high note diminishes the space between the thyroid and the cricoid; in other words, the thyroid is pulled downwards and forwards, as we have seen, on the cricoid by the crico-thyroid muscles (which take their name from this action), and by so doing stretches the vocal cords and produces the higher note.

The thyro-arytenoid opposes the crico-thyroid, and relaxes the vocal cords, by pulling the thyroid upwards and forwards, the upper and outer part pressing on the ventricle of the larynx and tending to empty it; the other portion straining and tightening the edge of the vocal cords, after the various muscles have fixed the thyroid and arytenoid. It is most likely this muscle which produces the difference between the quality of the head notes and the falsetto notes.

It is an open question whether the crack in the voice which sometimes happens when the singer is trying to produce a high note, is due to the over-straining of this muscle, or to the production of nodes by the touching of the vocal cords; the latter is the most likely.

The arytenoid muscle draws the arytenoid cartilages together, and hinders them from turning round.

The aryteno-epiglottidean muscle embraces the entire air tube, and draws the arytenoid cartilages together and forwards. The thyro-epiglottidean muscles are those by means of which the epiglottis is drawn on to the top of the vocal cords in the act of swallowing.

The action of the posterior crico-arytenoid muscle is represented in the following diagram (p. 245), in which the dotted lines show the action of the muscle. They pull backwards and downwards the outer portion of the arytenoid cartilages, and by turning them

(c c) on their axes (a a) divide the front angles, so as to separate the vocal cords.

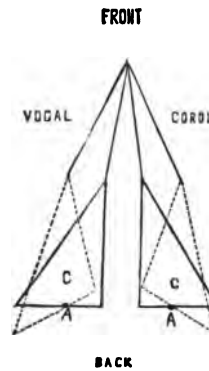


Fig 08

The lateral crico-arytenoids act in just the opposite direction, and pull forward the upper portions of the arytenoids, and turning them (c c) on their axes (a a) bring the vocal cords nearer together. Their action is represented in the following figure (p. 246).

The arytenoid muscle, above referred to, quite obliterates the triangular space between the cords.

The sterno-thyroids (so called because they issue from the sternum or breastbone) are inserted into the side of the thyroid, which cartilage they pull down, and help to steady the vocal cords.

The thyro-hyoid muscles arise from the hyoid bone at the base of the tongue, and are fastened to the sides of the thyroid, which cartilage they pull

upwards and turn round, thus loosening the vocal cords.

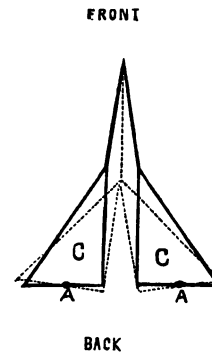


Fig. 99.

These are the separate parts which, by their combined action, produce the human voice.

The following admirable description of the action of the various parts of the larynx, together with the illustrations, are copied from Stainer and Barrett's Dictionary of Musical Terms:—

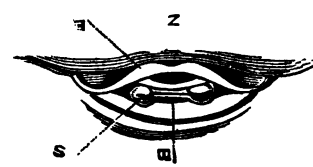


Fig. 99a.

Quiet breathing. Wide Glottis. Arytenoids apart and depressed. Epiglottis falling back so as to obscure the view into the larynx.

The same, but epiglottis raised by the pronunciation of *a* as in "fate," or *ee* as in "green," but the

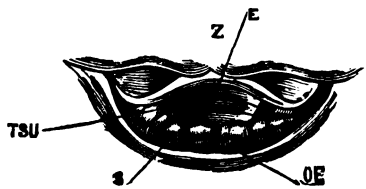


Fig. 99b.

actual sounding of the latter makes the tongue rise so high as to obscure the view.

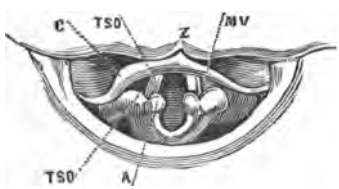


Fig. 99c.

The preparations for sounding the voice after quiet breathing, the process stopped halfway. The arytenoids project and approach one another with free and rapid movements. The glottis is narrowed.

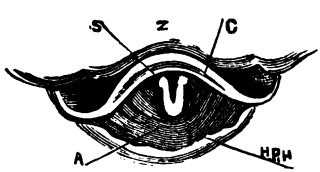


Fig. 99d.

Position during a deep chest note. The epiglottis lies back and obscures the view of the vocal cords.

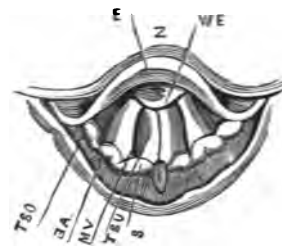


Fig. 99e.

Position during a very high note. Glottis very narrow, all the parts very tense, arytenoid cartilages, aryteno-epiglottidean folds, and epiglottis, forms a sort of additional tube above the floor of the larynx. In the highest possible notes, the epiglottis cushion is pressed on the front insertion of the vocal cords, shortening their vibrating length.

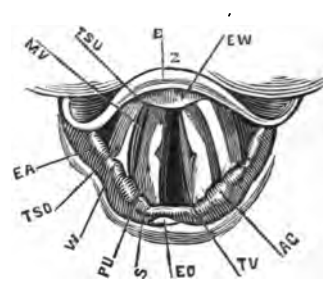


Fig. 99f.

Position of parts on taking a deep breath after

singing a high note. All the parts are relaxed and appear thicker, the arytenoid cartilages move apart, the *processus vocales* are turned out, the glottis is larger and diamond shaped.

Position in quiet breathing. The same as fig. 99f, but exaggerated. The glottis still larger, large enough to easily admit a finger. The parts do not move during quiet breathing with inspiration or expiration.

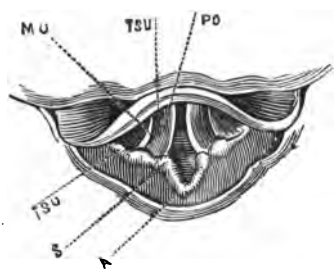


Fig. 99g.

Position during whispering. The arytenoids are seen near together, but not so near as the *processus vocales;* these last are, however, too far apart to cause a vocal sound. The *processus vocales,* being closer together than the rest of the arytenoids, produce a form of glottis the opposite of that shown in fig. 99f, namely, one approaching to that of two isosceles triangles, with their apices opposed; the whisper becomes louder as the *processus vocales* approach one another, until at last all that remains is

a triangular space (the hinder of the two triangles) between the arytenoids. In the louder, hoarser whisper, the cushion of the epiglottis presses on the front part of the vocal cords, and additionally prevents their being thrown into vibrations, though while any chink remains this cannot happen.

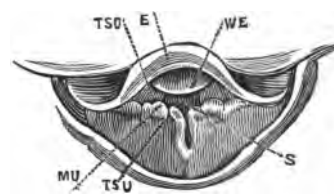


Fig. 99h.

Position in air-tight closure of the glottis; the process stopped halfway. The arytenoid and the vocal cords are firmly opposed, the false vocal cords are being approximated, the epiglottis with its cushion is being pressed down on the glottis.

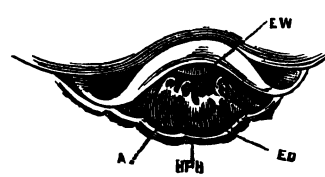


Fig. 99i.

The state of complete closure. The epiglottis pressed firmly on the glottis. The false vocal cords probably, the true vocal cords certainly, closely opposed. When the epiglottis is still farther pressed

back, we have a view similar to that during the sounding of a deep chest note (fig. 99e), except that a small space exists in the latter case, between the epiglottis and arytenoids, for the passage of air.

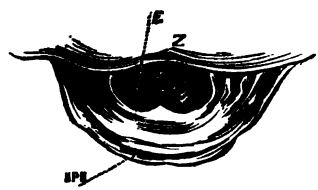


Fig. 99k.

Position at the commencement of the act of swallowing.

The voice is not like the trumpet or the trombone, instruments in which sounds are produced merely by the vibration of air in a tube, but its tones are produced by the passage of the air across the vocal cords, which by their greater or less tension produce high or low notes at the will of the singer or speaker. The following paragraph from the article in Stainer and Barrett, just referred to, is instructive:—

"The reed of the human voice differs from ordinary reeds in not being a stiff lamina fixed at one end, freely vibrating at another; it is a stretched membrane. Membranous tongues made elastic by tension may have three forms: 1st, a stretched band in an interval between two firm plates, leaving a chink on each side; 2d, it may be stretched

over part of the end of a tube, the other part being occupied by a solid plate, a narrow chink being left between the free edges; 3d, two elastic membranes may be stretched over the mouth of a tube leaving a chink between them. The last is obviously the case in point. But if the membranes are prolonged in a direction parallel with the current of air, not their edges only, but their whole surfaces are thrown into vibration. This resembles the larynx still more closely. An instrument on these principles has been constructed, and corresponds very closely in its behaviour with that of the larynx. In such an apparatus, pitch depends on the length, tension, and thickness of the membranes, and though their edges must be close together to produce sounds, the size of the chink has nothing to do with the pitch. A lower note is formed from a pair of such membranes than from one, their tension is heightened by increasing the strength of the current of air, thus they differ from rigid reeds in which the note is lowered by a similar proceeding."

The human voice is, therefore, produced by a pair of free reeds, with a resonance tube affixed. The mouth, pharynx, and all the spaces and air passages above the larynx forming part of the resonance tube, and this resonance tube plays a most important part in singing or speaking.

II.—The Organs which Modify Sound.

"The tongue, lips, and teeth," says Dr. Gordon Holmes, "superadded to the oral portion of the vocal tube, render possible articulate speech. The tongue and lips are almost wholly composed of muscular

substance, which, being capable of infinite combinations of contractions, bestows on them a great versatility of motion. They, with the soft palate and lower jaw, may be considered as the active organs of articulation. The teeth take a passive part, acting as a kind of fulcrum for the tongue and lips."

The organs which assist in the production of voice are the thorax, or chest; the pharynx, which extends from the top part of the lungs to the base of the skull; the mouth, which, of course, requires no special description; the cavity of the nose, which is an assemblage of six small tubes; the soft palate, which is attached to the back of the hard palate, and hangs down towards the back of the tongue, separating the pharynx from the mouth; and the lower jaw, which assists in augmenting the power of the voice by opening or shutting the mouth, so as to vary its capacity as a resonator. The following figure (p. 254) represents the relative positions of the organs of the voice.

A.—Cavities in bones of the head. B.—The channels of the nose. C.—Entrance of eustachian tube leading to the ear. D.—The uvula. E.—Pillars of the fauces with the tonsils between. F.—Epiglottis. G.—Thyroid cartilage. H.—Cricoid cartilage. J.—Ventricle of larynx. K.—Vocal band. L.—The windpipe. M.—The œsophagus or gullet.

The column of air rushing out of the lungs through the windpipe is converted into sound by

the vocal cords, without which man would be voiceless, or at best could speak only in whispers

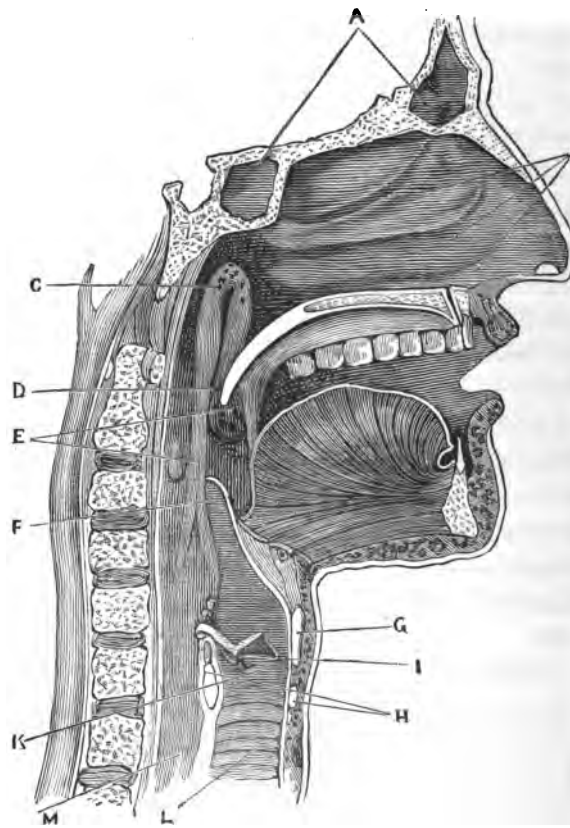
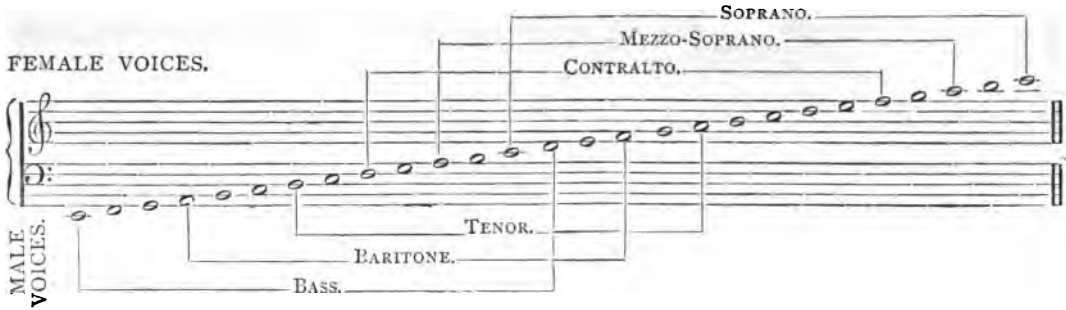


Fig. 100.

The comparative compass of the voices of men and women are represented in the following diagram. See also Appendix E.

COMPARATIVE SCALE OF DIFFERENT CLASSES OF VOICES.



NAMES OF NOTES— ENGLISH AND GERMAN.

E F G A B c d e f g a b c′ d′ e′ f′ g′ a′ b′ c″ d″ e″ f″ g″ a″ b″ c‴

FRENCH.

mi fa sol la si ut re mi fa sol la si ut re mi fa sol la si ut re mi fa sol la si ut

1 1 1 1 1 2 2 2 2 2 2 2 3 3 3 3 3 3 3 4 4 4 4 4 4 4 5

VIBRATIONS *—

80. 85. 96. 107. 120. 128. 144. 160. 170. 192. 214. 240. 256. 288. 320. 341. 384. 427. 480. 512. 576. 640. 682. 768. 854. 960. 1024.

* The vibrational numbers here given are those of the so-called natural scale, which assumes the lowest sound audible as a musical note, to have sixteen vibrations per second—fractions are omitted. In the French notation the above numbers would be doubled, as in France the to-and-fro motions of a vibrating body are reckoned separately.

The human voice, like most musical instruments, is not a simple tone, but is always attended with a greater or less number of upper partials. These were heard, without any artificial help, so long ago as 1726, by Rameau.

"Get a powerful bass voice to sing *e*♭ to the vowel O, in *more* (more like *aw* in *maw* than *ow* in *mow*), gently touch *b'*♭ on the piano, which is the Twelfth, or third partial tone of the note *e*♭, and let its sound die away while you are listening to it attentively. The note *b'*♭ on the piano will appear really not to die away, but to keep on sounding, even when its string is damped by removing the finger from the digital, because the ear unconsciously passes from the tone of the piano to the partial tone of the same pitch produced by the singer, and takes the latter for a continuation of the former. But when the finger is removed from the key, and the damper has fallen, it is, of course, impossible that the tone of the string should have continued sounding. To make the experiment for *g''* the fifth partial, or major Third of the second Octave above *e*♭, the voice should sing to the vowel A in *father*."—*Helmholtz.*

"The most perfect of reed instruments is the organ of voice. The vocal organ in man is placed at the top of the trachea or windpipe, the head of which is adjusted for the attachment of certain elastic bands which almost close the aperture. When the air is forced from the lungs though the slit which separates these *vocal cords*, they are thrown into vibration; by varying their tension, the rate of vibration is varied, and the sound changed in pitch. The vibrations of the vocal cords are practically unaffected by the resonance of the mouth, though we shall afterwards learn that this resonance,

by reinforcing one or the other of the tones of the vocal cords, influences in a striking manner the quality of the voice. The sweetness and smoothness of the voice depend on the perfect closure of the slit of the glottis at regular intervals during the vibration.

"The vocal cords may be illuminated and viewed in a mirror placed suitably at the back of the mouth. Varied experiments of this kind have been executed by Sig. Garcia. I once sought to project the larynx of M. Czermak upon a screen in this room, but with only partial success. The organ may, however, be viewed directly in the laryngoscope; its motions, in singing, speaking, and coughing, being strikingly visible. The roughness of the voice in colds is due, according to Helmholtz, to mucous flocculi, which get into the slit of the glottis, and which are seen by the means of the laryngoscope. The squeaking falsetto voice with which some persons are afflicted, Helmholtz thinks, may be produced by the drawing aside of the mucous layer which ordinarily lies under and loads the vocal cords. Their edges thus become sharper, and their weight less; while their elasticity remaining the same, they are shaken into more rapid tremors. The promptness and accuracy with which the vocal cords can change their tension, their form, and the width of the slit between them, to which must be added the elective resonance of the cavity of the mouth, renders the voice the most perfect of musical instruments." —*Tyndall.*

"On the other hand, in the *larynx* the tension of the vocal cords which here form the membranous tongues, is itself variable, and determines the pitch of the tone. The air chambers connected with the larynx are not adapted for materially altering the tone of the vocal cords. Their walls are so yielding that they cannot allow the formation

of vibrations of the air within them sufficiently powerful to force the vocal cords to oscillate with a period which is different from that required by their own elasticity. The cavity of the mouth is also far too short, and generally too widely open to serve as a resonance chamber which could have material influence on the pitch.

"In addition to the tension of the vocal cords (which can be increased not only by separating the points of their insertion in the cartilages of the larynx, but also by voluntarily stretching the muscular fibres within them), their density seems also to be variable. Much soft watery inelastic tissue lies underneath the proper elastic fibrils and muscular fibres of the vocal cords, and in the breast voice this probably acts to weight them and retard their vibrations. The head voice is probably produced by drawing aside the mucous coat below the cords, thus rendering the edge of the cords sharper, and the weight of the vibrating part less, while the elasticity is unaltered."—*Helmholtz.*

"The sound issuing from the larynx is produced under circumstances the most favourable possible for the production of overtones, and experiment shows them to be plentifully present. In a powerful, clear bass voice the harmonics can be clearly heard, by the aid of resonators, up to the sixteenth or the fourth octave above the fundamental; and in forced notes harmonics may be produced reaching up to nearly the highest notes of a modern pianoforte.

"The strength of the overtones, particularly of the higher ones, is subject to much variation in voices of different quality; being more powerful in clear and sharp voices than in weak and dull ones. The great differences in quality that are found, by everyday observation, to exist

between different voices may lie partly in this, and partly in other anatomical and physiological vibrations of structure and action that are difficult to describe, although Helmholtz makes an attempt at the description. If we could hear the tone of the larynx alone, unaffected by the resonance of the air passages, we should probably find that the loudness of the harmonic tones diminished gradually and evenly upwards, like those from any other reed; and in some vowels, such as the German *ä*, in which the mouth assumes a funnel shape, this is tolerably near the actual effect.

"In the majority of cases, however, the strength of the overtones is much varied by modifications in the form and contents of the air spaces; for in proportion as these are altered, either by the lips, the jaws, or the tongue, so the resonance is made to correspond with special notes; the consequence of which is to strengthen the overtones which approach nearest to these, and to damp the others. By experiments with resonators the first six or eight overtones are always found present, but in greatly varied strength, according to the position of the mouth, sometimes piercingly sharp in the ear, sometimes scarcely perceptible."—*Pole.*

"In order to understand the composition of vowel tones, we must, in the first place, bear in mind that the source of their sound lies in the vocal cords, and that when the voice is heard, these cords act as membranous tongues, and, like all tongues, produce a series of decidedly discontinuous and sharply separated pulses of air, which, on being represented as a sum of simple vibrations, must consist of a very large number of them, and hence be received by the ear as a very long series of partials belonging to a compound musical tone. With the assistance of resonators it is possible to recognise very high partials, up to the six-

teenth, when one of the brighter vowels is sung by a powerful bass voice at a low pitch, and, in the case of a strained forte in the upper notes of any human voice, we can hear, more clearly than on any other musical instrument, those high upper partials that belong to the middle of the four times accented octave (the highest on modern pianofortes), and these high tones have a peculiar relation to the ear, to be subsequently considered. The loudness of such upper partials, especially those of highest pitch, differs considerably in different individuals. For cutting, bright voices, it is greater than for soft and dull ones. The quality of tone in cutting, screaming voices may perhaps be referred to a want of sufficient smoothness or straightness in the edges of the vocal cords, to enable them to close in a straight narrow slit without striking one another. This circumstance would give the larynx more the character of striking tongues, and the latter have a much more cutting quality than the free tongues of the normal vocal cords. Hoarseness in voices may arise from the glottis not entirely closing during the vibrations of the vocal cords. At any rate, when alterations of this kind are made in artificial membranous tongues, similar results ensue. For a strong and yet soft quality of voice, it is necessary that the vocal cords should, even when most strongly vibrating, join rectilinearly at the moment of approach with perfect tightness, effectually closing the glottis for the moment, but without overlapping or striking against each other. If they do not close perfectly, the stream of air will not be completely interrupted, and the tone cannot be powerful. If they overlap, the tone must be cutting, as before remarked, because arising from striking tongues. On examining the vocal cords in action by means of a laryngoscope, it is marvellous to observe the accuracy with which they close, even when making vibrations occupying nearly the entire breadth of the cords themselves."—*Helmholtz.*

CHAPTER XII.

BEATS.

If the student will turn to the end of the chapter in which Helmholtz's theory of musical quality is explained, he will find that the form of a wave changes when one simple wave is superposed upon or added to another. If, therefore, a crest of a particular size and form coincides with another exactly like it, the result will be a crest double the height of each one, as shown in the following figure :—

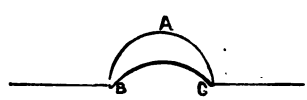


Fig. 101.

If a crest coincides with a trough, the result will be that the one will nullify the other, as in the following figure, the thick line showing the result :—

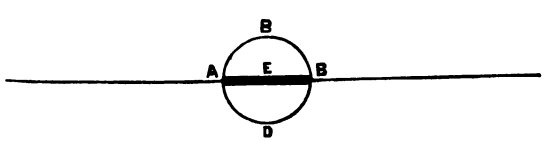


Fig. 102.

This is exactly what occurs when a tuning-fork, having been sharply struck, is turned round with the handle between the thumb and finger so that the prongs shall be near to the ear. Both prongs are sounding, but at certain points—four times in each complete revolution of the fork—the sound will lapse into silence, the tone of one prong destroying that of the other. At those points of the circuit which are midway between these points of silence, the sound will be augmented. The technical explanation of this simple phenomenon is, that at the points of augmentation the waves of the one prong are *added to* those of the other, whereas at the points of diminution the waves of the one are *subtracted from* the other; the result in the first case being a doubling of the tone, and in the last its complete destruction. It may seem paradoxical to say that one sound silences another, but this will not appear at all strange if it is remembered that sound is not something tangible thrown into the air by the motion of the vibrating body, but is merely a *form of motion* imparted to the air by such vibrations, and when one form is added to another there will be an augmentation of the amplitude of the wave, and equally when one form is subtracted from another, there will be a diminution of the amplitude. The following figure shows (p. 263) the points of increased sound and the points of diminution. A, B, are the ends of the fork; D, D, E, and E, the points of diminution; and 1, 2, 3, and 4, the points of augmentation:—

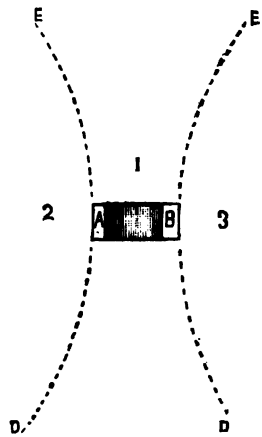


Fig. 103.

Another instance in which one sound will cancel another is that of two stopped organ pipes of exactly the same pitch, so placed that the air from the one passes into the other. Separately, the tone of each is clear; blown together, there is no sound heard, the waves of the one streaming into the other, and a listener at the end hears only the rushing of the air. That the conditions which produce sound are all present may be demonstrated by conveying a tube from the mouth of either of the pipes to the listener's ear, when its tone will be distinctly heard. In the case of the fork, the alternate augmentation and diminution produce a throbbing or beating, and the throbs are hence called *beats.* Beats of this class may be called *beats of simple tones of the same pitch.*

If two simple tones, varying very slightly in pitch,

are sounded together, the phenomenon arises which is now to be explained. Strike together on the pianoforte two notes a semitone apart, for instance, C and C sharp, or E and F, and the result will be a certain roughness of tone. This roughness is caused by the fact, speaking broadly, that the vibrations of the two notes *interfere with each other.* Let us, for the sake of an illustration, take two notes whose vibration numbers are nearer together than half a tone, vibrating respectively one hundred and one hundred and one times per second. When the first of these has made fifty vibrations the other will have made fifty and a half, and the first will be at a point of condensation while the second is at a point of rarefaction. At the hundredth vibration of the first tone, the second will have made one hundred and one vibrations. The one tone is, therefore, at this point exactly one vibration in advance of the other; but their condensations now occur at precisely the same moment, that is to say, the crest of the one wave will correspond with the crest of the other, and there will be a consequent augmentation of sound, whereas when each had made half its journey (fifty and fifty and a half vibrations respectively), the rarefaction of the half vibration nullified the effect of the condensation of the other, and a diminution of sound was the result. Thus there will occur once in every second one augmentation and one diminution, the one tone making, according to our hypothesis, one vibration per second more than the other. This law, therefore,

holds good, that *when two sounds produce beats, there are always as many beats per second as there are vibrations of the one tone per second more than the other;* or, more briefly, the difference between the vibration numbers of any two beating tones gives the number of beats per second. The following diagram (p. 266) illustrates graphically the beats of two simple tones, one of which makes twelve vibrations, and the other thirteen per second. Small numbers are used for the sake of simplicity, and because, if a very large number of vibrations were taken, they would have to be drawn on too small a scale to enable the student to notice the difference. From this figure it will be seen that the thick line gradually falls into the rear, and at the end of half a second completes its sixth vibration, while the thin line makes half its seventh, and that the two waves are therefore in opposition to each other.

The same figure reversed shows what goes on during the last half second, viz., that the operation is exactly reversed, the two sounds gradually increasing their power of reinforcing each other until the maximum augmentation of tone is reached; and if two such figures were joined together at A, we should have a full view of what would take place at the expiration of each second, with two simple tones beating respectively twelve and thirteen times per second.

"The law determining the number of beats in a second for a given imperfection in a consonant interval, results immediately from the law above assigned for the beats of simple tones. When two simple tones, making a small

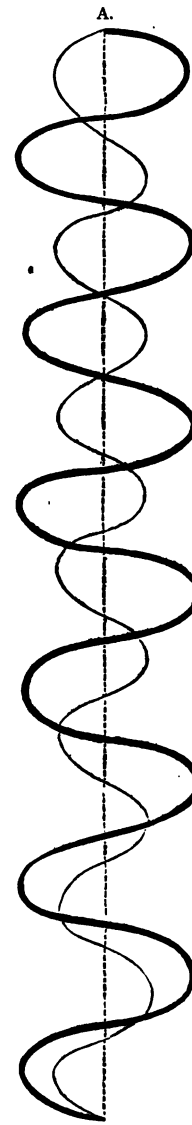


Fig. 104.

interval, generate beats, the number of beats in a second is the difference of their vibrational numbers. Let us suppose, by way of example, that a certain prime tone has the vibrational number 300. The vibrational number of the primes which make consonant intervals with it, will be as follows :—

Prime Tone = 300.			
Upper Octave	= 600	Lower Octave	= 150
,, Fifth	= 450	,, Fifth	= 200
,, Fourth	= 400	,, Fourth	= 225
,, Major Sixth	= 500	,, Major Sixth	= 180
,, Major Third	= 375	,, Major Third	= 240
,, Minor Third	= 360	,, Minor Third	= 250

"Now assume that the prime tone has been put out of tune by one vibration in a second, so that its vibrational number becomes 301, then calculating the vibrational number of the coincident upper partial tones, and taking their difference, we find the number of beats thus :—

Interval upwards.	Beating Partial Tones.		Number of Beats.
Prime . .	1 × 300 = 300	1 × 301 = 301	1
Octave . .	1 × 600 = 600	2 × 301 = 602	2
Fifth . .	2 × 450 = 900	3 × 301 = 903	3
Fourth . .	3 × 400 = 1200	4 × 301 = 1204	4
Major Sixth .	3 × 500 = 1500	5 × 301 = 1505	5
Major Third .	4 × 375 = 1500	5 × 301 = 1505	5
Minor Third .	5 × 360 = 1800	6 × 301 = 1806	6

Interval downwards.	Beating Partial Tones.		Number of Beats.
Prime . .	1 × 300 = 300	1 × 301 = 301	1
Octave . .	2 × 150 = 300	1 × 301 = 301	1
Fifth . .	3 × 200 = 600	2 × 301 = 602	2
Fourth . .	4 × 225 = 900	3 × 301 = 903	3
Major Sixth .	5 × 180 = 900	3 × 301 = 903	3
Major Third .	5 × 240 = 1200	4 × 301 = 1204	4
Minor Third .	6 × 250 = 1500	5 × 301 = 1505	5

"Hence the number of beats which arise from putting one of the generating tones out of tune to the amount of one vibration in a second, is always given by the two numbers which define the interval. The smaller number gives the number of beats which arise from increasing the vibrational number of the upper tone by 1. The larger number gives the number of beats which arise from increasing the vibrational number of the lower tone by 1. Hence if we take the major sixth *c a*, having the ratio 3 : 5, and sharpen *a* so as to make one additional vibration in a second, we shall have 3 beats in a second; but if we sharpen *c* so as to make one more vibration in a second, we obtain 5 beats in a second, and so on."—*Helmholtz.*

Referring to the two sounds above-mentioned—the one making one hundred, the other one hundred and one vibrations per second—there will, therefore, be once in every second a point of maximum strength and a point of minimum strength. These two, or more correctly, the former alone, constitute what is called the *beat.* These beats may very easily be made perceptible to the ear by fastening a small piece of sealing-wax to one of the prongs of a tuning-fork, and by increasing its weight and retarding its rate of vibration, making the one prong move more slowly than the other, and thus producing those alternate points of maximum and minimum strength.

The greater the weight added to the prong of the fork, the quicker will the beats become; and the reason of this is obvious, viz., that the difference between the rates of vibration being increased more and more

as more weight is added to the one prong, more points of maximum and minimum strength per second are reached, *i.e.*, more *beats* are made. Any two tones will occasion beats, whose distance from each other is not too great to permit of the ear taking in and recognising each beat. This limit is reached at about three semitones in the middle register of the pianoforte. If the experiment is made with notes in a higher register, it follows, of course, that the vibration numbers being already larger at starting than in the lower registers, the difference between them is proportionately larger than it is with lower tones, although their ratio to each other remains the same as before. Thus two tones, vibrating to each other as 50 : 51, will produce one beat per second, and so will two vibrating to each other as 400 : 401; and though the relative distance between the pairs would be exactly the same, the *absolute* distance to which it would be necessary to separate the lower pair to produce another beat per second would be a great deal more than would be necessary to make an additional beat between the higher tones; or, to put it in another way, the actual difference of pitch between two notes vibrating to each other as 50 : 52, would be very much greater than between two vibrating as 400 : 402, although, in each case, the beats would be two per second.

Beats still continue to take place even when they become too rapid for the ear to perceive them sepa-

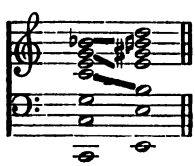
rately, but on this point we shall have more to say in the chapter on Consonance and Dissonance.

From what has been said, it will not be difficult to gather that the phenomena of beats can be studied to much greater advantage with low notes than with high ones. It is a well-known fact that in tuning pianofortes on the system called "equal temperament," the tuner, who is making his intervals swerve aside from strict accuracy, has absolutely nothing to guide him but the number of beats which his flattened or sharpened interval makes; and, starting with a perfect third or fifth, he flattens or sharpens his interval by exactly the number of beats required.

The great principle to be remembered in connection with the subject of beats is this:—that two tones which produce beats are the result of two sets of vibrations, one set travelling faster than the other, and their varying rates bring about, at regular intervals, first an access of tone when the crests of the waves correspond, and a diminution of tone when the crest of one coincides with the trough of the other.

Besides the beats of simple tones, there are beats produced between the upper partials of tones, whenever those upper partials are within beating distance of each other; that is to say, though two tones may perfectly correspond so far as their primes are concerned, there may be present upper partials which, being within beating distance of each other, will produce beats; and although these last are not always discernible by the ear, they can, when present, always

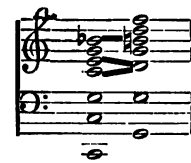
be detected by the use of suitable resonators. For instance, the notes C and E are not within beating distance of each other, but if they are compound tones, and there are present the first six upper partials in each tone as well as the prime, it will be seen, from the following diagram, that the third upper partial of C and the second of E will be within beating distance, so also will the fifth of C be within beating distance of the fourth of E, and the sixth of C within beating distance of the fifth of E.



Each of these pairs will therefore produce beats with each other, and although these beats, being situated in the higher regions of the tone, do not affect the consonance of their primes, they will yet affect the consonance of the upper partials, and there will be a certain amount of roughness when any tone is sounded together with its major third, both being of the same quality, that is, having the same number and the same intensity of upper partials.

If we take C and G, which form, except a note and its octave, the nearest consonant intervals to absolute unison, we shall find that though there are no beats between the primes there are between some of the upper partials. The diagram illustrates this,

the upper partials which are within beating distance being connected by lines:—



The next chapter contains diagrams which further illustrate the beats of upper partials, but the student is recommended to form them for himself, and join together by lines those which are within beating distance of each other. It will be seen in the next chapter that these beats between the upper partials of the different notes of the scale have a distinct effect upon their consonance.

The following extracts will serve to illustrate more fully the theory of beats:—

"We have alluded, in describing the experimental observations of Concords, to Beats. In the case of beats observed during the operation of bringing one string into unison with another, the explanation is simple. Suppose that while one string makes 100 vibrations the other makes 101 vibrations; or, which is the same thing, suppose that while there are 100 waves from one source there are superposed upon them 101 equal waves from another source. When the two waves are exactly in the same phase, one wave increases the other, and the amount of agitation produced is very great. But after 50 waves from the first source have passed, there have passed 50½ waves from the second source: the two waves are now in opposite phases: an advance of particles of air produced by one

wave is neutralised by a retreat of the same particles produced by the wave from the other source, and the particles are left absolutely at rest. And this rest continues sensibly through several waves. After this, the relative position of the two interfering waves changes, they begin to produce a real result, and after 100 waves from the first source the two waves are again united in the greatest force. The same change goes on in every successive 100 waves. Suppose that the first string under consideration produces 400 vibrations in a second of time. Then if the second string gives 101 waves for 100 of the first, their combination will produce a strong sound and a weak sound four times every second: if their tones are brought nearer, so that the second string gives 201 waves for 200 of the first, there will be a strong sound and a weak sound twice every second: if they are adjusted still more nearly so that 801 waves of the second string correspond to 800 of the first, there will be a strong and a weak sound every two seconds of time. These are the beats of notes nearly in unison."—*Airy.*

Dr. Pole, in an appendix to his "Philosophy of Music," gives this lucid exposition of the practical uses to which beats are put:—

"It will be gathered from the description in the text, that beats may arise (1) from two fundamental sounds that are nearly in unison: this kind may be called the *unison beat;* or (2) from two fundamental sounds which lie wider apart, but the overtones of which approach each other within beating distance: this kind may be called the *overtone beat.* The nature of the beats in each of these cases, and the rules governing their velocity, have been sufficiently illustrated by the examples given.

"There is, however, a third kind of beat which was pointed

out by the celebrated mathematician, Dr. Smith of Cambridge, in his learned work on Harmonics, published in 1749, and was afterwards further explained by Mr. De Morgan in the Cambridge Philosophical Transactions for 1858. It differs from the first mentioned kind of beat, in that it arises from the imperfection, not of *unisons*, but of wide-apart consonances, such as the third, fourth, fifth, sixth, and octave. It has been called the *beat of imperfect consonances*. It is well known practically to organ tuners, and is appreciable to any musical ear.

"Taking the fifth as an example, let two notes forming this interval be sounded on an organ or any instrument of sustained tones. If they are perfectly in tune, the united sound will be smooth and even, or at least will only be subject to the "roughness" naturally inherent in the interval as mentioned in the text. But if one of the notes be sharpened or flattened a little, a positive beat much more marked in character will be heard just as in the case of the imperfect unison, and will increase in rapidity as the note is made more and more out of tune.

"The theory of the consonance beat, as explained by Smith and De Morgan, is too complicated to be given here; but it may be said in general terms to be a beat of the second order, depending, not like the unison beat on a cycle of differing periods, but on a cycle of differing cycles.

"As this beat is of considerable practical use, it may be as well here to explain the rule for its frequency, *i.e.*, for finding how many beats per second will result from the concord being any given quantity out of tune, and *vice versâ*.

"Let n represent the denominator of the fraction expressing, in the lowest terms, the true ratio of the concord (*e.g.*, for the fifth $\frac{3}{2}$, $n = 2$; for the minor sixth $\frac{6}{5}$, $n = 5$, and so on); then let q = the number of vibrations per second of

the upper note, either in excess or deficiency of the number which would make the interval perfectly in tune. Also let β = the number of consonance beats per second; then, as a general formula $\beta = n\,q$.

"Applying this to the several consonances, we have the following results:—

For the unison or octave	. . .	$\beta = q$.
„ fifth	. . . . .	$\beta = 2\,q$.
„ fourth	. . . . .	$\beta = 3\,q$.
„ major third	. . . .	$\beta = 4\,q$.
„ minor third	. . . .	$\beta = 5\,q$.
„ major sixth	. . . .	$\beta = 3\,q$.
„ minor sixth	. . . .	$\beta = 5\,q$.

"It will now be easy to understand why beats are capable of such great utility, in a practical point of view, namely, as giving a means of measuring, with great ease and positive certainty, the most delicate shades of adjustment in the tuning of consonant intervals. To get, for example, an octave, a fifth, or a third perfectly in tune, the tuner has only to watch when the beats vanish, which he can observe with the greatest ease, and which will give him far more accuracy than he could possibly get by the ear alone. Whereas, if he desires to adopt any fixed temperament, he has only to calculate the velocity of beats corresponding to the minute error which should be given to each concord; and the required note may be tuned to its proper pitch with a precision and facility which would be impossible by the unaided ear.

"The delicacy of this method would hardly be believed if it did not rest on proof beyond question. For example, in tuning, say, a fifth above middle *C*, the difference between 95 and 100 beats per minute would be appreciable by any one with a seconds watch in his hand; and yet this would

correspond to a difference of only $\frac{1}{24}$ of a vibration per second, or in pitch less than $\frac{1}{600}$ of a semitone!

"This use of beats has been long practised by organ tuners to some extent, but its capabilities, as amplified by the aid of calculation, are certainly not appreciated or used as they might be.

"There is another practical use of beats, also very interesting, which has been alluded to on p. 32 of this work, namely, that they furnish a means of ascertaining the positive number of vibrations per second corresponding to any musical note. This may be done either by the unison or by the consonance beat, as follows:—

"For the unison beat:—Take two notes in unison on an organ, a harmonium, or other instrument of sustained sounds, and put one of them a little out of tune, so as to produce beats when they are sounded together. Let V and v represent the vibration-numbers of the upper and lower notes respectively. Then by means of a monochord it will be easy to determine the ratio $\frac{V}{v}$, which call m. Count the number of beats per second, which call β. Then, since $\beta = V - v$, we obtain the simple equation,

$$v = \frac{\beta}{m - 1}$$

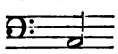
which gives the actual number of vibrations per second of the lower note of the two.

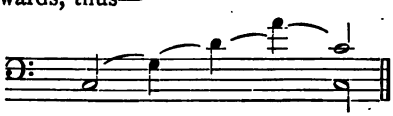
"The method of deducing the vibration number from the consonance beat was pointed out by Dr. Smith; but as this method, so far as I know, is not to be found anywhere, except buried under the mass of ponderous learning contained in his work, I give it here in a simple algebraical form. If $\frac{m}{n}$ represents the true ratio of the interval, N the actual number of vibrations per second of the lower note, and M

the same number for the upper one, the formula for the consonance beat becomes :—

$$\beta = \left(m - n\frac{M}{N}\right)N; \text{ or } N = \frac{\beta}{m - n\frac{M}{N}}$$

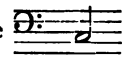
"Now as m and n are both known for any given concord, if we can tell by any independent means the actual ratio of the notes $\frac{M}{N}$, we may, by simply counting the beats, calculate the actual number of vibrations N of the lower note. If the interval is too flat, β must be + ; if too sharp it must be −. The following example will show how this may be done :—

"Let it be required to determine how many vibrations per second are given by the note  on an organ. Tune three perfect fifths upwards, and then a perfect major sixth downwards, thus—



which will give the C an octave above the original note, But, by the laws of harmonics, we know that this octave will not be in tune; the upper C will be too sharp, the ratio being $\frac{81}{40}$ instead of $\frac{2}{1}$, as it ought to be. Hence $\frac{M}{N} = \frac{81}{40}$, and $\frac{m}{n} - \frac{2}{1}$. Count the beats made by this imperfect octave, and suppose them = 192 per minute, or 3·2 per second, then, as the interval is sharp—

$$N = \frac{-3{\cdot}2}{2 - \frac{81}{40}} = 128;$$

i.e., the note  is making 128 double vibrations per second.

"This method has the advantage of dispensing with the use of the monochord, which was necessary in the former case."—*Pole.*

It may be remarked, parenthetically, that although accurate tempering of intervals on the pianoforte is only possible to the practised tuner, it is yet within the power of any person of ordinary intelligence, by a careful study of the theory of beats at the pianoforte, to acquire as much knowledge of tempering and tuning as will suffice to keep a piano in tolerably good tune, when a tuner may not be within easy reach.

Helmholtz applied the syren in his investigations into the subject of beats, and in Chapter VIII. of his great work (from which frequent quotations have been already made), we find the following passage:—

"So far we have combined tones of precisely the same pitch, now let us inquire what happens when the tones have slightly different pitch. The double syren just described is also well fitted for explaining this case, for we can slightly alter the pitch of the upper tone by slowly revolving the upper box by means of the handle, the tone becoming flatter when the direction of revolution is the same as that of the disc, and sharper when it is opposite to the same. The periodic time of a tone of the syren is equal to the time required for a hole in the rotating disc to pass from one hole in the wind-box to the next. If the hole of the box advances to meet the hole of the disc through the

rotation of the box, the two holes come into coincidence sooner than if the box were at rest; and hence the periodic time is shorter, and the tone sharper. The converse takes place when the revolution is in the opposite direction. These alterations of pitch are easily heard when the box is revolved rather quickly. Now produce the tones of twelve holes on both discs. These will be in absolute unison as long as the upper box is at rest. The two tones constantly reinforce or enfeeble each other according to the position of the upper box. But on setting the upper box in motion, the pitch of the upper tone is altered, while that of the lower tone, which has an immovable wind-box, is unchanged. Hence we have now two tones of slightly different pitch sounding together. And we hear the so-called *beats* of the tones, that is, the intensity of the tone will be alternately greater and less in regular succession. The arrangement of our syren makes the reason of this readily intelligible. The revolution of the upper box brings it alternately in positions which, as we have seen, correspond to stronger and weaker tones. When the handle has been turned through a right angle, the wind-box passes from a position of loudness through a position of weakness to a position of strength again. Consequently every complete revolution of the handle gives us four beats, whatever be the rate of revolution of the discs, and hence however low or high the tone may be. If we stop the box at the moment of maximum loudness, we continue to hear the loud tone; if at a moment of minimum force, we continue to hear the weak tone.

"The mechanism of the instrument also explains the connection between the number of beats and the difference of the pitch. It is easily seen that the number of the puffs is increased by one for every quarter revolution of the handle. But every such quarter revolution corresponds

to one beat. Hence *the number of beats in a given time is equal to the difference of the numbers of vibrations executed by the two tones in the same time.* This is the general law which determines the number of beats for all kinds of tones. This law results immediately from the construction of the syren; in other instruments it can only be verified by very accurate and laborious measurements of the numbers of vibrations."—*Helmholtz.*

And further on in the same chapter:—

"Beats are easily produced on all musical instruments, by striking two notes of nearly the same pitch. They are heard best from the simple tones of tuning-forks or stopped organ pipes, because here the tone really vanishes in the pauses. A little fluctuation in the pitch of the beating tone may then be remarked. For the compound tones of other instruments the upper partial tones are heard in the pauses, and hence the tone jumps up an octave, as in the case of interference already described. If we have two tuning-forks of exactly the same pitch, it is only necessary to stick a little wax on to the end of one, to strike both, and bring them near the same ear, or to the surface of a table, or sounding-board. To make two stopped pipes beat, it is only necessary to bring a finger slowly near to the lip of one, and thus flatten it. The beats of compound tones are heard by striking any note on a pianoforte out of tune, when the two strings belonging to the same note are no longer in unison; or if the piano is in tune it is sufficient to attach a piece of wax, about the size of a pea, to one of the strings. This puts them sufficiently out of tune. More attention, however, is required for compound tones because the enfeeblement of the tone is not so striking. The beat, in this case, resembles a fluctuation in pitch and quality. This is very striking on the syren, according as the brass

resonance cylinders are attached or not. These make the prime tone relatively strong. Hence when beats are produced by turning a handle, the decrease and increase of loudness in the tone is very striking. On removing the resonance cylinders, the upper partial tones are relatively very strong, and, since the ear is very uncertain when comparing the loudness of tones of different pitch, the alteration of force during the beats is much less striking than that of pitch and quality of tone."—*Helmholtz.*

If the student compares the figure given above (p. 266), which illustrates what happens when two sets of twelve and thirteen vibrations per second respectively start together, he will be able to imagine, without difficulty, what happens when two strings of a pianoforte vibrate respectively 100 and 101 times per second. These vibrations are too rapid to be seen, but it is not impossible to make beats visible :—

"It is possible to render beats visible by setting a suitable elastic body into sympathetic vibration with them. Beats can then occur only when the two exciting tones lie near enough to the prime tone of the sympathetic body for the latter to be set into sensible sympathetic vibration by both the tones used. This is most easily done with a thin string which is stretched on a sounding-board on which have been placed two tuning-forks, both of very nearly the same pitch as the string. On observing the vibrations of the string through a microscope, or attaching a fibril of a goose feather to the string, which will make the same excursions on a magnified scale, the string will be clearly seen to make sympathetic vibrations with alternately large and small excursions, according as the tone of the fork is at its maximum or minimum."—*Helmholtz.*

"A variety of experiments will suggest themselves to the reflecting mind, by which the effect of interference may be illustrated. It is easy, for example, to find a jar which resounds to a vibrating plate. Such a jar, placed over a vibrating segment of the plate, produces a powerful resonance. Placed over a nodal line, the resonance is entirely absent; but if a piece of pasteboard be interposed between the jar and plate, so as to cut off the vibrations on one side of the nodal line, the jar instantly resounds to the vibrations of the other. Again, holding two forks, which vibrate with the same rapidity, over two resonant jars, the sound of both flows forth in unison. When a bit of wax is attached to one of the forks, powerful beats are heard. Removing the wax, the unison is restored. When one of these unisonant forks is placed in the flame of a spirit-lamp its elasticity is changed, and it produces long loud beats with its unwarmed fellow. If while one of the forks is sounding on its resonant case, the other be excited and brought near the mouth of the case, loud beats declare the absence of unison. Dividing a jar by a vertical diaphragm, and bringing one of the forks over one of its halves, and the other fork over the other; the two semi-cylinders of air produce beats by their interference. But the diaphragm is not necessary; on removing it, the beats continue as before, one half of the same column of air interfering with the other.

"The intermittent sound of certain bells, heard more especially when their tones are subsiding, is an effect of interference. The bell, through lack of symmetry, as explained in the fourth chapter, vibrates in one direction a little more rapidly than in the other, and beats are the consequence of the coalescence of the two different rates of vibration."—*Tyndall.*

"Each of the two forks now before you executes exactly

256 vibrations in a second. Sounded together, they are in unison. Loading one of them with a bit of wax, it vibrates a little more slowly than its neighbour. The wax, say, reduces the number of vibrations to 255 in a second: how must their waves affect each other? If they start at the same moment, condensation coinciding with condensation, and rarefaction with rarefaction, it is quite manifest that this state of things cannot continue. At the 128th vibration their phases are in complete opposition, one of them having gained half a vibration on the other. Here the one fork generates a condensation where the other generates a rarefaction: and the consequence is, that the two forks, at this particular point, completely neutralise each other. From this point onwards, however, the forks support each other more and more, until, at the end of a second, when the one has completed its 255th, and the other its 256th vibration, condensation again coincides with condensation, and rarefaction with rarefaction, the full effect of both sounds being produced upon the ear.

"It is quite manifest, that under these circumstances we cannot have the continuous flow of perfect unison. We have, on the contrary, alternate reinforcements and diminutions of the sound. We obtain, in fact, the effect known to musicians by the name of *beats*, which, as here explained, are a result of interference.

"I now load this fork still more heavily, by attaching a fourpenny-piece to the wax; the coincidences and interferences follow each other more rapidly than before; we have a quicker succession of beats. In our last experiment, the one fork accomplished one vibration more than the other in a second, and we had a single beat in the same time. In the present case, one fork vibrates 250 times, while the other vibrates 256 times in a second, and the number of beats per second is 6. A little reflection will make it plain, that in

the interval required by the one fork to execute one vibration more than the other, a beat must occur; and inasmuch as, in the case now before us, there are six such intervals in a second, there must be six beats in the same time. In short, *the number of beats per second is always equal to the difference between the two rates of vibration.*"—*Tyndall.*

"The most direct way of studying the beats of simple tones experimentally is to take two unison tuning-forks and attach a small pellet of wax to the extremity of a prong of one of them. The fork so operated on becomes slightly heavier than before; its vibrations are therefore proportionately retarded, and its pitch lowered. When *both* forks are struck and held to the ear beats are heard. These will be most distinct when the fork's tones are exactly equally loud, for in this case the minima of intensity will be equal to zero, and the beats will therefore be separated by intervals of absolute, though but momentary, silence. The increase, in rapidity, of the beat, as the interval between the beating-tones widens, may be shown by gradually loading one of the forks more and more heavily with wax pellets, or by a small coin pressed upon them. If it is desired to exhibit these phenomena to a large audience, the forks should first be mounted on their resonance-boxes, and, after the pellets have been attached, stroked with the resined bow, care being taken to produce tones as nearly as possible equal in intensity. Slow beats may also be obtained from any instrument capable of producing tones whose vibration-numbers differ by a sufficiently small amount. Thus, if the strings corresponding to a single note of the pianoforte are not strictly in unison, such beats are heard on striking the note. If the tuning is perfect, a wax pellet attached to one of the wires will lower its pitch sufficiently to produce the desired effect. Beats not too fast to be readily

counted arise between adjacent notes in the lower octaves of the harmonium, or, still more conspicuously, in those of large organs. They are also frequently to be heard in the sounds of church bells, or in those emitted by the telegraph wires when vibrating powerfully in a strong wind. In order to observe them in the last instance, it is best to press one ear against a telegraph-post and close the other: the beats then come out with remarkable distinctness. It should be noticed that, when we are dealing with two composite sounds, several sets of beats may be heard at the same time, if pairs of partial tones are in relative positions suited to produce them. Thus, suppose that two clangs co-exist, each of which possesses the first six partial tones of the series audibly developed. Since the second, third, &c., partial tones of each clang make twice, three times, &c., as many vibrations per second as their respective fundamentals, it follows that the difference between the vibration-numbers of the two second-tones will be *twice*, that between those of the two-third tones *three times*, &c., as great as the difference between the vibration-numbers of the two fundamentals. Accordingly, if the fundamental tones give rise to beats, we may hear, in addition to the series so accounted for, five other sets of beats, respectively twice, three, four, five, and six times as rapid as they. In order to determine the number of beats per second, for any such set, we need only multiply the number of the fundamental beats by the *order* of the partial tones concerned. The beats of two simple tones necessarily become more rapid if the higher tone be sharpened, or the lower flattened; *i.e.*, if the interval they form with each other be widened. The beats may, however, also be accelerated without altering the interval, by merely placing it in a higher part of the scale. In this way greater vibration-numbers are obtained, and the difference of these is proportionally large,

though their ratio to each other remains what it was before. Thus the rapidity of the beats due to an assigned inter l depends jointly on two circumstances, the width of the interval, and its position in the musical scale; in other words, on both the *relative* and *absolute* pitch of the tones forming the given interval."—*Taylor.*

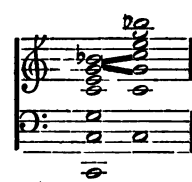
"Accidental illustrations of beats, may be heard whenever a large organ tuned according to equal temperament is played. They are very audible in the sound of large bells, from unavoidable want of symmetry in the figure. A good mechanical example of interference may be noted when the big bell at the House of Commons tolls the hour. The first blow of the hammer falls on the metal in a quiescent state, and a sound of medium loudness is elicited. But as the clockwork lifts and drops its weight at regular periods, before the first vibration is extinguished, two conditions may occur; either the hammer may meet the bell in the same phase as its own, in which case an extremely loud toll results, or it may fall on it in the opposite phase, when great part of the momentum is employed in cancelling the interfering vibrations already set up, and a very feeble sound is given out. These differences of intensity in the successive strokes can be plainly heard at 11 o'clock or midnight when the air is still."—*Stone.*

CHAPTER XIII.

HELMHOLTZ' THEORY OF CONSONANCE AND DISSONANCE.

WE have now to look at the question of the relation between the notes which form the various intervals used in music. We have seen before that beats are still made even though the primes which make them have separated beyond the beating distance. This harshness, then, ceases not because the beats cease, but because they occur too rapidly to be noted by the ear as beats; but that the ear does note them remains now to be shown, and their effect is that which is usually called discord. When two tones are simple there can only be dissonance when they are within beating distance, and the point where the harshness becomes greatest will depend entirely upon the register of the two notes between which the beats occur. That distance is, as we have said, for the middle register about a minor third, for high tones it is less, for low ones more. This is why the ear notes sooner a slight change of pitch in a high than in a low tone, and it is more difficult to tune two pianoforte wires in consonance at the right end of the piano than it is at the left. This would be the case if the tones were

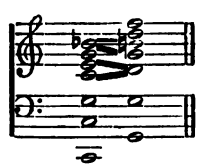
merely simple tones, but when they are compound, and have therefore upper partials as well as the prime, the case is much more complicated. About the tenth upper partial, the interval between them is something like a tone; about the sixteenth, it is a semitone; and the higher we proceed in the series, the more closely do they come into contact with each other; and where the high upper partials are well developed, there is therefore a certain amount of roughness of tone, and, practically speaking, all the upper partials above the eighth or ninth do not add to the musical quality of the tone, but simply increase its roughness. If we take now the different intervals used in music, and compare the upper partials of each pair of notes with each other, we shall see why it was that the intervals whose ratios were represented by the smallest numbers were regarded as the most consonant. Take, for instance, the octave, where the agreement is most perfect—the following diagram will show that there



are no upper partials in the two notes which can produce beats with each other to an extent which will interfere with the consonance of the two sounds: for though the sixth upper partial of the lower note is within "beating distance" of the second and third upper partials of the higher note, yet the latter are,

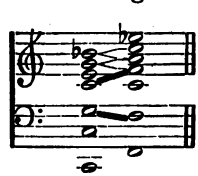
for most instruments, so much more powerful than the sixth of the lower tone, that the beats produced would be practically inaudible. We see, however, from this diagram, that *absolute* purity (freedom from upper-partial beats) can only be possible with two sounds actually in unison—when, that is to say, not only do their primes agree in all respects, but they have exactly the same number of and intensity in their upper partials.

In the next place, take the fifth, which vibrates to the tonic in the proportion of 3 : 2 :—



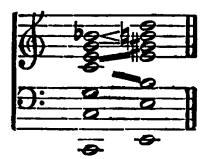
Here D, the second upper partial of G, comes into contact with both C and E, the third and fourth of C; and B♭, the sixth of C, makes beats with G, and B♮ the third and fourth of G. The interval of the fifth, then, though it is next to the octave in order of consonance, is thus seen to be considerably rougher than the octave, because more of the pairs of the upper partials of the tones constituting the fifth come into contact with each other so as to produce beats.

The interval of the fourth gives this result :—



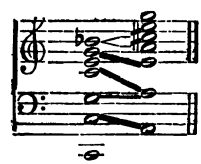
Here the disagreement is still more marked, as the first, third, fourth, and fifth upper partials of F produce beats with the second, fourth, fifth, and sixth of C, proving the fourth to be still less consonant than the fifth.

The remaining intervals, of which we give diagrams below, are still less consonant than the preceding, and the student should analyse them for himself, and note those pairs of upper partials which clash so as to produce beats and thus increase the dissonance. The major third, for instance, comes out thus :—

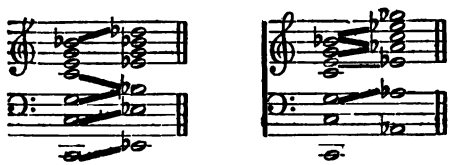


This interval, which is regarded æsthetically as one of the pleasantest of all, is actually one of the roughest of all, and it is probably to this roughness that its telling effect is to be attributed.

The major sixth comes out thus :—



The only two intervals now remaining less than the octave which are regarded as consonant, are the minor third and the minor sixth. These two come out thus :—



It will be seen that we have by gradual steps arrived on the very verge of the line of demarcation between consonance and dissonance. The minor sixth can hardly be called consonant at all, especially on instruments tuned to equal temperament.

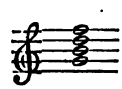
Helmholtz has well illustrated the consonance and dissonance of the various notes of the scale by a graphic illustration of the whole scale, in which dissonances are drawn as elevations, and consonances as depressions. Mr. Sedley Taylor, in his work on Sound and Music, very aptly says, speaking of this illustration, "If we regarded the outline as that of a mountain chain, the discords would be referred by *peaks*, and the concords by *passes*. The lowness and narrowness of a greater pass would measure the smoothness and definition of the corresponding musical interval."

It must not be forgotten that the illustrations given above in musical symbols of the dissonances between the upper partials of the various intervals of the scale, are only applicable to tones whose upper partials are confined to the first six; and it will be easily understood that to introduce higher numbers would bring in complications and place the subject outside our present purpose.

It would probably have rendered much easier the work of the student had the *actual number of beats per second* between all the notes within beating distance been calculated for him; but having furnished him in a former chapter with the materials for finding these out for himself, it will be much more profitable for him to make these calculations, taking the vibrations of the note at sixty-four as a basis. As the vibration number of the prime is to those of the upper partials in the ratio of the series 1 2 3 4 5 6 7, his work will be easy. Thus, in the first diagram in this chapter, the sixth upper partial of C (seventh of the whole series, including the prime) vibrates seven times as rapidly as the prime, or $64 \times 7 = 448$, and the second and third of G (third and fourth of the series) vibrate respectively three and four times as rapidly as the prime. Then

$128 \times 3 = 384$, and $448 - 384 = 64$ beats per sec.
$128 \times 4 = 512$, and $512 - 448 = 64$ „

These beats, though too rapid to be counted, yet considerably affect the perfect consonance of the pair. It must be borne in mind that what Helmholtz calls "the normal position of discords" is that which arranges their constituent parts in thirds from the roots; thus the dominant seventh in any key, or either mode, consists of the root, and its third, fifth, and seventh:—



where each note is a third (major or minor) from the one above it. A fifth is composed of two thirds; inverted this gives the fourth. A seventh is made up of three thirds; its inversion is the second. We can thus obtain all the intervals of the scale from intervals (or their inversions) made up of thirds.

The following table will show at one view the relative consonance and dissonance of the different degrees within an octave, the intervals all being calculated for just intonation:—

Interval.	No.	Notation.	Ratio.	No. of Beats per Second.*	Comparative Roughness.†
Unison .	1		1:1	0	0
Minor 2nd	2		16:15	$8\frac{8}{15}$	70
Major 2nd	3		10:9	$14\frac{2}{9}$	38
Minor 3rd	4		6:5	$25\frac{3}{5}$	20
Major 3rd	5		5:4	32	8
Perfect 4th	6		4:3	$42\frac{2}{3}$	2
Sharp 4th	7		25:18	$49\frac{7}{9}$	32
Flat 5th .	8		36:25	$56\frac{8}{25}$	35
Perfect 5th	9		3:2	64	0
Minor 6th	10		8:5	$76\frac{4}{5}$	20
Major 6th .	11		5:3	$185\frac{1}{3}$	3
Minor 7th	12		9:5	$102\frac{2}{5}$	55
Major 7th	13		15:8	112	42
Octave .	14		2:1	0	0

* These are the differences between the vibration numbers of the pairs of notes, and are not to be regarded as audible beats which may be counted by the ear, but only as affecting the roughness of the dissonances. The note C is taken at 128 vibrations per second.

† The roughness is calculated in hundredths of an inch above a horizontal line which is taken as the position of perfect smoothness. Thus two notes in unison would both be on that line; a minor second would be $\frac{70}{100}$ths (70ths) of an inch above it; a perfect fourth $\frac{2}{100}$ths of an inch above it; a minor third $\frac{38}{100}$th (38ths) of an inch above it, and so on.

This table only deals with those intervals the vibration-numbers of which are calculated for just intonation, in which every interval bears its just proportion to the tonic; but there are numerous varieties of nearly every interval mentioned in the table, and the consonance or dissonance of a chord will depend often upon whether the intervals of which it is built up are just intervals, or whether they come within the category of those variations.

The general result of this investigation is that consonance or dissonance depend on the presence or absence of beats:—

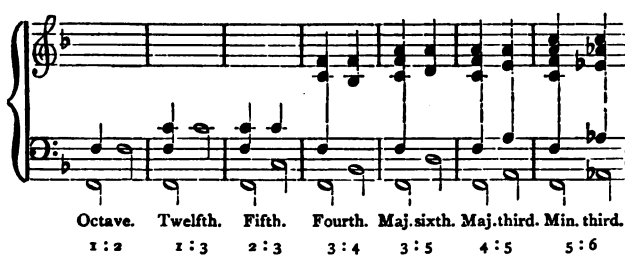
"In the middle and upper portions of the musical scale the beats are most grating and harsh when they succeed each other at the rate of 33 per second. When they occur at the rate of 132 per second, they cease to be sensible.

"The perfect consonance of certain musical intervals is due to the absence of beats. The imperfect consonance of other intervals is due to their existence. And here the overtones play a part of the utmost importance. For though the primaries may sound together without any perceptible roughness, the overtones may be so related to each other as to produce harsh and grating beats. A strict analysis of the subject proves that intervals which require large numbers to express them, are invariably accompanied by overtones which produce beats; while in intervals expressed by small numbers the beats are practically absent." —*Tyndall.*

"The following is the order of the consonant intervals beginning with those distinctly characterised, and then proceeding to those which have their limits somewhat blurred so to speak, by the weaker beats of the higher upper partial tones:—

1.	Octave	1 : 2
2.	Twelfth	1 : 3
3.	Fifth	2 : 3
4.	Fourth	3 : 4
5.	Major Sixth	3 : 5
6.	Major Third	4 : 5
7.	Minor Third	5 : 6

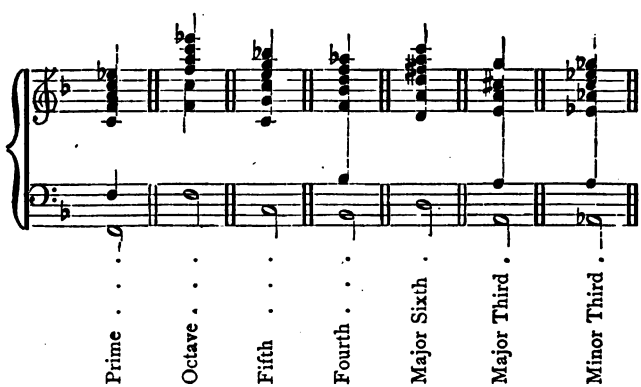
The following examples in musical notation show the coincidences of the upper partials. The primes are as before represented by minims, and the upper partials by crotchets. The series of upper partials is continued up to the common tone only:—



"We have hitherto confined our attention to beats arising from intervals which differ but slightly from those o perfect consonances. When the difference is small the beats are slow, and hence easy both to observe and count. Of course beats continue to occur when the deviation of the two coincident upper partials increases. But as the beats then become more numerous, their real character is more easily concealed than even the more rapid beats of dissonant primes, under the overwhelming mass of sound of the louder primes. These more rapid beats give a rough effect to the whole mass of sound, but the ear does not readily recognise its cause, unless the experiments have

been conducted by gradually increasing the imperfection of an harmonic interval, so as to make the beats gradually more and more rapid, thus leading the observer to mark the intermediate steps between the numerable rapid beats on the one hand, and the roughness of a dissonance on the other, and hence to convince himself that the two phenomena differ only in degree."—*Helmholtz.*

We now give, at one view, a musical representation of all the above intervals (except the twelfth), showing the partials of each prime as far as the sixth upper partial (seventh partial), and enabling the student to see where there is any clashing of the upper partials, just as the last example shows their coincidences.



The "roughness" of the various intervals of the major and minor scales may be thus summarised by the figures giving the roughnesses on the same principles as those in the table :—

MAJOR SCALE.

Thirds and Sixths.

1. Justly intoned major third ($\frac{5}{4}$), roughness 8, its inversion the justly intoned minor sixth ($\frac{8}{5}$), roughness 20.

2. Justly intoned minor third ($\frac{6}{5}$), roughness 20; its inversion the justly intoned major sixth ($\frac{5}{3}$), roughness 3.

These intervals are by Helmholtz all considered consonant.

3. The Pythagorean minor third ($\frac{32}{27}$), roughness 26. This interval is rather less harmonious than the just minor third. The Pythagorean major sixth ($\frac{27}{16}$), roughness 24; the inversion of the Pythagorean minor third is a comma (1 vib. per second in 80) wider than the justly intoned major sixth, to which it is greatly inferior in harmoniousness.

Fifths and Fourths.

4. The justly intoned fifth ($\frac{3}{2}$), roughness 0, is made up of a just major and just minor third ($\frac{5}{4} \times \frac{6}{5} = \frac{3}{2}$). Its inversion is the just fourth ($\frac{4}{3}$), roughness 2. Both these are consonant.

5. The imperfect fifth ($\frac{40}{27}$), roughness 44, is made up of a Pythagorean minor third and a just major third ($\frac{32}{27} \times \frac{5}{4} = \frac{40}{27}$), and is therefore one comma (see above) less than a just fifth. Its inversion is the sharp fourth ($\frac{27}{20}$), roughness 27.

6. The "false" fifth (all fifths not just are really

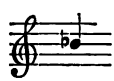
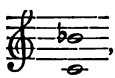
false) is composed of a just minor third and a Pythagorean minor third ($\frac{6}{5} \times \frac{32}{27} = \frac{64}{45}$), roughness 28. This is a harsh dissonance, and nearly as rough as a major second. Its inversion, the "false" fourth, or tritone ($\frac{45}{32}$), roughness 20, is about a comma less than the false fifth, and consists of three whole tones. A comma being $\frac{81}{80}$, the difference between these intervals (which on the pianoforte are identical) is less than that, being $\frac{98}{88}$, or $\frac{10}{11}$ of a comma. This difference is quite enough to destroy the perfection of the interval to any ordinary ear.

7. The superfluous fifth, or extreme sharp fifth of the minor scale ($\frac{25}{16}$), roughness 39, consists of two just major thirds, $\frac{5}{4} \times \frac{5}{4} = \frac{25}{16}$. This interval is, on the pianoforte, the same as the minor sixth, but in just intonation it is clearly two commas flatter. Its inversion, the diminished fourth ($\frac{32}{25}$), roughness 25, is about two commas sharper, and considerably rougher than the just major third. The major third and diminished fourth are on the pianoforte the same.

Sevenths and Seconds.

8. The diminished seventh of the minor scale consists of a Pythagorean minor third and two just minor thirds ($\frac{6}{5} \times \frac{6}{5} \times \frac{32}{27} = \frac{128}{75}$), roughness 24. It is two commas wider than the just major sixth, and is a very harsh dissonance. Its inversion is the superfluous second, roughness 24. Its ratio is $\frac{75}{64}$, or nearly $\frac{7}{6}$, actually $\frac{7}{6} \times \frac{225}{224}$.

9. The close minor seventh, consisting of a just

major, just minor, and Pythagorean minor third, has for its ratio $\frac{16}{9}$ ($\frac{5}{4} \times \frac{5}{6} \times \frac{32}{27} = \frac{16}{9}$), roughness 23. This is a mild dissonance, and lies very close to the seventh partial tone (sixth upper partial): thus, the sixth upper partial of C is B♭  D, and the interval  , if tuned as a *close* minor seventh, has a B♭ as close to the sixth upper partial of C as $\frac{7}{4} : \frac{16}{9}$. The inversion of this close minor seventh is the major second ($\frac{9}{8}$), roughness 32, which is a strong dissonance.

10. The wide minor seventh ($\frac{9}{8}$), roughness 25, is a comma ($\frac{81}{80}$) wider than the last interval, and being nearer the octave it is perceptibly more harsh. It consists of a just major third and two just minor thirds; or $\frac{5}{4} \times \frac{6}{5} \times \frac{6}{5} = \frac{9}{5}$. Its inversion is the minor tone ($\frac{10}{9}$), roughness 38, which is harsher than the major tone ($\frac{9}{8}$), roughness 32.

11. The major seventh ($\frac{15}{8}$), roughness 42, is made up of two just major thirds and one just minor third; $\frac{5}{4} \times \frac{5}{4} \times \frac{6}{5} = \frac{15}{8}$. Its inversion is the minor second or semitone ($\frac{16}{15}$), roughness 70, which is the roughest dissonance in the scale.

12. The extreme sharp sixth ($\frac{225}{128}$), roughness 15, is made up of two just major thirds and one just major tone ($\frac{5}{4} \times \frac{5}{4} \times \frac{9}{8} = \frac{225}{128}$). Its inversion is the diminished third ($\frac{256}{225}$), roughness 30.

From what has been said on beats, and on con-

sonance and dissonance, the student will have gathered that *the various degrees of dissonance are produced by beats*, those which result from two tones within a minor third of each other being rougher, and therefore more easily detected by the ear; the others, which lie outside that distance, are none the less real, but make their presence known by affecting the consonance of the interval. The beats between the primes are the most palpable to the ear; but the beats between the upper partials still affect the consonance of the interval when they can no longer be detected as beats, their effect being then as real as in the more obvious (because slower) beats.

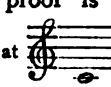
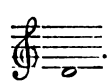
"Two musical tones, therefore, which stand in the relation of a perfect Octave, a perfect Twelfth, or a perfect Fifth, go on sounding uniformly without disturbance, and are thus distinguished from the next adjacent intervals, imperfect Octaves and Fifths, for which a part of the tone breaks up into distinct pulses, and consequently the two tones do not continue to sound without interruption. For this reason the perfect Octave, Twelfth, and Fifth will be called *consonant intervals* in contradistinction to the next adjacent intervals, which are termed *dissonant.* Although these names were given long ago, long before anything was known about upper partial tones and their beats, they give a very correct notion of the essential character of the phenomenon which consists in the undisturbed or disturbed co-existence of sounds.

"Since the phenomena just described form the essential basis for the construction of normal musical intervals, it is advisable to establish them experimentally in every possible form.

"We have stated that the beats heard are the beats of those partial tones of both compounds which nearly coincide. Now it is not always very easy on hearing a Fifth or an Octave which is slightly out of tune, to recognise clearly with the unassisted ear which part of the whole sound generates the beats. On listening we are apt to feel that the whole sound is alternately reinforced and weakened. Yet an ear accustomed to distinguish upper partial tones, after directing its attention on the common upper partials concerned, will easily hear the strong beats of these particular tones, and recognise the continued and undisturbed sound of the primes. Strike the note d', * attend to its upper partial a'', and then strike a tempered fifth a'; the beats of a'' will be clearly heard. To an unpractised ear the resonators already described will be of great assistance. Apply the resonator for a'', and the above beats will be heard with great distinctness. If, on the other hand, a resonator, tuned to one of the prime tones d' or a', be employed, the beats are heard much less distinctly, because the continuous part of the tone is then reinforced.

"This last remark must not be taken to mean that no other simple tones beat in this combination except a''. On the contrary, there are other higher and weaker upper partials, and also combinational tones, which beat, as we shall learn in the next chapter, and these beats coexist with those already described. The beats of the lowest common upper partials are the most prominent, simply because these partials are the loudest and slowest of all.

"Secondly, a direct experimental proof is desirable

* The "once-accented (2-ft.) octave" starts at  , and d' is therefore  .

that the numerical ratios here deduced from the vibrational numbers are really those which give no beats. This proof is most easily given by means of the double syren. Set the discs in revolution and open the series of 8 holes on the lower and 16 on the upper, thus obtaining two compound tones which form an Octave. They continue to sound without beats as long as the upper box is stationary. But directly we begin to revolve the upper box, thus slightly sharpening or flattening the tone of the upper disc, beats are heard. As long as the box was stationary the ratio of the vibrational numbers was exactly 1 : 2, because exactly 8 pulses of air escaped on one rotation of the lower, and 16 on one rotation of the upper disc. By diminishing the speed of rotation of the handle this ratio may be altered as slightly as we please, but however slowly we turn it, if it move at all, beats are heard proclaiming the falsification of the interval.

"Similarly with the Fifth. Open the series of 12 holes above, and 18 below, and a perfectly unbroken Fifth will be heard as long as the upper windbox is at rest. The ratio of the vibrational numbers, fixed by the holes of the two series, is exactly 2 to 3. On rotating the windchest, beats are heard. We have seen that each revolution of the handle increases or diminishes the number of vibrations of the tone due to the 12 holes by 4. When we have the tone of 12 holes on the lower discs also, we thus obtain 4 beats. But with the Fifth from 12 and 18 holes each revolution of the handle gives 12 beats, because the vibrational number of the third partial tone increases on each revolution of the handle by $3 \times 4 = 12$, when that of the prime tone increases by 4, and we are now concerned with the beats of this partial tone."—*Helmholtz.*

Those intervals which are usually called consonant

are not all harmonious in a like degree. We have before said that the larger the ratio of any interval, the farther is it removed from absolute consonance or perfect agreement between the vibrations of the notes of which it consists. There are dissonances between the upper partials of most of the intervals which are called consonant, and Helmholtz says that *in each consonant interval those upper partials form a dissonance, which coincide in one of the adjacent intervals,* and Mr. A. J. Ellis, in a note, offers this explanation of what Helmholtz means :—

"That is, in intervals which differ from the first by raising or depressing one of its tones by a Semitone (either $\frac{16}{15}$ or $\frac{25}{24}$) or even a tone ($\frac{9}{8}$). Thus for Fifth, $\frac{3}{2} \times \frac{16}{15} = \frac{8}{5}$ a minor Sixth; and $\frac{3}{2} \times \frac{8}{9} = \frac{4}{3}$ a Fourth. For Fourth, $\frac{4}{3} \times \frac{15}{16} = \frac{5}{4}$ a major Third; and $\frac{4}{3} \times \frac{9}{8} = \frac{3}{2}$ a Fifth. For major Third $\frac{5}{4} \times \frac{16}{15} = \frac{4}{3}$ a Fourth; and $\frac{5}{4} \times \frac{24}{25} = \frac{6}{5}$ a minor Third. For minor third $\frac{6}{5} \times \frac{25}{24} = \frac{5}{4}$ a major Third, and $\frac{6}{5} \times \frac{15}{16} = \frac{9}{8}$ a major Tone. The adjacency of the consonant intervals is best shown in fig. 60,* A where it appears that the order may be taken at; (1) Unison, (2) minor Third, (3) major Third, (4) Fourth, (5) Fifth, (6) minor Sixth, (7) major Sixth, 8) Octave. In the Table,† other intervals, not perfectly consonant, are intercalcated among these."

Remembering, then, that even *the consonances are not all entirely consonant,* and that *the purity of their consonance depends upon the absence of dissonance among their upper partials,* the student will be pre-

* The figure before referred to, in which Helmholtz represents dissonance by peaks, and consonance by passes.

† Given in the next quotation.

pared to profit by the following extract from Helmholtz, containing the table just referred to by Mr. Ellis:—

"The following table gives a general view of this influence of the different consonances on each other. The partials are given up to the ninth inclusive,* and corresponding names assigned to the intervals arising from the coincidence of the higher upper partial tones. The third column contains the ratios of their vibrational numbers, which at the same time furnish the number of the order of the coincident partial tones. The fourth column gives the distance of the separate intervals from each other, and the last a measure of the relative strength of the beats resulting from the mistuning of the corresponding interval, reckoned for the quality of tone of the violin. The degree to which any interval disturbs the adjacent intervals increases with this last number.

Intervals.	Notation.	Ratio of the Vibrational Numbers.	Relative Distance.	Intensity of Influence.
Unison	*C*	1 : 1		
			8 : 9	100·0
Second	*D*	8 : 9		
			63 : 64	1·4
Super-Second . .	*D*+	7 : 8		
			48 : 49	1·8
Sub-minor Third .	*E*♭—	6 : 7		
			35 : 36	2·4
Minor Third . .	*E*♮	5 : 6		
			24 : 25	3·3
Major Third . .	*E*	4 : 5		
			35 : 36	5·0
Super-major Third	*E*+	7 : 9		
			27 : 28	1·6
Fourth	*F*	3 : 4		
			20 : 21	8·3
Sub-Fifth . . .	*G*♭—	5 : 7		
			14 : 15	2·8
Fifth	*G*	2 : 3		
			15 : 16	16·7
Minor Sixth. . .	*A*♭	5 : 8		
			24 : 25	2·5
Major Sixth. . .	*A*	3 : 5		
			20 : 21	6·7
Sub-minor Seventh	*B*♭—	4 : 7		
			35 : 36	316
Minor Seventh . .	*B*♭	5 : 9		
			9 : 10	2.2
Octave	*c*	1 : 2		50·0

* Eighth upper partial.

"The most perfect chord is the *Unison*, for which both compound tones have the same pitch. All its partial tones coincide, and hence no dissonance can occur except such as is contained in each compound separately.

"It is much the same with the *Octave*. All the partial tones of the higher note of this interval coincide with the even-partial tones of the deeper, and reinforce them, so that in this case also there can be no dissonance between two upper partial tones, except such as already exists, in a weaker form, among those of the deeper note. A note accompanied by its Octave consequently becomes brighter in quality, because the higher upper partial tones, on which brightness of quality depends, are partly reinforced by the additional Octave. But a similar effect would also be produced by simply increasing the intensity of the lower note without adding the Octave; the only difference would be, that in the latter case the reinforcement of the different partial tones would be somewhat differently distributed.

"The same holds for the *Twelfth* and *double Octave*, and generally for all those cases in which the prime tone of the higher note coincides with one of the partial tones of the lower note, although as the interval between the two notes increases, the difference between consonance and dissonance tends towards obliteration.

"The case hitherto considered, where the prime of one compound tone coincides with one of the partials of the other, may be termed *absolute consonance*. The second compound tone introduces no new element, but merely reinforces a part of the other.

"Unison and Octave disturb the next adjacent intervals considerably, in the sense assigned to this expression above, so that the minor Second *C D♭*, and the major Seventh *C B*, which differ from the Unison and Octave by a Semitone respectively, are the harshest dissonances in

our scale. Even the major Second *C D*, and the minor Seventh *C B♭*, which are a whole Tone apart from the disturbing intervals, must be reckoned as dissonances, although, owing to the greater interval of the dissonant partial tones, they are much milder than the others. In the higher regions of the scale their roughness is materially lessened by the increased rapidity of the beats. Since the dissonance of the minor Seventh is due to the second partial tone, which in most musical qualities of tone is much weaker than the prime, it is still milder than that of the major Second, and hence lies on the very boundary between dissonance and consonance.

"To find additional good consonances we must consequently go to the middle of the Octave, and the first we meet is the *Fifth*. Immediately next to it within the interval of a Semitone there are only the intervals 5 : 7 and 5 : 8 in our table, and these cannot much disturb it, because in all the better kinds of musical tones the 7th and 8th partials are either very weak or entirely absent. The next intervals with stronger upper partials are the Fourth 3 : 4 and the major Sixth 3 : 5. But here the interval is a whole Tone, and if the tones 1 and 2 of the interval of the Octave could produce very little disturbing effect in the minor Seventh, the disturbance by the tones 2 and 3, or by the adjacency of the Fifth for the Fourth and major Sixth must be insignificant, and the reaction of these two intervals with the tones 3 and 4 or 3 and 5 on the Fifth must be entirely neglected. Hence the Fifth remains a perfect consonance, in which there is no sensible disturbance of closely adjacent upper partial tones. It is only in harsh qualities of tone (harmonium, double-bass, violoncello, reed organ pipes) with high upper partial tones, and deep primes, when the number of beats is small, that we remark that the Fifth is somewhat rougher than the Octave. Hence the

Fifth has been acknowledged as a consonance from the earliest times and by all musicians. On the other hand, the intervals next adjacent to the Fifth are those which produce the harshest dissonances after those next adjacent to the Octave. Of the dissonant intervals next the Fifth, those in which the Fifth is flattened, that is, which lie between the Fifth and Fourth, and are disturbed firstly by the tones 2 and 3, and secondly by the tones 3 and 4, are more decidedly dissonant than those in which the Fifth is sharpened and which lie between the Fifth and major Sixth, because for the latter the second disturbance arises from the tone 4 and the weaker tone 5. The intervals between the Fifth and Fourth are consequently always considered dissonant in musical practice. But between the Fifth and major Sixth lies the interval of the *minor Sixth*, which is treated as an imperfect consonance, and owes this preference mainly to its being the inversion of the major Third. On keyed instruments, as the piano, the same keys will strike notes which at one time represent the consonance *C* $A\flat$, and at another the dissonance *C* $G\sharp$."—*Helmholtz.*

Proceeding on this plan Helmholtz analyses all the intervals of the scale, the results being those given in this chapter, and thus furnishes a scientific basis for facts the causes of which had been, until within the past twenty years, guessed at, but never accurately known. The following summary of his work in this direction will form a fitting close to the present chapter :—

"When two musical tones are sounded at the same time, their united sound is generally disturbed by the beats of the upper partials, so that a greater or less part of the whole

mass of sound is broken up into pulses of tone, and the joint effect is rough. This relation is called *Dissonance.*

"But there are certain determinate ratios between vibrational numbers, for which this rule suffers an exception, and either no beats at all are formed, or at least only such as have so little intensity that they produce no unpleasant disturbance of the united sound. These exceptional cases are called *Consonances.*

"1. The most perfect *consonances* are those that have been here called *absolute*, in which the prime tone of one of the combined notes coincides with some partial tone of the other. To this group belong the *Octave*, *Twelfth*, and *double Octave.*

"2. Next follow the *Fifth* and the *Fourth*, which may be called *perfect consonances*, because they may be used in all parts of the scale without any important disturbance of harmoniousness. The Fourth is the less perfect consonance, and approaches those of the next group. It owes its superiority in musical practice simply to its being the defect of a Fifth from an Octave, a circumstance to which we shall return in a later chapter.

"3. The next group consists of the *major Sixth* and the *major Third*, which may be called *medial consonances.* The old writers on harmony considered them as imperfect consonances. In lower parts of the scale the disturbance of the harmoniousness is very sensible, but in the higher positions it disappears, because the beats are too rapid to be sensible. But each, in good musical qualities of tone has an independent character, because any little defect in its intonation produces sensible beats of the upper partials, and consequently each interval is clearly separated from all adjacent intervals.

"4. The *imperfect consonances*, consisting of the *minor Third* and *minor Sixth*, have not in general an independent

character, because in good musical qualities of tone the partials on which their definition depends are often not found for the minor Third, and are generally absent for the minor Sixth, so that small imperfections in the intonation of these intervals do not necessarily produce beats. They are all less suited for use in lower parts of the scale than the others, and they owe their precedence as consonances over many other intervals which lie on the boundaries of consonance and dissonance, essentially to their being indispensable in the formation of chords because they are defects of the major Sixth and major Third from the Octave or Fifth. The sub-minor Seventh, 4 : 7, is very often more harmonious than the minor Sixth, 5 : 8; in fact, it is always so when the third partial tone of the note is strong compared with the second, because then the Fifth has a more powerfully disturbing effect on the intervals distant from it by a Semitone, than the Octave on the sub-minor Seventh, which is rather more than a whole Tone removed from it. But this sub-minor Seventh when combined with other consonances in chords produces intervals which are all worse than itself, as 6 : 7, 5 : 7, 7 : 8, &c., and it is consequently not used as a consonance in modern music.

"5. By increasing the interval by an Octave, the Fifth and major Third are improved on becoming the Twelfth and major Tenth. But the Fourth and major Sixth become worse as the Eleventh and major Thirteenth. The minor Third and minor Sixth, however, become still worse as the minor Tenth and minor Thirteenth, so that the latter intervals are far less harmonious than the sub-minor Fourteenth, 2 : 7, and sub-minor Tenth, 3 : 7."

CHAPTER XIV.

COMBINATIONAL TONES.

If upon a loud-toned harmonium two notes, say at an interval of a major third or a perfect fourth or fifth, are held down together, and the tone be steadily sustained, a third sound will be heard; and the pitch of this third sound depends always upon the distance between the two notes which by their combined vibrations produce it. These third tones were found out as long ago as 1740, by a German organist named Sorge, who wrote a treatise in which he endeavoured to explain them. Tartini, the Italian violinist, was also accustomed to instruct his pupils when playing double stops that they could always be sure they were playing accurately in tune if this third tone could be heard, but not otherwise. "These tones," says Helmholtz, "are heard whenever two musical tones of different pitches are sounded together loudly and continuously. The pitch of the combinational tone is generally different from that of either the generating tones or their harmonic upper partials. In experiments, then, the combinational are readily distinguished from the upper partial tones by not being heard when only one generating tone is sounded,

and by appearing simultaneously with the second tone. Combinational tones are of two kinds. The first class discovered by Sorge and Tartini I have termed *differential tones*, because their vibrational number is the *difference* of the vibrational numbers of the joint tones. The second class of *summational tones*, having their vibrational numbers equal to the sum of the vibrational numbers of the generating tones, were discovered by myself."

There are, of course, not only the combinational tones of the prime tones of the pair, but also—though these latter are heard only with very great difficulty—those combinational tones which result from the union of the upper partials. These, however, we need not consider at present, as, if the student is familiar with the law which operates to produce the combinational tones of primes, he will without difficulty be able to calculate for himself what are those produced between any pair of upper partials.

Taking then, to begin with, a major third, for instance, tenor C and the E next above it, we have this result. The vibration number of the note C being 264, that of its major third is 330; the difference between these two numbers will give the vibration number of the combinational tone, and therefore its pitch. This number is sixty-six, and a calculation will show that the pitch of this note is two octaves deeper than the lower of the two tones by which it is generated.

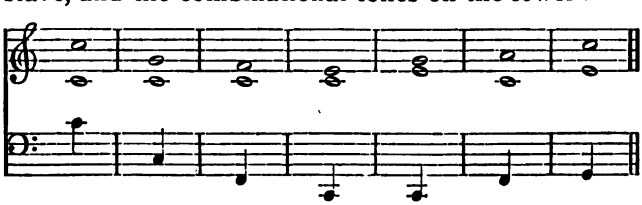
Now take a perfect fifth. If the same note C be taken as the lower of the two, the vibration number

of the G above it will be 396, and the vibration number of the combinational tone will be the difference between 264 and 396, *i.e.*, 132, or a note exactly an octave below that of the lower of the two generating tones.

The following table will show the different combinational tones derived from the usual intervals when combined with the tenor C:—

C = 264.	Vibration Numbers.	Vibration Numbers of Combinational Tone.	Combinational Tone, lower than the less Generated Tone by
Octave . . .	528	264	Unison.
Fifth . . .	396	132	Octave.
Fourth . . .	352	88	Twelfth.
Major Third .	330	66	Two Octaves.
Minor Third .	$316\frac{4}{5}$	$52\frac{4}{5}$	Two Octaves and Major Third.
Major Sixth .	440	176	Fifth.
Minor Sixth .	$422\frac{2}{5}$	$158\frac{2}{5}$	Major Sixth.

Put in musical symbols this table would show the following results, the generators being on the upper stave, and the combinational tones on the lower:—



The student should calculate the vibrational num-

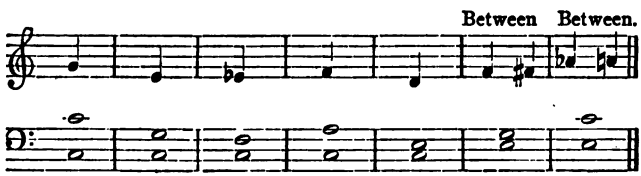
bers of the combinational tones resulting from the sounding of these intervals in different octaves of the harmonium.

"When the ear has learned to hear the combinational tones of pure intervals and sustained tones, it will be able to hear them from inharmonic intervals and in the rapidly dying notes of a pianoforte. The combinational tones from inharmonic intervals are more difficult to hear, because these intervals beat more or less strongly, as we shall have to explain hereafter. Those arising from tones which rapidly die off, as those of the pianoforte, are not strong enough to be heard except at the first instant, and die off more rapidly than the generating tones. Combinational tones are also in general easier to hear from the simple tones of tuning-forks and stopped organ pipes than from compound tones where a number of other secondary tones are also present. These compound tones, as has been already said, also generate a number of differential tones by their harmonic upper partials, and these easily distract attention from the differential tones of the primes. Combinational tones of this kind, arising from the upper partials, are frequently heard from the violin and harmonium.

"*Example.*—Take the major Third $c'e'$, ratio of vibrational numbers 4 : 5. First difference 1, that is C. The first harmonic upper partial of c' is c'', vibrational number 8. Ratio of this and e', 5 : 8, difference 3, that is g. The first upper partial of e' is e'', vibrational number 10; ratio for this and c', 4 : 10, difference 6, that is g'. Then again $c''e''$ have ratio 8 : 10, difference 2, that is c. Hence, taking only the first upper partials, we have the series of combinational tones 1, 3, 6, 2 or C, g, g', c. Of these the tone 3, or g, is often easily perceived."—*Helmholtz.*

What Helmholtz has called differential tones are

only one species of combinational tone, the others being discovered by himself, namely, summational tones, which result usually in a higher tone than either of the two generating tones, the differentials resulting usually in a lower tone. These summational tones are obtained by subtracting the lesser vibrational number from the greater of the two generators. Summational tones, like differential tones, may also arise from the upper partials as well as from the primes. The following table gives the summational tones in musical type, the generators being on the lower stave:—



There are some instruments which give combinational tones so powerful as to be easily heard by the unaided ear. The condition under which they are produced, is that the same body of air is agitated by two simple tones at the same time. The syren, which has been before described, illustrates the theory of combinational tones better than anything else, as its sounds are at once simple and powerful.

Mr. A. J. Ellis, in a note on p. 231 of his translation of Helmholtz, says:—

"I have found that combinational tones can be made quite audible to a hundred people at once, by means of

two flageolet fifes or whistles, blown as strongly as possible. I choose very close dissonant intervals, because the great depth of the low tone is very striking, being very far below anything that can be touched by the instrument itself. Thus g'''' being sounded loudly on one pipe by an assistant, I give $f''''\sharp$, when a deep note is instantly heard which, if the interval were pure, would be g, and is sufficiently near to g to be recognised as extremely deep. As a second experiment, the g'''' being held as before, I give first $f''''\sharp$ and then e'''' in succession. If the intervals were pure, the combinational tones would jump from g to c'', and in reality, the interval is very nearly the same and quite appreciable. The differential tones are well heard on the English concertina, especially if the ear be placed against the bellows and the interval be small. Semitones tell well. The tones $f'''\sharp$ and g''', and e''' and g''' are here also instructive, but are not so well fitted for a large audience as the fifes. It is also convenient to choose these dissonant intervals for first examples, in order to dissipate the notion that the 'grave harmonic' is the 'true fundamental bass' of the 'chord.' Again, to dissipate the notion that the combinational tones arise from the number of beats produced, I find it best to use on the English concertina g'' and $g''\sharp$, or f'' and $f''\sharp$ as generators. By proper attention it is then quite possible to hear the *booming* of the very low differential tones in the contra octave, and the *rattle* of the beats, *at the same time.* Generally when the ear has come to recognise the combinational tones, the performer himself will find them inconveniently prominent on the English concertina."

The harmonium is also a good instrument for their production, and on some harmoniums the combinational tones, being almost as powerful as the generat-

ing tones themselves, become exceedingly unpleasant. These combinational tones are always produced more perfectly by two notes sounding on one instrument, than by two instruments or two singers together.

Beside those produced between the primes, there are also with compound tones combinational tones generated between the primes and their upper partials; but some of these are only heard with extreme difficulty, though they materially affect the smoothness of a chord. Combination tones of the *second order* are those generated between one of the primes and the *first* order combination tone, and to discover their vibration number and pitch will be an excellent exercise for the student. These grow very rapidly weaker as we proceed, and it is doubtful whether those of the *fourth* order have ever been heard at all.

"Particular instruments give very powerful combinational tones. The condition for their generation is that the same mass of air should be violently agitated by two simple tones simultaneously. This takes place most powerfully in the polyphonic syren, in which the same rotating disc contains two or more series of holes which are blown upon simultaneously from the same windchest. The air of the windchest is condensed whenever the holes are closed; on the holes being opened, a large quantity of air escapes, and the pressure is considerably diminished. The mass of air in the windchest, and partly even in the bellows, as can be easily felt, comes into violent vibration. If two rows of holes are blown, vibrations arise in the air of the windchest corresponding to both tones, and each row of openings gives vent not to a stream of air uniformly supplied, but

to a stream of air already set in vibration by the other tone. Under these circumstances the combinational tones are extremely powerful, almost as powerful, indeed, as the generators. Their objective existence in the mass of air can be proved by vibrating membranes tuned to be in unison with the combinational tones. Such membranes are set in sympathetic vibration immediately upon both generating tones being sounded simultaneously, but remain at rest if only one or other of them is sounded. Indeed, in this case the summational tones are so powerful that they make all chords extremely unpleasant which contain Thirds or minor Sixths. Instead of membranes it is more convenient to use the resonators already recommended for investigating harmonic upper partial tones. But resonators are also unable to reinforce a tone when no pendular vibrations actually exist in the air; they have no effect on a tone which exists only in auditory sensation, and hence they can be used to discover whether a combinational tone is objectively present. They are much more sensitive than membranes, and are well adapted for the clear recognition of very weak objective tones.

"The conditions in the harmonium are similar to those in the syren. Here, too, there is a common windchest, and when two keys are pressed down, we have two openings which are closed and opened rhythmically by the tongues. In this case also the air in the common receptacle is violently agitated by both tones, and air is blown through each opening which has been already set in vibration by the other tongue. Hence in this instrument also the combinational tones are objectively present, and comparatively very distinct, but they are far from being as powerful as on the syren, probably because the windchest is very much larger in proportion to the openings, and hence the air which escapes during the short opening of an exit by the

oscillating tongue cannot be sufficient to diminish the pressure sensibly. In the harmonium also the combinational tones are very clearly reinforced by resonators tuned to be in unison with them, especially the first and second differential and the first summational tone. Nevertheless, I have convinced myself by particular experiments, that even in this instrument the greater part of the force of the combinational tone is generated in the ear itself. I arranged the portvents in the instrument so that one of the two generators was supplied with air by the bellows moved below by the foot, and the second generator was blown by the reserve bellows, which was first pumped full and then cut off by drawing out the so-called expression-stop, and I then found that the combinational tones were not much weaker than for the usual arrangement. But the objective portion which the resonators reinforce was much weaker. The noted examples given above will easily enable any one to find the digitals which must be pressed down in order to produce a combinational tone in unison with a given resonator."—*Helmholtz.*

CHAPTER XV.

CONSONANT CHORDS.

WHEN more than two tones are sounded together they are called a chord; and it is in this chapter our object to examine the harmoniousness of different chords, just as we have previously examined the harmoniousness of two tones sounded at once.

There are within the octave only three triads or chords of three notes which are consonant; they are as follows:—

C, E, G
C, E flat, G

C, F, A
C, F, A flat

C, E flat, A flat
E, C, A

These triads are called major or minor, according as the third from the fundamental note is composed of three semitones or four, that is—a minor or major third.

The first, C, E, G, is a major triad, the second is a minor triad, the third is called a chord of the 6: 4 (figured thus, $\frac{6}{4}$), and the fourth is also a chord of the $\frac{6}{4}$. The fifth is a chord of the fundamental, its third and

sixth, the sixth in this case being minor; and the last is an ordinary chord of the sixth.

These chords will be more or less harmonious according to the consonance of the intervals they contain. We have seen that the fifth is more agreeable than the fourth, and that the major third or sixth is more agreeable than the minor third or sixth.

The following list will show the intervals of which these several triads and their various inversions are formed:—

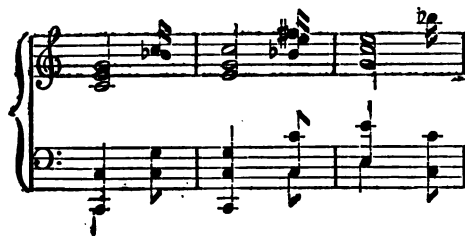
C, E, G has a 5th, major 3d, minor 3d.
E, G, C has a 4th, minor 3d, minor 6th.
G, C, E has a 4th, major 3d, major 6th.
C, E, F has a 5th, minor 3d, major 3d.
E flat, G, C has a 4th, major 3d, major 6th.
G, C, E flat has a 4th, minor 3d, minor 6th.

In just intonation thirds and sixths are less harmonious than fourths, in spite of the fact that the latter were always classed by the ancients as discords and the former as "imperfect concords." The minor triad, C, E flat, G, is decidedly less harmonious than the major triad C, E, G, although both contain a major and a minor third.

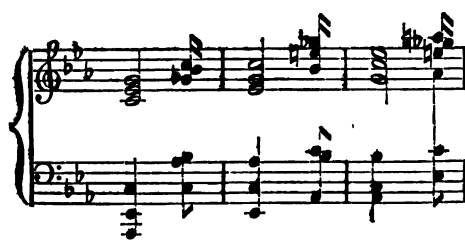
We have already seen that although two notes sounded together may produce no beats if simple tones, yet that the combinational tones of their upper partials may produce beats, and it follows that in order to determine fully and correctly the consonance of the various triads and their inversions, we must

find out what combinational tones they make in their various positions.

First of all, let us take the combinational tone of the major triad C, E, G, and its two inversions, E, G, C, and G, C, E.



Now let us look at the combinational tones of the minor triad C, E flat, G, and its first and second inversions.



In each of these examples the crotchets are the combinational tones resulting from the primes, those from the primes and the first upper partials being represented by quavers and semiquavers. It is thus easy to see that combinational tones affect the harmoniousness of the minor triads far more than that

of the major triads, because the combinational tones of the latter, whether of the first or second order (which are written as crotchets and quavers respectively), are only defects in the lower octaves of the notes of the triad itself. The second order of tones (semiquavers) are weak, and can scarcely be said to affect the general harmoniousness of the chord, these latter not having as yet been distinctly heard even when the ear has been assisted by resonators. They may therefore be left altogether out of account. But this is not so with the minor triads, the combinational tones of which of the first order being very audible, although not near enough to generate beats, are yet out of harmony. The second order of combinational tones, however, are so near to the tones of the triad that beats necessarily arise, while the corresponding tones of the major triads are themselves a part of the chord itself. An inspection of the musical examples given above will be sufficient to elucidate this subject.

The combinational tones which have the most influence are, of course, of the first order; and although in the case even of a minor chord they are not strong enough to altogether spoil the harmony, they are strong enough to give to it that veiled effect, for which the hearer cannot account, but which he cannot help feeling. Minor chords are, therefore, peculiarly fitted to express "mysterious obscurity or harshness;" and all the shades of feeling which minor chords represent are more or less "veiled." "Modern harmonists,"

says Helmholtz, "are unwilling to acknowledge that the minor triad is less consonant than the major. They have probably made all their experiments with tempered instruments, on which, indeed, this distinction may perhaps be allowed to be a little doubtful. But on justly-intoned instruments, and with a moderately piercing quality of tone, the difference is very striking and cannot be denied. The old musicians, too, who composed exclusively for the voice, and were consequently not driven to debilitate consonances by temperament, show a decided feeling for the difference. This must, I think, have been the reason for their avoidance of a minor chord at the close. The mediæval composers, down to Sebastian Bach, used for their closing chords either exclusively major chords or doubtful chords without the third; and even Handel and Mozart occasionally conclude a minor piece of music with a major chord. Of course, other considerations besides the degree of consonance have great weight in determining the final chord, such as the desire to mark the prevailing tonic or keynote with distinctness, for which purpose the major chord is decidedly superior."

Let us now look at the consonant triads which lie outside the compass of the octave. As a rule, consonant intervals remain consonant when one of their tones is transposed an octave higher or lower. In all the consonant chords, therefore, hitherto found, consonant intervals may be so transposed, and transposition sometimes improves rather than injures the harmoni-

ousness of the chord. Some of the intervals, however, have their harmoniousness affected by transposition. A major tenth is better than a major third, but minor tenths are not better than minor thirds, but worse; as also major and minor fifteenths are worse than minor sixths. The rule is this—*Those intervals in which the smaller of the two numbers expressing the ratios of the vibrational numbers is* EVEN, *are improved by having one of their tones transposed by an octave.*

The reason for that rule is, that the numbers expressing the ratio of the two notes are by transposition diminished; and it is a fact which ought by this time to be known to the student, that the less is the ratio between the vibration numbers of any two notes, the nearer to absolute purity of consonance do they become. For instance—

The 5th, 2 : 3, becomes the 12th, 4 : 3.

Major 3d, 4 : 5, becomes major 10th, 2 : 5.

Sub-minor 3d, 6 : 7, becomes sub-minor 10th, 3 : 7.

Another rule, or rather a part of the same rule, is—*That those intervals, in which the smaller of the two numbers expressing the ratio of the vibrational numbers is* ODD, *are made worse by having one of their tones transposed by an octave.* The reason for this rule is the reverse of that for the former. The result for the odd numbers is thus as follows:—

The perfect 4th, 3 : 4, becomes the 11th, 3 : 8.
Minor 3rd, 5 : 6, becomes minor 10th, 5 : 12.
Minor 6th, 3 : 5, becomes major 13th, 3 : 10.
Minor 6th, 5 : 8, becomes minor 13th, 5 : 16.

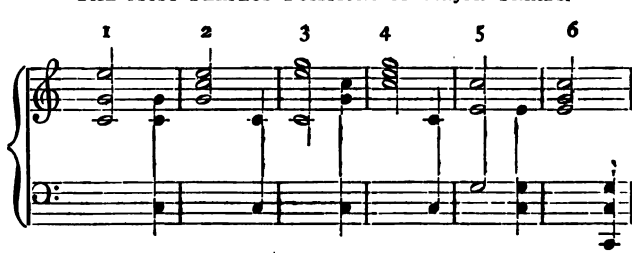
It will be found that the combinational tones of the

octave, fifth, twelfth, fourth, and major third are transpositions of one or other of the primary tones, and bring in, therefore, no tones foreign to the chord.

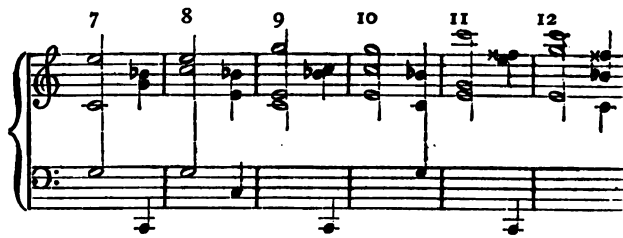
These can for this reason be used in all sorts of consonant triads, their combinational tones not interfering with the harmonious effect. The following intervals are injurious to minor chords by reason of their combinational tones, but not to major chords: —eleventh, minor third, major tenth, major sixth, minor sixth; but it will be found that neither the minor tenth nor the major or minor thirteenth can be introduced into a chord without imparting harshness by reason of their combinational tones.

Let us see now how this consideration affects the triads, and first the major triads. These can all be arranged so that the combinational tones form part of the chord; but it must be remembered that all minor thirds and sixths could be included herein, but that no minor tenths or thirteenths are admissible. The only possible positions of the major chords which are entirely free from disturbances from combinational tones are these six:

THE MOST PERFECT POSITIONS OF MAJOR TRIADS.



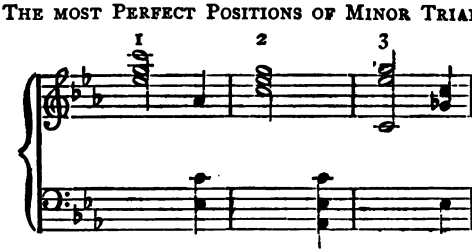
THE LESS PERFECT POSITIONS OF MAJOR TRIADS.



and it is doubtless unnecessary to point out, that these latter positions are those which are much less in use than the former.

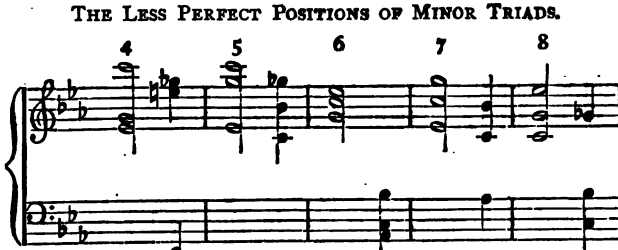
As concerns the minor triads, they can never be free from combinational tones. The best positions of minor triads are these—

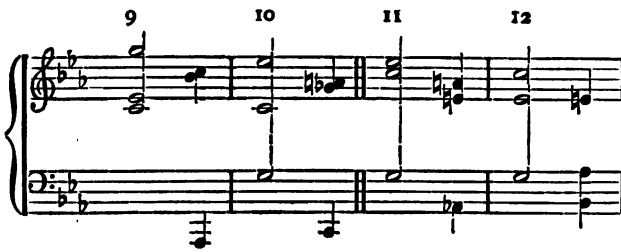
THE MOST PERFECT POSITIONS OF MINOR TRIADS.



These positions are not so good—

THE LESS PERFECT POSITIONS OF MINOR TRIADS.





It is apparent from its construction, that this simple fact of a chord being major or minor when started may make it more or less harmonious. It is not what is seen on paper which affects the resultant harmoniousness of major and minor chords; it is simply and entirely due to the combinational tones, which are developed by those chords and their inversions in different positions, which make the one or the other more or less acceptable and agreeable to the ear.

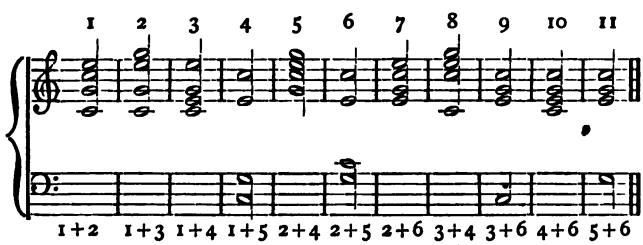
With respect to four-part chords, all such, to be consonant, must necessarily be made from the major and minor triads, and the one or the other of their notes doubled in the octave; and every consonant four-note chord will become a consonant triad by removing any one of its tones.

This can be done in either of these four ways:— C E G C may become either C E G, C E C, E G C, C G C; every consonant chord, therefore, of three prime tones must be major or minor, and may be formed from the triads by adding the octave of one of the three tones. To obtain the best positions of four-note chords, care must be taken that no minor

tenths or minor thirteenths occur, and the rule is—*That those major chords are most harmonious in which the fundamental tone, or the fifth, does not lie more than a sixth above the third, or the fifth does not lie more than a sixth below it.*

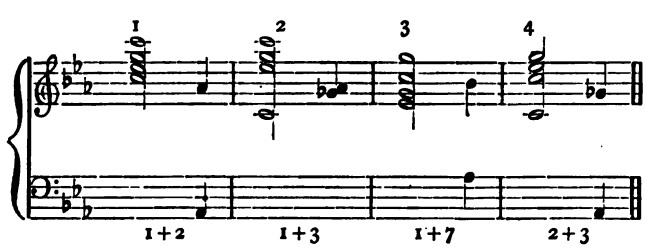
Within the compass of two octaves, the most perfect positions of four-note major chords are these—

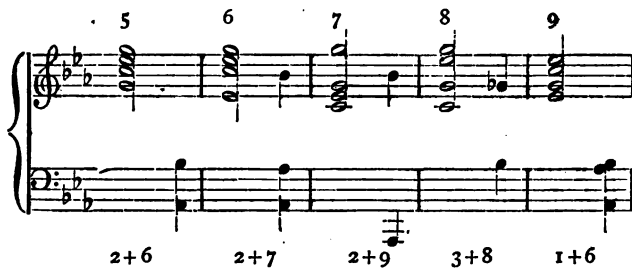
THE MOST PERFECT POSITIONS OF THE MAJOR TETRADS WITHIN THE COMPASS OF TWO OCTAVES.



and the best positions within the same compass of minor four-note chords are these—

BEST POSITIONS OF MINOR TETRADS.





It will be readily observed that those chords can be used most freely which have their fundamental note in the bass.

Helmholtz winds up his chapter on this subject with the following apt remarks:—

"The subject has been treated here at such length, in order so show that a right view of the cause of consonance and dissonance leads to rules for relations which previous theories of harmony could not contain. The propositions we have enunciated agree, however, with the practice of the best composers, of those, I mean, who studied *vocal music principally* (before the great development of instrumental music necessitated the general introduction of tempered intonation), as any one may easily convince himself by examining those compositions which aimed at producing an impression of perfect harmony. Mozart is certainly the composer who had the surest instinct for the delicacies of his art. Among his vocal compositions, the *Ave verum corpus* is particularly celebrated for its wonderfully pure and smooth harmonies. On examining this little piece, as one of the most suitable examples for our purpose, we find in its first clause, which has an extremely soft and sweet effect, none but major chords and chords of the dominant seventh. All these major chords belong to those which we have noted as having

the more perfect position. Position 2 occurs most frequently, and then 8, 10, 1, and 9. It is not till we come to the final modulation of this first clause, that we meet with two minor chords and a major chord in an unfavourable position. It is very striking, by way of comparison, to find that the second clause of the same piece, which is more veiled, longing, and mystical, and laboriously modulates through bolder transitions and harsher dissonances, has many more minor chords, which, as well as the major chords scattered among them, are for the most part brought into unfavourable positions, until the final chord again restores perfect harmony.

"Precisely similar observations may be made on those choral pieces of Palestrina, and of his contemporaries and successors, which have a simple harmonic construction without any involved polyphony. In transforming the Roman church music, which was Palestrina's task, the principal weight was laid on harmonious effect, in contrast to the harsh and unintelligible polyphony of the older Dutch system; and Palestrina and his school have really solved the problem in the most perfect manner. Here also we find an almost uninterrupted flow of consonant chords, with dominant sevenths, or dissonant passing notes, charily interspersed. Here also the consonant chords wholly or almost wholly consist of those major and minor chords which we have noted as being in the more perfect positions. But in the final cadence of a few clauses, on the contrary, in the midst of more powerful and more frequent dissonances, we find a predominance of the unfavourable positions of the major and minor chords. Thus that expression which modern music endeavours to attain by various discords and an abundant introduction of dominant sevenths, was obtained in the school of Palestrina by the much more delicate shading of various transpositions of consonant chords. This

explains the deep and tender expressiveness of the harmony of these compositions, which sound like the songs of angels, with hearts affected but undarkened by human grief in their heavenly joy. Of course such pieces of music require fine ears, both in singer and hearer, to let the delicate gradation of expression receive its due, now that modern music has accustomed us to modes of expression so much more violent and drastic.

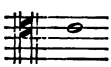
"The great majority of major tetrads in Palestrina's *Stabat Mater* are in the positions 1, 10, 8, 5, 3, 2, 4, 9, and of minor tetrads in the positions 9, 2, 4, 3, 5, 1. For the major chords, one might almost think that some theoretical rule led him to avoid the bad intervals of the minor tenth and thirteenth. But this rule would have been entirely useless for minor chords. Since the existence of combinational tones was not then known, we can only conclude that his fine ear led him to this practice, and that the judgment of his ear exactly agreed with the rules deducted from our theory.

"These authorities may serve to lead musicians to allow the correctness of my arrangement of consonant chords in the order of their harmoniousness. But any one can convince himself of their correctness on any justly-intoned instrument. The present system of tempered intonation certainly obliterates somewhat of the more delicate distinctions, without, however, entirely destroying them."

CHAPTER XVI.

SCALES AND TEMPERAMENTS.

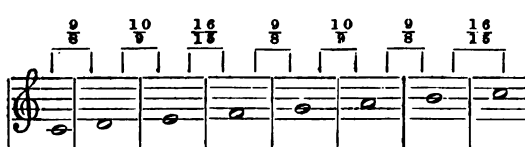
(*A.*) *Construction of the Musical Scale.*

If the reader will refer to that part of Chapter V. which treats of "Pitch and Rapidity of Vibration," he will find a table which gives the vibration numbers of each note of the scale, commencing on the note  . Let us now look more closely into the materials of which this scale is composed.

"It follows from what has been said that the number of possible notes, all differing from each other in pitch, is theoretically unlimited, inasmuch as any difference in the vibration-number will certainly give rise to a different sound. Practically, also, the number is very large, depending only on the sensitiveness of the ear for minute differences of pitch; a skilful pianoforte tuner, for example, is obliged, in the exercise of his art, to distinguish between a true and an equally tempered fifth, the difference being only about *one-fiftieth of a semitone*, which would give 600 distinguishable sounds in the octave! The pianoforte has only 12 sounds in the octave, but the intervals between them may be easily divided into several parts, and it is usual to estimate that from 50 to 100 sounds in the octave may be distinguished by ordinary ears."—*Pole.*

It will be found, upon examination, that those notes whose vibration numbers stand to each other in the smallest ratios are the most consonant; or, putting it in another way, that the smaller the ratio in which the vibration numbers of any two notes stand to each other, the less discordance will there be between them. The only sounds which are, correctly speaking, absolutely consonant, are two sounds in unison with each other, whose vibration numbers, that is, are the same. The nearest approach to this is found in the octave, the vibration number of which is to the tonic as 2 : 1; and the tonic vibration number being 256, that of its octave will be $256 \times 2 = 512$. If a string is divided in half, each half will vibrate twice as rapidly as the whole string, and give a note an octave higher than the fundamental note. If now the string be touched at *one-third* of its length, the longer portion will vibrate half as rapidly again as the fundamental note, or in the proportion of 3 to 2; that is, its vibration number will be $\frac{256 \times 3}{2} = \frac{768}{2} = 384$. This note is G, the fifth, or dominant of the scale, and is next to the octave in order of consonance. If the string is touched at *one-fourth* of its length, the vibrations of the longer part will be to the whole length in the proportion of 4 to 3, or $\frac{256 \times 4}{3} = \frac{1024}{3} = 341\frac{1}{3}$. This is F, the fourth, or subdominant of the scale. In the same way we discover that the third of the scale (E) vibrates to the tonic in the proportion of 5 to 4, or $\frac{256 \times 5}{4} = \frac{1280}{4} = 320$; the sixth of the scale (A), in the proportion of 5 to

3, or $\frac{256 \times 5}{3} = \frac{1280}{3} = 426\frac{2}{3}$; the second of the scale (D), in the proportion of 9 to 8, or $\frac{256 \times 9}{8} = \frac{2304}{8} = 288$; and the major seventh of the scale (B), in the proportion of 15 to 8, or $\frac{256 \times 15}{8} = \frac{3840}{8} = 480$. We have, then, for the complete scale, the following results:—



Technical Name.	Tonic.	Supertonic.	Mediant.	Sub-dominant.	Dominant.	Sub-mediant.	Leading Note.	Octave of Tonic.
Interval from Tonic.		Major Second.	Major Third.	Perfect Fourth.	Perfect Fifth.	Major Sixth.	Major Seventh.	Octave.
Proportion to Tonic.	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
Actual Vibration Number per Second.	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512

The upper row of fractions shows the proportion of each note of the scale to that which precedes it: thus while D is to C as $\frac{9}{8}$: 1, E is to D as $\frac{10}{9}$: $\frac{9}{8}$. These fractions are obtained by dividing each fraction in the last line but one of the above table by that which precedes it. By this process we find that there are in

an exact scale not only the two kinds of interval—tone and semitone—commonly recognised, but that there are also two kinds of tone, a greater and a lesser tone. The distance from C to D is greater than that from D to E, by the difference between $\frac{9}{8}$ and $\frac{10}{9}$. The greater tones of the scale are from C to D, F to G, and A to B; the smaller tones from E to F, and from G to A; and the semitones, which are alike, are between E and B and B and C. There are two kinds of semitone: the *chromatic* C to C♯ ($\frac{25}{24}$), and the diatonic, E to F, or B to C ($\frac{16}{15}$), as above.

This is the construction of the major scale when all its notes bear their just and exact proportion to the tonic. There are three other intervals: the minor third, which is to the tonic as 6 : 5; the minor sixth, 8 : 5; and the minor seventh, 16 : 9. With the aid of these figures the student can easily calculate for himself the vibration numbers of any major or minor scale; and it will be an admirable exercise for him to calculate the vibration numbers of scales beginning on different notes, before proceeding any further.

"Why need there be any particular selection or limitation of the sounds to be used? Why cannot melody be made by using any we please out of the infinite number of sounds possible? Why is it necessary to proceed by steps, and forbidden to progress by continuous transitions? The question is a curious one. It appears to be a fact that all nations, in all times, who have made music have adopted such a selection, although they have not always selected the same series of sounds.

"Helmholtz is the only person who has attempted to give an answer to this question. His explanation is somewhat metaphysical and difficult, but it appears to be essentially as follows:

"He believes that the reason is a psychological one, and is of the same nature as the feeling which has led to rhythmical division in poetry and music.

"The essence of melody is motion; and this motion, in order to produce its proper effect, must be effected in such a manner that the hearer can easily, clearly, and certainly appreciate the character of that motion by *immediate perception.* But this is only possible when the steps of this motion—their rapidity and amount—are also exactly *measurable* by immediate perception. Therefore the distance between the various successive notes must be definite and positive, and the alterations in pitch must proceed by regular and easily appreciable degrees.

"It may be objected that continuous curved lines in design, addressed to the eye, not only produce a pleasing effect, but are usually considered more beautiful than angular stepped transitions of form; and by this analogy the continuous progression of sound might be supposed to be more pleasing to the ear than abrupt change.

"But Helmholtz has an ingenious answer to this. He says that the eye which contemplates curves can take in and compare all parts at once; or can at least return backwards and forwards, so as to get a comprehensive simultaneous idea of the whole. But the individual parts of a melody reach the ear in *succession;* we cannot observe backwards and forwards at pleasure. Hence, for a clear and sure measurement of the change of pitch, no means is left but progression by determinate degrees. When the wind howls, and its pitch rises or falls, in continuous gradation, we have nothing to define the variations of

pitch, nothing by which we can compare the later with the earlier sounds, and comprehend the extent of the change; the whole phenomenon produces a confused impression, which, whatever else may be its character, is certainly not music. The musical scale (or definite series of notes) is, as it were, a divided rod by which we measure progression in pitch, just as rhythm measures progression in time.

"Helmholtz shows, by a quotation from Aristotle, that the ancients had this idea of the analogy between the scale of tones and the scale of rhythms; and further remarks, that we consequently find the most complete agreement among all nations that use music at all, from the earliest to the latest times, as to the separation of certain determinate degrees of pitch, these degrees forming the *scale* in which the melody moves.

"Whether this explanation is fully satisfactory or not, at any rate it is novel and ingenious, and probably the best that can be given."—*Pole.*

Professor Airy, writing, in his work "On Sound," upon the "simple scale of music," explains it thus:—

"The notes in the greater part of the tunes, songs, hymns, &c., sung by persons not acquainted with artificial music, are included within the compass of an Octave. And, whatever notes are adopted within any Octave, if we adopt similar notes in the Octaves above and below it (meaning, by 'similar notes,' notes whose number of vibrations bear to the number of vibrations in the first note of the series the same proportion in one Octave as in the other), we infallibly secure a series of strong concords, and also give great facilities for notation. These appear to be the principal reasons which have induced mankind to use, in keyed instruments (as the pianoforte and organ) or

in stringed instruments where no alteration is made, during musical performance, in the length of strings (as the harp), a series of notes defined in one Octave by the concords given in Article 87, with some additions; and to repeat them in other Octaves above and below; and even to mark them with the same letters.

"The letters are A, B, C, &c., to G. Apparently at some time in the history of Music, A was considered the fundamental note. But in modern Music, C is always considered the fundamental, in the same sense in which it is taken in Article 87.* Using then the proportions in that Article, we have for the notes (omitting those called Minor) the following proportionate number of vibrations :—

Fundamental.	Major Third.	Fourth.	Fifth.	Major Sixth.	Octave.
1	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	2
C	E	F	G	a	c

These notes are sufficient for common music. (The relations of adjacent notes are not very harmonious; for instance, the proportion of E : F is 15 : 16; that of F : G is 8 : 9; but all are closely related to C, which has acquired the name of 'key-note.') But, with a very small extension of musical desires, we find that other notes are required. Having sounded any note, perhaps we desire to associate with it the Third above it. We must multiply the fraction for the Third, or $\frac{5}{4}$, by the fraction for the note. This, applied to the Third and Sixth, gives $\frac{5}{4} \times \frac{5}{4}$ or $\frac{25}{16}$, and $\frac{5}{4} \times \frac{5}{3}$ or $\frac{25}{12}$, in which the numbers are too large (Art. 87); applied to the Fourth, or $\frac{4}{3}$, it produces $\frac{5}{4} \times \frac{4}{3}$, or $\frac{5}{3}$, or the

* Article 87 in Airy treats of the octave, fifth, fourth, third, &c., as deduced from the tonic.

Sixth; applied to the Fifth, it produces $\frac{5}{4} \times \frac{3}{2}$, or $\frac{15}{8}$, in which the numbers are not excessively large, and which falls well between the Sixth and the Octave. This proportion $\frac{15}{8}$ is therefore adopted as Seventh, with the letter b. If we desire to associate with any note its Fourth, whose factor is $\frac{4}{3}$; applied to the Fourth or $\frac{4}{3}$ it gives $\frac{16}{9}$ (which we may consider as a 'flat Seventh'); applied to the Fifth or $\frac{3}{2}$ it produces $\frac{4}{3} \times \frac{3}{2}$, or $\frac{2}{1}$, or the Octave; applied to the Sixth, it produces $\frac{20}{9}$, or (as referred to the Octave) $\frac{10}{9}$ (which when we have found a note preferable for adoption as Second we may consider as a 'flat Second'). If we desire to associate with any note its Fifth; the application to the Fifth gives $\frac{9}{4}$ or (as referred to the Octave) $\frac{9}{8}$. This falls well between C and E; and the note which is an octave below it is adopted in the same place in the first Octave as Second, with the letter D. The Fifth applied to the Sixth gives $\frac{5}{2}$, or (referred to the Octave) $\frac{5}{4}$ which is the Third or e. Thus we find that only two new notes, namely $\frac{9}{8}$ and $\frac{15}{8}$, are to be inserted in our series; and it now stands thus:

Fundamental.	Second.	Major Third.	Fourth.	Fifth.	Major Sixth.	Seventh.	Octave.
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
C	D	E	F	G	a	b	c

"The reason for the term *Octave* is now obvious. The scale which we have thus obtained is called the 'Major Scale,' or sometimes the 'Diatonic Scale.' It is universally recognised as the foundation of Music.

"Sometimes the Minor Third and Minor Sixth, $\frac{6}{5}$ and $\frac{8}{5}$, are substituted for the Major Third and Major Sixth, producing the 'Minor Scale.' These two new notes, though

well connected together, are not well related to the other notes; and they produce a partially discordant music, of peculiar character, usually melancholy."

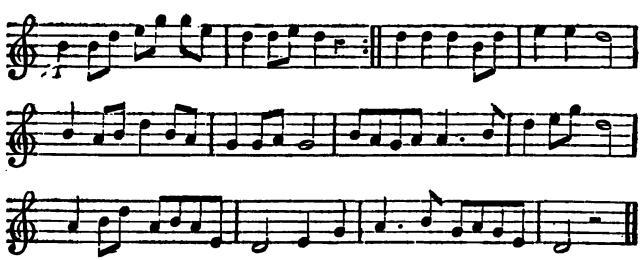
It is not quite correct to say that the major scale is universally recognised as the basis of music, seeing that whole nations (as, for instance, the Scotch and the Chinese) have used scales differing in material points from the major and minor modes which are the foundation of modern music. Helmholtz says:—

"Among the Chinese, a certain prince Tsay-yu is said to have introduced the scale of seven notes amid great opposition from conservative musicians. The division of the Octave into twelve semitones, and the transposition of scales have also been discovered by this intelligent and skilful nation. But the melodies transcribed by travellers mostly belong to the scale of five notes. The Gaels and Erse have likewise become acquainted with the diatonic scale of seven tones by means of psalmody, and in the present form of their popular melodies the missing tones are sometimes just touched as appoggiature or passing notes. These are, however, in many cases merely modern improvements, as may be seen on comparing the older forms of the melodies, and it is usually possible to omit the notes which do not belong to the scale of five tones without impairing the melody. This is not only true of the older melodies, but of more modern popular airs which were composed during the last two centuries, whether by learned or unlearned musicians. Hence the Gaels as well as the Chinese, notwithstanding their acquaintance with the modern tonal system, hold fast by the old. And it cannot be denied that by avoiding the semitones of the

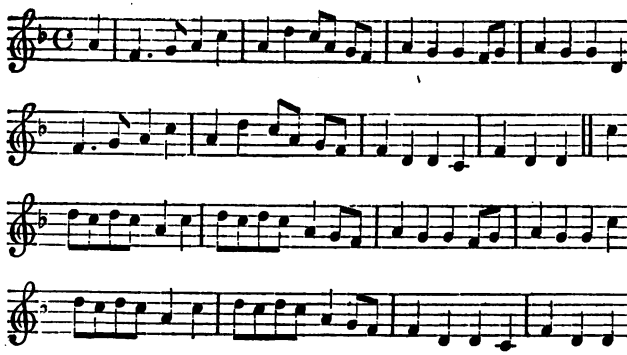
diatonic scale, Scotch airs receive a peculiarly bright and mobile character, although we cannot say as much for the Chinese. Both Gaels and Chinese make up for the small number of tones within the Octave by great compass of voice."

Helmholtz, in his chapter on "The Tonality of Homophonic Music," enumerates five scales which differ more or less from our modern major scale. These all consist of five tones selected from the seven now in use.

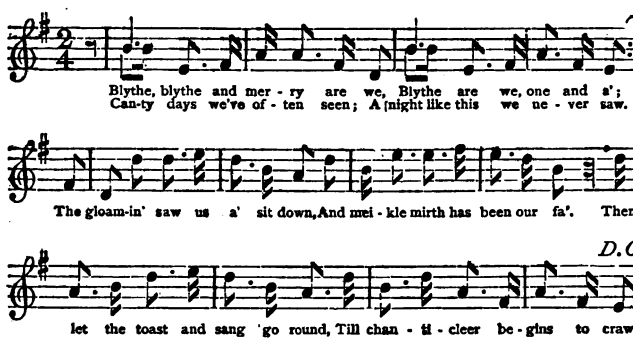
The first is a scale without a third or seventh, used, according to Barrow, by the Chinese, as will be seen from the following melody:—



Another has no second or sixth, and to this scale belong most Scotch airs of a minor character. "In the modern forms of these airs one or other of the missing tones," he says, "is often transiently touched." The older form of the air "Cockle Shells" is a specimen of this scale:—

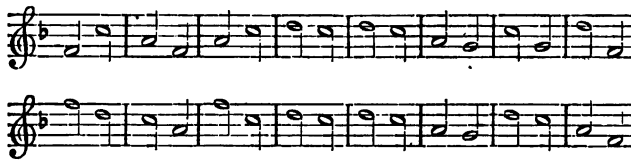


A third scale is without third or sixth, and is also Scotch :—



Mr. Ellis, in a note, refers to another tune of a similar character, which occurs in "Songs of Scotland," vol. iii. p. 10, and in which the sixth occurs but once.

A fourth scale has no fourth or seventh, and most Scotch melodies in the major are in this scale. The specimen given by Helmholtz in this scale is Chinese, and being in F, has no B♭ or E :—



Of the fifth scale, without second or fifth, Helmholtz says he has found no perfectly pure examples, but alludes to several tunes in which the second and fifth are but transiently touched.

It would be apart from our purpose to enter into a discussion respecting the gradual development of the Greek scales or modes. Most students are aware that the modern major and minor scales are those which have survived because they are the fittest for the purposes of modern harmony, with its frequent changes of key, and its constantly-present sensitive or leading note. The effect of this note and its great importance in modern music are elucidated in an admirable manner in Professor Macfarren's "Six Lectures on Harmony." It will be sufficient here to allude to the Greek scales or modes for the purpose of showing the source of the two modes now in use. Helmholtz tabulates them thus:—

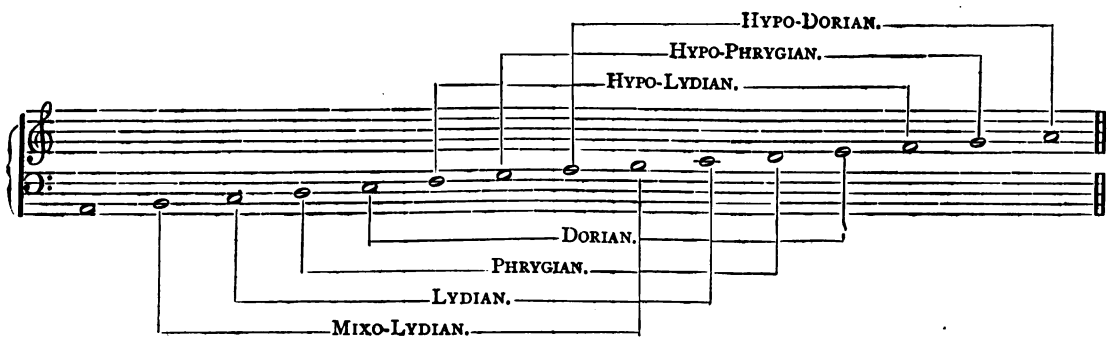
Ancient Greek Names	Scales beginning with *c*	Glarean's Ecclesiastical Names	Proposed new Names
			Mode of the
1. Lydian .	*c–d –e –f –g –a –b –c′*	Ionic	First (*major*)
2. Ionic . .	*c–d –e –f –g –a –b♭–c′*	Mixo-lydian	Fourth
3. Phrygian .	*c–d –e♭–f –g –a –b♭–c′*	Doric	minor Seventh
4. Eolic . .	*c–d –e♭–f –g –a♭–b♭–c′*	Eolic	minor Third (*minor*)
5. Doric . .	*c–d♭–e–f♭–g –a♭–b♭–c′*	Phrygian	minor Sixth
6. Mixo-lydian.	*c–d♭–e♭–f –g♭–a♭–b♭–c′*	Lydian	minor Second
7. Syntono-lydian .	*c–d –e –f♯–g –a –b –c′*		Fifth

This is Dr. Pole's table of modes:—

Church Name.	Name given by Glareanus.	Original name of the corresponding Greek octave-forms.	Limiting notes of the octave, when expressed on the white keys of the pianoforte.
First Mode, Authentic	*Dorian*	*Phrygian*	D.
Second ,, Plagal of the 1st	*Hypo-Dorian*	None corresponding.	—
Third ,, Authentic	*Phrygian*	*Dorian*	E.
Fourth ,, Plagal of the 3d	*Hypo-Phrygian*	None corresponding.	—
Fifth ,, Authentic	*Lydian*	*Hypo-Lydian*	F.
Sixth ,, Plagal of the 5th	*Hypo-Lydian*	None corresponding.	—
Seventh ,, Authentic	*Mixo-Lydian*	*Hypo-Phrygian*	G.
Eighth ,, Plagal of the 7th	*Hypo-Mixo-Lydian*	None corresponding.	—
Modern major. No church mode corresponding	*Ionian*	*Lydian*	C.
Modern minor. No church mode corresponding	*Æolian*	*Hypo-Dorian*	A.
None corresponding	None corresponding	*Mixo-Lydian*	A.

The accompanying diagram gives the Greek greater scale,* and shows the derivation of our major and minor scales:—

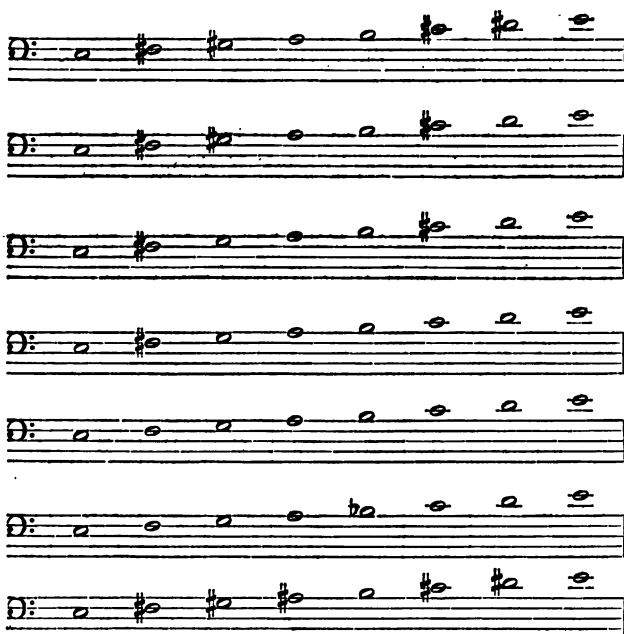
DIAGRAM *Illustrating the Nature of the Succession of Intervals in the various Greek Modes or Octave-Forms.*



* For an admirable and well-nigh exhaustive explanation of Greek music, see Stainer and Barrett's Dictionary *in loco.*

Helmholtz' table gives the modes with C as the starting point. They are here given in musical characters in the same order, but starting from E :—

THE SEVEN GREEK MODES, OR OCTAVE-FORMS, APPLIED TO THE SAME PITCH.



(*B.*) *Exact and Tempered Intonation.*

The student will now be ready to examine how the scale is affected by temperament.

Let us take F as a tonic, and see what the vibration numbers will be. The scale will contain F, G,

A, B flat, C, D, E, F. The vibration numbers of these notes, all of which, except the B flat, are contained by name in the scale of C, are as follows:—F 352, G 396, A 440, B flat $455\frac{1}{9}$, C 528, D 576, E 660, F 704. It will be seen, however, on calculating the vibration numbers in the scale of F, according to the just proportions above given, that the numbers will not all agree. Thus, the vibration numbers of the correct scale of F, the tonic of which vibrates 352 times in a second, are as under:—F 352, G 396, A 440, B flat $469\frac{1}{3}$, C 528, D $563\frac{1}{5}$, E 660, F 704. The discrepancies of these two calculations—the one containing notes taken from the C scale, the other calculated from their own tonic, F—are seen when the vibration numbers of the two scales are placed together, thus—

	F	G	A	B♭ *	C	D *	E	F
Tonic C...	352	396	440	$455\frac{1}{9}$	528	576	660	704
Tonic F...	352	396	440	$469\frac{1}{3}$	528	$563\frac{1}{5}$	660	704

The vibration numbers of the scales which can be taken on each note of the scale of C as a tonic, are as follows:—

Tonic	C 264.	D 297.	E 330.	F 352.	G 396.	A 440.	B 495.
Second . . .	297	$334\frac{1}{8}$	$371\frac{1}{4}$	396	$445\frac{1}{2}$	495	$556\frac{7}{8}$
Major Third .	330	$371\frac{1}{4}$	$412\frac{1}{2}$	440	495	550	$618\frac{3}{4}$
Fourth . . .	352	396	440	$469\frac{1}{3}$	528	$586\frac{2}{3}$	660
Fifth	396	$445\frac{1}{2}$	495	528	594	660	$742\frac{1}{2}$
Major Sixth .	440	495	550	$586\frac{2}{3}$	660	$733\frac{1}{3}$	825
Major Seventh.	495	$556\frac{7}{8}$	$618\frac{3}{4}$	660	$742\frac{1}{2}$	825	$928\frac{1}{8}$

It will thus be seen that no single scale, if it be itself exactly tuned, can furnish complete materials for any other scale. Remembering the proportions which the vibration numbers of each note of the scale bear to their tonic, we shall have, whatever note be taken for that tonic, a scale on the principle of what is called "exact" or "just intonation," that is, a justly-intoned scale; and the figures which we have given will show, for instance, that the major third of any scale will not serve as the supertonic of a scale beginning a note higher, because the vibration number of, say, the note E in the above scale of C, when serving as a major third with C as a tonic, and vibrating 330 times in a second, will not exactly represent the E required for the second note in the scale of which D is the tonic, because this latter E should vibrate $334\frac{1}{8}$

times per second. The moving of the tonic unhinges the scale, so to speak, and renders some compromise necessary.

This compromise is called "Temperament;" that is to say, the various notes of one scale, not furnishing materials for justly-intoned scales in all cases, must necessarily be tempered to enable them to serve as nearly as may be all the purposes for which they may be required in other scales.

"All or some of these reasons, then, made it necessary for musicians to have free command over the pitch of the tonic, and hence even the later Greeks transposed their scales on to all degrees of the chromatic scale. For singers these transpositions offer no difficulties. They can begin with any required pitch, and find in their vocal instrument all such of the corresponding degrees as lie within the extreme limits of their voice. But the matter becomes much more difficult for musical instruments, especially for such as only possess tones of certain definite degrees of pitch. The difficulty is not entirely removed even on bowed instruments. It is true that these can produce every required degree of pitch; but players are unable to hit the pitch as correctly as the ear desires without acquiring a certain mechanical use of their fingers, which can only result from an immense amount of practice.

"The Greek system was not accompanied with great difficulties, even for instruments, so long as no deviations into remote keys were permitted, and hence but few marks of sharps and flats had to be used. Up to the beginning of the seventeenth century musicians were content with two signs of depression for the notes *B*♭ and *E*♭, and with the sign ♯ for *F*♯, *C*♯, *G*♯, in order to have the leading

tones for the tonics G, D, and A. They took care to avoid the enharmonically equivalent tones $A\sharp\sharp$ for $B\flat$, $D\sharp\sharp$ for $E\flat$, $G\flat$ for $F\sharp\sharp$, $D\flat$ for $C\sharp\sharp$, and $A\flat$ for $G\sharp\sharp$. By help of $B\flat$ for B every tonal mode could be transposed to the key of its subdominant, and no other transposition was made.

"In the Pythagorean system, which maintained its predominance over theory to the time of Zarlino in the sixteenth century, tuning proceeded by ascending Fifths, thus—

$$C \quad G \quad D \quad A \quad E \quad B \quad F\sharp\sharp \quad C\sharp\sharp \quad G\sharp\sharp \quad D\sharp\sharp \quad A\sharp\sharp \quad E\sharp\sharp \quad B\sharp\sharp$$

"Now if we tune two Fifths upwards and an Octave downwards, we make a step having the ratio $\frac{3}{2} \times \frac{3}{2} \times \frac{1}{2} = \frac{9}{8}$, which is a major second. This gives for the pitch of every second tone in the last list—

$$\begin{array}{ccccccc} C & D & E & F\sharp\sharp & G\sharp\sharp & A\sharp\sharp & B\sharp\sharp. \\ 1 & \frac{9}{8} & (\frac{9}{8})^2 & (\frac{9}{8})^3 & (\frac{9}{8})^4 & (\frac{9}{8})^5 & (\frac{9}{8})^6 \end{array}$$

"Now if we proceed *down*wards by Fifths from C we obtain the series—

$$C \; F \; B\flat \; E\flat \; A\flat \; D\flat \; G\flat \; C\flat \; F\flat \; B\flat\flat \; E\flat\flat \; A\flat\flat \; D\flat\flat.$$

"If we descend two Fifths and rise an Octave, we may obtain the tones—

$$\begin{array}{ccccccc} \mathrm{C} & \mathrm{B\flat} & \mathrm{A\flat} & \mathrm{G\flat} & \mathrm{F\flat} & \mathrm{E\flat\flat} & \mathrm{D\flat\flat} \\ 1 & (\frac{8}{9}) & (\frac{8}{9})^2 & (\frac{8}{9})^3 & (\frac{8}{9})^4 & (\frac{8}{9})^5 & (\frac{8}{9})^6 \end{array}$$

Now the interval $(\frac{8}{9})^6 = \frac{262144}{531441} = \frac{1}{2} \times \frac{524288}{531441}$.
or, approximately $(\frac{8}{9})^6 = \frac{1}{2} \times \frac{73}{74}$
$(\frac{8}{9})^6 = 2 \times \frac{74}{73}$.

"Hence the tone $B\sharp\sharp$ is higher than the Octave of C by the small interval $\frac{74}{73}$, and the tone $D\flat\flat$ is lower than the Octave below C by the same interval. If we ascend by

perfect Fifths from *C* and *D*♭♭, we shall find the same constant difference between

C G D A E B F♯ C♯ G♯ D♯ A♯ E♯ B♯ and
D♭♭ A♭♭ E♭♭ B♭♭ F♭ C♭ G♭ D♭ A♭ E♭ B♭ F C.

"The tones in the upper line are all higher than those in the lower by the small interval $\frac{74}{73}$. Our staff notation had its principles settled before the development of the modern musical system, and has consequently preserved these differences of pitch. But for practice on instruments with fixed tones the distinction between degrees of tone which lie so near to each other was inconvenient, and attempts were made to fuse them together. This led to many imperfect attempts, in which individual intervals were more or less altered in order to keep the rest true, producing the so-called *unequal temperaments*, and finally to the system of *equal temperament*, in which the Octave was divided into twelve precisely equal degrees of tone. We have seen that we can ascend from *C* by 12 perfect Fifths to *B*♯♯, which differs from *C* by about $\frac{1}{5}$ of a semitone, namely by the interval $\frac{74}{73}$. In the same way we can descend by 12 perfect Fifths to *D*♭♭, which is as much lower than *C*, as *B*♯♯ is higher. If, then, we put $C = B\sharp\sharp = D\flat\flat$, and distribute this little deviation of $\frac{74}{73}$ equally among all the 12 Fifths of the circle, each Fifth will be erroneous by about $\frac{1}{60}$ of a semitone [or $\frac{1}{11}$ of a comma], which is certainly a very small interval. By this means all varieties of tonal degrees within an Octave are reduced to twelve, as on our modern keyed instruments."—*Helmholtz.*

We shall now proceed to describe some of the devices which have been adopted in order to overcome this difficulty of temperament. A difficulty it verily is, for no less than seventy-six notes are required to

each octave to furnish a justly-intoned scale for all the keys used in modern music. Seeing that with our present mechanical appliances this number of notes is, for keyed instruments, not practicable, some other course must be taken; and as to tune any one scale perfectly would be to make the rest intolerable, many different expedients have from time to time been devised to overcome the difficulty, and strike a balance between the various major and minor keys.

Mr. A. J. Ellis, in a very able paper "On the History of Musical Pitch," read before the Society of Arts on March 3d, 1880, and reprinted in the Society's *Journal* of March 5th, thus clearly sets out the chief temperaments or tunings, just intonation being regarded as one :—

"1. *Just Intonation*, where all the Fifths and Thirds are perfect, used only by singers and theorists.

"2. *Pythagorean Temperament*, in which the Fifths of the series, E *flat*, B *fl.*, F, C, G, D, A, E, B, F *sharp*, C *sh.*, G *sh.* only, are perfect, and the major Thirds E *fl.*, G, B *fl.*, D, FA, CE, GB, DF *sh.*, AC *sh.*, EG *sh.* only, are a comma, or V1 in V* 80 too sharp.

"3. *Meantone Temperament*, in which all the major Thirds specified in (2) are perfect, but the Fifths specified in (2) are a quarter of a comma, or V1 in V 322 too flat.

"4. *Equal Temperament*, in which every Fifth, without exception, is one-eleventh of a comma, or V1 in V 885 too flat, and every major Third, without exception, is seven-elevenths of a comma, or V1 in V 126, too sharp."—*Journal of the Society of Arts.*

* This "V," in Mr. Ellis' paper, means "Vibrations."

Another way was that of having additional keys to the octave, which was the case with the organ presented to the Foundling Chapel by Handel, and with the organ built by Father Smith in the Temple Church in 1688. This latter had two distinct keys for A flat and G sharp, and for E flat and D sharp. This device widened the extent of playable keys considerably. Helmholtz has also devised a means of adding new sounds without undue complication, which gives twenty-four tones to the octave, and is explained at p. 633 of his work.

General Perronet Thompson, Mr. Alexander Ellis, Mr. H. W. Poole (an American), Mr. Bosanquet, and Mr. Colin Brown have all devised keyboard instruments intended to obviate, more or less, the very great difficulties which attend the construction of an instrument which will provide just intonation in all keys. (See Appendix B.)

Mr. A. J. Ellis, in his paper "On the History of Musical Pitch," just referred to, writes of the different temperaments as follows :—

"Just intonation is due to Ptolemy, the astronomer, A.D. 136. Meantone temperament was perfected by Salinas, A.D. 1577. Equal temperament is said to have been proposed by Aristoxenus, a pupil of Aristotle, and to have been in use in China for centuries earlier. It seems to have been used, in intention, in North Germany, as early as 1690, and to have remained on many organs. It was recommended by P. E. Bach, and is believed to have been used by J. S. Bach. But, throughout Europe

generally, meantone temperament was used till about 50 years ago. It is still retained generally on Spanish organs, and in England on Green's organs, at St. George's Chapel, Windsor; St. Katharine's, Regent's Park; and Kew Parish Church, and on a few other organs; but equal temperament is now generally aimed at, though seldom really attained. Messrs. Broadwood did not use it on any of their pianos till 1840, and it was generally introduced in their works, under the superintendence of Mr. Hipkins, from 1844 to 1846. The organ of St. Nicholas, Newcastle-upon-Tyne, was tuned in equal temperament in 1842, on the occasion of a great musical festival. In the Great Exhibition of 1851, no English organ was tuned in equal temperament. In July 1852, while making alterations in the Exeter Hall organ, Messrs. J. W. Walker & Sons put it into equal temperament, and it was first used in that tuning in November 1869. In the meantime, in September 1852, Mr. George Herbert, a barrister and amateur, then in charge of the organ at the Roman Catholic Church in Farm Street, Berkeley Square, had that organ tuned equally by Hill, its builder. Though much opposed, it was visited and approved by many, and, among others, by Mr. Cooper, who had the organ in the hall of Christ's Hospital tuned equally in 1853. The first organ built and tuned originally in equal temperament, by Messrs. Gray & Davison, was for Dr. Fraser's Congregational Chapel at Blackburn, in 1854 (since burned). Messrs. Walker and Mr. Willis also sent out their first equally tempered organs in 1854. Hence, in England, equal temperament is barely 40 years old.

"Before, and indeed, after 1577, many unequal temperaments were used, and the meantone temperament itself is commonly called *unequal*, whereas, when expressed on 12 notes, it is merely defective, because it requires 27 notes to the octave for its full development, as is shown in ordinary

musical notation. The law followed in these unequal temperaments is generally so unknown that the exact value of the notes cannot be calculated. There was, however,—at least on the old bonded or fretted clavichord—a semi-meantone temperament, in which the natural notes C, D, E, F, G, A, B were tuned in meantone temperament, and the chromatics were interpolated at intervals of half a meantone. This was very like equal temperament in most keys. But, in the calculations of this paper, none but the Just, Meantone, and Equal systems of tuning will be regarded, and all the unequal temperaments, which were slight variations of the meantone system, will be treated as belonging to that species of tuning."—*Journal of the Society of Arts.*

(*C.*) *Equal Temperament.*

The principle usually adopted at the present day for all keyed instruments is that called "Equal Temperament," which professes to divide the octave into twelve exactly equal parts, though it does not actually so divide it. It seems somewhat strange that nearly all writers on temperament, with the exception of Mr. A. J. Ellis, should describe it as dividing the octave into twelve precisely equal intervals, without explaining that these semitones are not *absolutely* equal, as the language used in each case would seem to imply.

Helmholtz (Ellis' Translation, edition 1875, p. 486) says:—"This led . . . . finally to the system of *equal temperament,* * in which the octave was divided into twelve precisely equal degrees of tone."

* The italics are given as they stand in the works referred to.

Dr. Pole ("Philosophy of Music," edition 1879, p. 150) says:—"By this system the octave is divided into *twelve equal semitones*, each semitone being tuned alike to an interval = $\frac{1}{12}$ of an octave." He gives also a plate showing the vibration numbers of a scale divided by equal temperament, which shows the following results:—

	C♯		D♯			F♯
C	D♭	D	E♭	E	F	G♭
256	271.2	287.3	304.4	322.5	341.7	362
	G♯			A♯		
G	A♭	A	B♭	B	C	
383.6	406.4	430.5	456.1	483.2	512	

An examination of these numbers will show at once that no interval in the octave is precisely "$\frac{1}{12}$ of an octave," to which each semitone is said to be alike tuned.

Mr. Sedley Taylor, at page 204 of his "Sound and Music," says, "The octave of which C 264 is the lowest note, will contain, on the equal-temperament system, the following sounds:—The vibration numbers are given true to the nearest integer. When a note is slightly sharper than that so indicated, this is shown by the sign + attached to the vibration number in question; when slightly flatter, by the sign —. For the sake of comparison, the perfect intervals of the same scale are written below the tempered ones." He then gives the following list of vibration numbers, the upper line, as stated in the

paragraph just quoted, containing the "tempered" intervals, the lower the perfect:—

C	C♯	D	D♯	E	F	F♯	G	G♯	A	A♯	B
264	280—	296+	314—	333—	352+	373+	395+	419+	444—	470+	498+
264		297	E♭ 317—	330	352		396	A♭ 422+	440	B♭ 469+	495

The intervals in the upper row are "tempered," it is true; but they are not "equally" tempered, nor do they in any single instance answer Mr. Taylor's own description of "*twelve precisely equal intervals.*"

Twelve perfect fifths, as Pythagoras discovered, make a comma more than seven octaves. Six major tones also exceed an octave; and it is vain to hope, by any system of "equal" intervals, to settle the question of temperament. As Mr. A. J. Ellis, in one of his appendices to his translation of Helmholtz, says, it is impossible to form octaves by just fifths or just thirds, or both combined, or to form just thirds by just fifths; because it is impossible, by multiplying any one of the numbers $\frac{8}{5}$, or $\frac{5}{4}$, or 2, each by itself, or one by the other, any number of times, to produce the same result as by multiplying any *other* of these numbers by itself any number of times. The octave and the fifth are, in fact, as Dr. Stone says in his admirable little work on "Sound," incommensurable, just as are the diameter and circumference of a circle. It is strange that Dr. Stone, while he says in

one place that "equal temperament aims at dividing the octave into equal parts or semitones," and goes on to say that "If it so happened that the octave could be divided thus, and the other intervals, such as the fifth and third, retained in tune, it would be a great boon," but, "unfortunately nature has not so ordained it," should remark a little later on that, "according to the equal temperament, the octave is divided into twelve perfectly similar equal intervals, seven of which are taken for the fifth, although its real measure is $7\frac{1}{61}$ of these." It is impossible that a system of temperament which divides the octave into twelve *perfectly similar* intervals can have a fifth $\frac{1}{61}$ sharper than the sum of seven of the twelve equal divisions; because if the intervals are equal, the fifth will include seven only, and will be too flat for an endurable fifth at all. It is palpably a misuse of language to call this equal temperament, and is calculated to produce misconception on the subject.

Take, for instance, the octave between C = 256 and C = 512, and the octave above it—C = 512 to C = 1024. It is obvious that, in tuning "equally," we must practically calculate so that the semitone above C = 512 shall be precisely as wide as that above C = 256—or in other words, it must be as truly and as exactly an octave from one C♯ to another as from one C to another, and every semitone must have its octave perfect. If these conditions are not fulfilled, not only will the tempering not be equal, but music will be impossible. The octaves must be in perfect accord

all over the instrument—that is to say, the Cs must agree, and so with the A flats, the F sharps, &c., but the fulfilment of this condition also becomes impossible if the octave is divided into twelve exactly equal parts.

Mr. Ellis, in a note to the paper already referred to, gives the following valuable information :—

"*Calculation of Temperaments.*—It is absolutely necessary, for all investigations on historical musical pitches, to be able to calculate A from C, and C from A, and sometimes from other notes, and often to find the V to all the notes in any system of temperament when the V of one is known.

"To find C from A.

"(1) In Just Intonation, increase the V of A by one-fifth. Thus, to A 440, add one-fifth, or 88, to find JC* 528.

"(2) In Meantone Temperament, first find JA, and then subtract 3 in 1000 and 1 in 10,000, working to two places of decimals, and finally retaining one. Thus, from A 440, find JC 528, and then subtract 1·58 or 3 in 1000, and also ·05 or 1 in 10,000; that is, 1·63 on the whole, giving 526·37, whence MC 526·4.

"(3) In Equal Temperament, first find JA, and then subtract 1 in 111. Thus, for A 440, we find JC 528, which, divided by 111, gives 4·76, and subtracting this we obtain 523·24, whence EC† 523·2.

"To find A from C.

"(1) In Just Intonation, subtract one-sixth. Thus, one-sixth of JC 528 is 88, which, subtracted, gives JA 440.

"(2) In Meantone Temperament, find JA, and increase

* That is "just C," or C in just intonation.

† "Equal C," or C in equal temperament.

the result by 3 in 1000 and 1 in 10,000. Thus, one-sixth of MC* 526·4 is 87·73, which, subtracted, leaves 438·67, and this increased by 1.31, or 3 in 1000, and ·04 or 1 in 10,000 gives 440·02, whence MA 440.

"(3) In Equal Temperament, find JA, and increase the result by 1 in 110. Thus, from C 523·24 we find JA 436·03, and, adding the 110th part, or 3·97, the result is EA 440.

"A Justly Intoned scale can be formed by adding one-eighth for the major tones C to D, F to G, and A to B; one-ninth for the minor tones D to E, and G to A; and one-fifteenth for the diatonic semitones E to F, and B to C.

"A Pythagorean scale can be made from a series of Fifths up, adding one-half for each Fifth, and dividing by 2 when necessary to keep within the Octave; or a series of Fifths down, subtracting one-third for each Fifth, and doubling the result where necessary to keep within the Octave. Work up to G *sharp* and down to E *flat*, beginning anywhere.

"A Meantone scale can be formed by taking the perfect Fifths, as in the last case, and then diminishing each upward and increasing each downward Fifth, as it is calculated, by 31 in 10,000. Thus, the perfect Fifth above C 256 is found by adding one-half, or 128, to be JG 384; taking 3 in 1000, and 1 in 10,000, we have 1·19, which being subtracted, gives MG 382·81; and the perfect Fourth below C 256 is found, by subtracting one-third, or 85·33, to be JF 170·67, double which is 341·34; and then, taking 3 in 1000 and 1 in 10,000, we have 1·06, adding which we have MF = 342·40. Begin anywhere, and work up to G *sharp* and down to E *flat*. Make two places of decimals and keep one.

* "Meantone C," or C in meantone temperament.

"A scale in Equal Temperament can be made by first forming a series of equal tones by continually adding 12¼ per cent., the proof being that the Sixth Tone thus found is scarcely more than double the first; then, the semitones may be found by adding 6 per cent., and subtracting 1 in 2000 to each of the tones. The result ought not to be wrong by one-tenth of a vibration anywhere.

"The above are chiefly close approximations, very convenient for those who can use decimal fractions but do not understand logarithms."—*Journal of the Society of Arts.*

And, in another note :—

"*How to Tune Equally.*—In my translation of Helmholtz, p. 785, I gave a rule for tuning sensibly in equal temperament, and I put it into a thoroughly practical form in the *Musical Times* for 1st October 1879, pp. 520, 521. It may be epitomised thus. Tune the bearings in the one-foot Octave of an organ or harmonium in the order C, G, D, A, E, B, F*sh.*, C*sh.*, D*sh.*, A*sh.*, E*sh.* Make all the Fifths too close, and all the Fourths too wide, so as to beat the Fifths 'up,' CG, DA, EB, C*sh.*, G*sh.*, D*sh.*, A*sh.*, *twice* in a second, and the Fourths 'down,' GD, AE, BF*sh.*, F*sh.*, C*sh.*, G*sh.*, D*sh.*, A*sh.*, E*sh.*, three times in a second. The Fourth CF is not tuned. The pitch is unimportant. The beats hardly last long enough to be available for the piano, which should be tuned to a harmonium."—*Journal of the Society of Arts.*

It is clearly wrong to call any system of temperament "equal temperament," which does not divide the octave into twelve precisely similar intervals; because if the intervals are not precisely similar, the temperament is clearly not equal, and has no right to be called equal; nor can any useful purpose be served by apply-

ing that term to any other system; and seeing that equal temperament in that sense can never be used for any scale which can ever be called musical, according, at any rate, to the demands of modern harmony, as developed since the time of J. S. Bach, the term "equal temperament" ought to be at once and for ever banished from treatises on acoustics. What is called equal temperament has a fifth which is neither perfect nor made up of equal intervals, but which is flatter than a perfect fifth, while its major thirds are far too sharp, the sixth being extremely sharp; the seventh is also flattened, and the fourth has an interval which is altered less than either of those already mentioned. The largest departure from just intonation is in the second note of the scale. If the semitones *are* equal—if, that is, the distance from C to C is divided into twelve equal parts—music would be impossible.

The vexed question of temperament is certain to be decided, one day or the other, by the gradual development of a system of mechanism, which will bring within the reach of players on keyboard instruments scales in all possible keys. It is possible that this day is at present very far distant, but that that day will ultimately arrive we are convinced. Singers who have been properly trained use just intonation now and then; and the difference between an exact scale and the compromise which the so-called equal temperament endeavours to effect, is sometimes most painfully manifest when

a well-trained singer is accompanied on the pianoforte. The difference between the quality of sound which is produced by the same means, when just intonation and tempered intonation are adopted, is far greater than those who have never tested the question would believe. To hear a well-trained quartette sing *Quando Corpus* from Rossini's *Stabat Mater*, and then to hear it played on a pianoforte tuned to so-called equal temperament, makes that difference very plain. Stringed instruments can play in just intonation, and frequently do; the chief difficulty seems to lie with the keyboard instruments, where it will continue to lie until advancing science discovers a mode of bringing under the player's fingers the requisite number of sounds in an octave to obtain, at any rate, something like an approach to just intonation.

CHAPTER XVII.

SYSTEMS OF PITCH NOTATION.

MUCH has been written on this subject which, when analysed, reduces itself simply to this:—That that system is the most perfect, other things being equal, which shows most plainly the relationship of every note of the scale to the tonic. Numerous attempts have been made to overcome the difficulties which exist, or which are supposed to exist, in connection with tonal relationship in the ordinary musical notation, or, as it is called, the "staff notation." The chief of these are the tonic sol-fa system, the letter-note system, and the Galin-Chevé method. These methods we will briefly describe. The tonic sol-fa system uses the syllables *do, re, mi, fah, soh, lah, ti,* to represent respectively the tonic, supertonic, mediant, subdominant, dominant, submediant, and leading note of the scale, and the initial letters of these words are used to represent the tones. Changes of key, if permanent, are indicated by the substitution of *do* for the note in the former scale which becomes *do* in the new one. If the change of key is only transient, signs representing *sharps* and *flats* are used, having the same effect as the

signs used in the old notation. The time-marks do not, of course, affect the question now under consideration, which is merely that of pitch notation.

The letter-note system adopts the plan of printing music in the same way as that used for the staff notation, with the addition of a letter engraved on the head of each note to indicate whether it is *do, re, mi,* &c., of the scale; in all other respects it is like the old notation.

The Galin-Chevé method, which is in many respects similar to that known in England as "Waite's method," employs figures to represent the seven sounds of the scale; 1 representing the tonic; 2, the supertonic, and so on up to the figure seven. Changes of tone are indicated, as in the tonic sol-fa system, by the substitution of 1 for the note in the old scale which becomes the tonic in the new one, accidentals being marked by diagonal strokes through the figures, downwards from right to left for sharps, and upwards from right to left for flats.

It is no doubt true that the old notation presents difficulties, to some extent, as regards its power of indicating the relation of all notes to the tonic, especially when, by means of accidentals, a key remote from the original key has been reached; but it is also equally true that every system which has yet been devised as a substitute for the staff notation, and for the purpose of remedying the defects in that notation with regard to tonality, has worse faults than, though they may not be of the same kind as, those of the system which it is intended to remedy; and the remedy is thus, in

our opinion, worse than the disease. These varied systems are all admirable in their way as means to an end, that end being the better understanding and more intelligent use of the staff notation; but as permanent systems, they are not for a moment to be compared with that which they are intended to supplant, or rather the defects of which they are intended to supply. Even the tonic sol-fa system, which is incomparably superior to any other yet invented as a substitute for music proper, leaves very much to be desired before it can become adapted as a permanent system for use by musicians generally. These endeavours to make the relationship of every note to the tonic more plain are admirable so far as they show the connection of each note of the scale with the keynote, and here we cannot but think their use ceases; they are, that is to say excellent stepping-stones to the better understanding of the old notation by those persons who have not been endowed by nature with that peculiar quality which enables a musician (born, not made) to *feel* what key he is in. There are two advantages which the old notation possesses, and which every other system wants—(1) the position of every note shows its pitch, and (2) its shape shows its duration; and if for no other reason than this, we must insist on preferring the old staff notation, until some plan is devised which embraces these two features, and at the same time places the tonality of the music in a better light. These systems are, after all, only "broken lights" of the staff

notation. When people have arrived at mature age without musical training of any sort, the tonic sol-fa method affords an excellent portal by which to lead them into the inner temple of music proper; but when musical culture begins where it ought to begin—in the nursery—and music is imbibed at the same time as a knowledge of letters, words, and figures, so that tonality becomes a thing fixed in the mind as firmly as the fact that two and two make four, there is never any difficulty in using the staff notation for all purposes. In such cases these "little systems" do not "have their day and cease to be;" they never begin to be, because the necessity for them does not exist. It is folly—and, withal, a very short-sighted folly—which blames the staff notation for faults which are so clearly the result of a bad method of teaching it.

Mr. Sedley Taylor, in the last chapter of his work on "Sound and Music," says:—

"The essential requisite for a system of vocal notation, therefore, is that whenever it specifies any sound it shall indicate, in a direct and simple manner, the relationship in which that sound stands to its tonic for the time being; a method by which this criterion is very completely satisfied shall now be briefly described."

He then proceeds to describe the tonic sol-fa system, and further says:—

"In fact, I am prepared to maintain that the complicated repulsiveness of the pitch notation, in the old system, must be held responsible for the humiliating fact that, of the large number of musically well-endowed persons of the opulent

classes who have undergone at school an elaborate instrumental and vocal training, comparatively few are able to play, and still fewer to sing, even the very simplest music at sight. Set an average young lady to accompany a ballad, or to sing a psalm-tune she has never seen before, and we all know what the result is likely to be. Now, there is no more inherent difficulty in teaching a child with a fairly good ear to *sing* at sight, than there is in making him *read* ordinary print at sight. A vocalist who can only sing a few elaborately prepared songs, ought to be regarded as on a level with a school boy who should be unable to read except out of his own book.

"If evidence be wanted to make good this assertion, it is at once to hand in the fact, that the youngest children, when well trained on the tonic sol-fa system, soon obtain a power of steady and accurate sight singing, and will even tell you whether a new tune pleases them or not, after merely glancing through it, without uttering a note." *

He takes care, however, to point out that the alleged superiority of the tonic sol-fa system is confined to its benefits as a *singing* system.

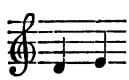
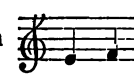
We certainly think that, in the last quotation we have made, Mr. Taylor blames the staff notation for what is, in ninety-nine cases in every hundred, solely attributable to the manner of teaching it. The "average young lady," of course, knows nothing at all about key relationship; but that is not because the system of notation from which she plays or sings is defective,

* The Report presented to Government by Dr. Hullah, embodying the results of his tour on the Continent in 1879, abundantly proves that when "well-trained," children can read equally well from the staff notation.

but because she has never been taught all that the system is capable of doing. The flats, sharps, and naturals of that system, which are usually the *bête-noir* of the said "average young lady," would not be enemies to reading at sight, but friends, if she had only been taught what purpose they served, and that, in fact, they are indispensable in any system of notation. So long as accidentals are required by composers, they must be marked in some way or other; and, however they are marked, the fact still remains that, starting with a key of C, it is possible to modulate into any major or minor key known to our modern system of tonality. This being so, it seems to us that no notation can modulate into either near or remote keys with such facility, *and at the same time preserve the idea of the original key*, as the staff notation; indeed, the very fact of starting from a new note as *doh*, especially if it be often repeated, quickly banishes the original tonality altogether from the mind; the beauties of form, so far as they depend on change of key, are altogether lost sight of; every *doh* is a new point of departure, which has no reference to the original key except that it is "two removes," or "three removes," as the case may be, from that key. The staff notation preserves to the eye, as well as the ear, that relationship of key to key which all its substitutes must intrust to the memory; and while those substitutes preserve (in a mode which we contend is no easier than that of the staff notation when the latter is properly taught) the relationship of all notes to their

tonic for the time being, they fail to preserve what is, for all the higher purposes of music, quite as important—the relationship of any extraneous key to the key in which the piece is written. While every credit is due, therefore, to the promoters of the tonic sol-fa system for spreading abroad a knowledge of and love for music, where, probably, they would not have reached by means of the staff notation; and while all genuine musicians will welcome the teachers of that system as fellow-workers in the same good cause, it is, we think, still true that their system will not, for the higher purposes of music, bear comparison with the staff notation. The sharps and flats, which (though only to the ill-taught) are a bar to the understanding of the position which any note holds in its scale, are, as yet, the only signs given to the world which will enable musicians to appreciate at once, through the media of eye and ear, those beauties in classical works which arise from the connection of key with key, and of every key, however remote, with the original tonic.

The objections which are urged against the accepted method of notation seem, to us, to arise entirely from the slipshod manner in which it is taught, and which renders sharps, flats, and naturals so many stumbling-blocks in the way of understanding key-relationship, instead of making them, as they should be and might easily be, the main helps to its comprehension. The groundwork of all musical knowledge lies in knowing that any "scale" consists of seven sounds which bear an invariable relation one to another, whatever

be the position of the first of the seven; and in this respect we agree most cordially with our friends the tonic sol-faists, that the relation of every note to *doh* must be known before any other knowledge can be of use. It does not appear to us to matter at all whether *mi* and *fah*, *ti* and *doh*, are called by those names, or by any others, so long as it be remembered where the "semitones" occur. In reading a line of sol-fa music, there is nothing at all to *show* that *m* and *f* are nearer to each other than *f* and *s*, any more than there is in the old notation to show that  is a larger interval than  . This fact must be *remembered* by the student of either notation. This being so, we cannot see that *fe* is any easier to understand than or *mi* than ; nor can we see why there is any difficulty in finding the position of *doh* when the key is seven flats or sharps, any more than when the key is one sharp or flat, or none at all. How to find *doh;* what is the effect of accidentals upon the various notes of the scale; how notes, affected by accidentals, are restored to their original status—these and the other "difficulties," which are supposed to stand in the way of the universal acceptance of the staff notation by the people at large, are, *cæteris paribus*, as easily and quickly acquired, *when once the construction of the scale*

is understood, as the modulator and the notation marks of the tonic sol-fa system. It is this thorough comprehension of the nature of a scale, and this alone, which can secure to the student the advantages of any notation, and which, when once understood, would render any other than the staff notation quite unnecessary.

We do not think, therefore, that anything is wanting in the staff notation—but only in the method of teaching it—to render it perfect as an indicator of key relationship; nor do we think that, even were a substitute required, any system at present before the world could effectually take its place, or do more than act as an aid to its better understanding and wider dissemination. We have many friends amongst the adherents of the tonic sol-fa system, and we are sure that, on a full consideration of the question of pitch-notation in all its bearings, they will agree with us in our conclusions.

APPENDIX A.

THE requirements of the Universities of Cambridge in Acoustics are thus set out in the University regulations—

II. THE EXAMINATION FOR THE DEGREE OF MUS. BAC. consists of three parts—

(1.) A preliminary Examination in Acoustics, Counterpoint, and Harmony.

(2.) The Exercise.

(3.) A more advanced Examination in Musical Science.

The following are the Subjects for Examination (1) :—

(*a.*) Acoustics.

Sensation and external cause of sound. Mode of its transmission. Nature of wave-motion in general. Application of the wave-theory to sound. Elements of a musical sound. Loudness and extent of vibration. Pitch and rapidity of vibration. Measures of absolute and of relative pitch. Resonance. Analysis of compound sounds. Helmholtz's theory of musical quality. Motion of sounding strings. The pianoforte and other stringed instruments. Motion of sounding air-columns. Flue and reed stops of the organ. Orchestral wind-instruments. The human voice. Interference. Beats. Helmholtz's theory of consonance and dissonance. Combination tones. Consonant chords. Construction of the musical scale. Exact and tempered intonation. Equal temperament. Systems of pitch-notation.

No knowledge of mathematics beyond that of Arithmetic will be required to satisfy the Examiners in this subject.

The University of London, among other things, examines Candidates for Musical Degrees in the following branches of Acoustics:—

B. MUS.

The relations between musical sounds and the vibrations of sonorous bodies, as affecting the *pitch* of the sounds.

The simple properties of stretched strings, and the sounds produced by them. Compound vibrations. Nodes.

The nature of harmonics.

The general theory and simpler phenomena of compound sounds.

The theoretical nature and values of musical intervals.

The theoretical construction of the modern scales.

Temperament.

The phenomena of sound in general, and the general nature of aerial sound-waves.

The special characteristics of musical sounds; the physical causes determining their *pitch*, *loudness*, and *quality*. Standards of pitch.

The more elaborate phenomena of compound sounds.

The theoretical nature of the sounds of musical instruments of various kinds, including the human voice. The principles of stretched strings.

The theoretical nature of musical intervals, and the philosophical modes of defining and representing them. The theoretical values of the various intervals used in music.

The theoretical construction of the modern scales.

The theory of temperament and its various practical applications.

The phenomena attending the combinations of two sounds. The various theories proposed for the explanation

of consonance and dissonance. Beats. Resultant or combination tones.

The theoretical nature of chords generally, and in particular of the various concords and discords in ordinary use; also of discords arising accidentally.

The Cambridge Higher Local Examinations also include "The Laws of Acoustics which determine the loudness, pitch, and quality of musical sounds."

APPENDIX B.

EFFORTS TO SECURE JUST INTONATION ON KEY-BOARD INSTRUMENTS.

Dr. Stone, in his "Elementary Lessons on Sound,"* summarises as follows the attempts which have been made to construct instruments on which justly-intoned scales could be played in all keys:—

"Even as early as the time of Handel the advantage to be derived from additional keys was obviously appreciated, for it is known that he presented to the Foundling Chapel an organ thus furnished. The original organ in the Temple Church, built by Father Smith in 1688, possessed fourteen sounds to the octave instead of twelve, the A♭ and G♯ as well as the E♭ and D♯ being distinct and divided. The keys themselves were split across in the middle, the back halves rising above the front portions, so that the finger could be placed on either at the player's discretion. The range of good keys on the unequal system was thus materially extended. The device of additional keys was, however, carried to its fullest development by Colonel

* Macmillan & Co., 1879.

Perronet Thompson in his enharmonic organ, which may still be seen at the South Kensington Museum. He used the large number of seventy-two to the octave, which were further distributed on three different keyboards, but which also differed among themselves in colour, shape, appearance, and in name. Besides the ordinary digitals there were others termed *Flutals*, *Quarrills*, and *Buttons*. By this means, though still retaining the ordinary arrangement of the keyboard, he was enabled to produce accurately twenty-one scales with a minor to each of them. He employs a cycle of fifty-three sounds, of which he uses about forty, the full cycle being discontinued at a certain point.

"The difficulty of adding new sounds without undue mechanical complication has been attacked in a different way by Helmholtz. The keyboards are in this case increased to two, so as to obtain twenty-four instead of twelve notes to the octave. They are of half the usual depth, placed one above the other, as in the organ. This has always seemed to the writer a practical and simple system. The instrument made for Helmholtz was so tuned that all the major chords from F♭ to F♯ could be played on it. On the lower manual were the scales from C♭ major to G, and on the upper those from E♭ major to B major. To modulate beyond B major on one side and C♭ major on the other it was necessary to make the enharmonic change between these two notes, which perceptibly alters the pitch by the interval of a comma, $\frac{81}{80}$. The minor modes on the lower manual were B or C♭ minor, on the upper D♯ or E♭ minor.

"The same idea has been carried out with slight variation in an instrument shown at South Kensington, namely, Gueroult's modification of Helmholtz's harmonium, of which the following is the maker's own description.

"This instrument has a front and back keyboard, each

divided into twelve semitones, like that of a piano, and each possessing five octaves. They are both tuned to true fifths, but the back keyboard is throughout a comma flatter than the front, which is on the normal diapason. The black keys on each keyboard therefore do duty for a flat and a sharp, but not in the same series. On the front keyboard, for instance, E♭ represents the D♯ of the back. Considered as flats, the black keys of the second keyboard represent sharps of a third board which would be tuned a comma lower than the second. By thus fusing the flat of one series with the sharp of the other, an error is committed equal to the interval $\frac{846}{845}$, which is at the extreme limit of audible differences.

"On the front keyboard, starting from A, the following notes are tuned to true fifths, so as to give no beat whatever; A, E; E, B; D, A; G, D; C, G; F, C; B♭, F; E♭, B♭. The perfect chords D, F♯, A; A, C♯, E; E, G♯, B are also made. The fifths D, A; A, E; E, B are those previously determined; the F♯, C♯, and G♯ are the thirds which give no beat in the perfect chords.

"On the back keyboard B, E, A, D, G, F, B♭ are in succession fixed by taking these notes as true thirds in the perfect chords G, B, D; C, E, G; F, A, C; B♭, D, F; E♭, G, B♭; D♭, F, A♭; G♭, B♭, D♭; of which the fifths C, G, F, &c., are taken on the front keyboard. The chords D, F♯, A; A, C♯, E; E, G♯, B, are formed on the back keyboard, using for the fifths D, A, E, B sounds already found and tuning the thirds without beats.

"The B of the back keyboard forms a true fifth with the F♯ or G♭ of the front. The D♯ or E♭ of the back keyboard is got by taking it as the true third of the perfect chord B, D♯, F♯, the two first notes being taken on the back and the third on the front board. The G♯ or A♭ of the front board gives, with the E♯ of the same board, a fifth which

is not quite true, being exactly equal to the tempered fifth. The C of the back board is determined by so taking it that the resultant tones of the two thirds about it should be free from beats. In the six major scales of C, F, B♭, E♭, A♭, D♭ the fingering is the same, the third, sixth, and seventh are on the back keyboard, all the others on the front. The keys of G, D, A are played with the sharpened notes on the front board. The key of G has thus only B and E on the back board, of D only B, and A has none. A can be played entirely on the back board also. The key-notes of all minor scales are on the back board. For the minor scales of A, D, G, C, F, and B♭ the third and sixth alone are on the front keyboard.

"A somewhat simpler method of working Helmholtz's system has been suggested by Mr. Alexander Ellis, and carried out by Mr. Saunders. The keyboard is single, but communicates with two rows of vibrators tuned according to the method given above, or in the following series:—

Back row .	B♯♯	D♭	C♯♯♯	E♭	F♭	E♯♯	G	F♯♯♯	A♭	G♯♯♯♯	B♭	C♭
Front row .	C	C♯♯	D	D♯♯	E	F	F♯♯	G	G♯♯	A	A♯♯	B

"When no stops are drawn out, the arrangement is that of the front series, the white notes being naturals and the black sharps. On pulling out a stop, the vibrators of its name in the front series of the instrument are damped, and the corresponding vibrators of the back series come into action, until the notes speaking are those of the old-fashioned manual. Between these extremes any required combination of notes can be produced, from seven flats to seven sharps, according to the keys employed. This method, which entirely removes the difficulties of complex

fingering, has the disadvantage of requiring a constant alteration of stops, which in transitory modulations is occasionally laborious.

"The last class of contrivance for producing true intonation does away with the ordinary form of keyboard altogether. It is impossible here to give full details of these instruments, which practically introduce a new principle into musical execution. Poole's, Bosanquet's, and Colin Brown's forms may be taken as typical representatives of many less perfect devices. In all, the series of tones are arranged diagonally one beyond another, so that 'the form of a chord of given key relation is the same in every key. But the notes are not all symmetrical, and the same chord may be struck in different forms according to the view which is taken of its key relationship.' They therefore possess the great advantage of similarity of manipulation, although this is quite different from that ordinarily taught. It would appear, however, that the new systems are far from difficult to learn by any person who has obtained some experience on the older form of instrument.

"The first attempt in this direction was made by H. W. Poole, of South Danvers, Massachusetts, U.S. The instrument appears to have been constructed, and is described in *Silliman's Journal* for 1850. His organ was intended to contain 100 pipes to the octave, and the scale to consist of just fifths and thirds in the major chords, and also the natural or harmonic sevenths.

"According to Mr. Bosanquet's notation here used, notes are arranged in series in order of successive fifths. Each series contains twelve fifths from F♯ up to B. One series is *unmarked.* It contains the standard C. Each note of the next series of twelve fifths up is affected with the mark /, which is called a mark of elevation, and is drawn

upwards in the direction of writing. The next series has the mark //, and so on. The series below the unmarked series is affected with the mark \, which is called a mark of depression, and is drawn downwards, in the direction of writing; the succeeding series is marked \\, and so on. Where, as in Poole's keyboard perfect thirds are tuned independently of the fifths, they are here represented by the note eight fifths distant in the series; this is a close approximation to the perfect third, according to a relation which has been called Helmholtz's Theorem. Thus C—/E means a perfect Third; \E—\G♭ is also a perfect third (chord of dominant of \A minor). The places of harmonic sevenths are marked by circles (Q).*

"*Bosanquet's Generalised Keyboard.*—In the enharmonic harmonium exhibited at the Loan Collection of Scientific Instruments, South Kensington, 1876, there was a keyboard which can be employed with all systems of tuning reducible to successions of uniform fifths; from this property it has been called the generalised keyboard. It will be convenient to consider it first with reference to perfect fifths. These are actually applied in the instrument in question to the division of the octave into fifty-three equal intervals, the fifths of which system differ from perfect fifths by less than the thousandth part of an equal temperament semitone.

"It will be remembered that the equal temperament semitone is the twelfth part of an octave. The letters E. T. are used as an abbreviation for the words 'equal temperament.'

"The arrangement of the keyboard is based upon E. T. positions taken from left to right, and deviations or departures from those positions taken up and down. Thus the notes nearly on any level are near in pitch to the notes of

* *Proceedings of the Musical Association*, 1874–75, p. 14.

an E. T. series; notes higher up are higher in pitch; notes lower down lower in pitch.

"The octave is divided left to right into the twelve E. T. divisions, in the same way, and with the same colours, as if the broad fronts of the keys of an ordinary keyboard were removed, and the backs left.

"The deviations from the same level follow the series of fifths in their steps of increase. Thus G is placed one-fourth of an inch further back, and one-twelfth of an inch higher, than C; D twice as much, A three times, and so on, till we come to /C, the note to which we return after twelve fifths up; this note is placed three inches further back, and one inch higher than the C from which we started.

"With the system of perfect fifths the interval C—/C is a Pythagorean comma. With the same system, the third determined by two notes eight steps apart in the series of fifths (C—\E) is an approximately perfect third. With the system of fifty-three the state of things is very nearly the same as with the system of perfect fifths.

"The principal practical simplification which exists in this keyboard arises from its arrangement being strictly according to intervals. From this it follows that the position-relation of any two notes forming a given interval is always exactly the same; it does not matter what the key relationship is, or what the names of the notes are. Consequently a chord of given arrangement has always the same form under the finger; and, as particular cases, scale passages as well as chords have the same form to the hand in whatever key they are played, a simplification which gives the beginner one thing to learn, whereas there are twelve on the ordinary keyboard.

"The keyboard has been explained above with reference to the system of perfect fifths and allied systems; but there

is another class of systems to which it has special applicability, the meantone and its kindred systems. In these the third, made by tuning four fifths up, is perfect or approximately perfect. The meantone system is the old unequal temperament. The defects of that arrangement are got rid of by the new keyboard, and the fingering is remarkably easy. The unmarked naturals in the diagram present the scale of C when the meantone system is placed on the keys.

"*Colin Brown's Natural Fingerboard with Perfect Intonation.*—The digitals consist of three separate sets, of which those belonging to four related keys, representing the notes 2, 5, 1, 4, are white; those belonging to three related keys, and representing 7, 3, 6, are coloured: the small round digitals represent 7 *minor*, or the major seventh of the minor scale. These are the same in all keys.

"This fingerboard can be made to consist of any number of keys. The scales run in the usual order in direct line horizontally from left to right *along* the fingerboard.

"The keys are at right angles to the scales, and run vertically *across* the keyboard, from \C♭ in the front to /C♯ at the back, C being the central key.

"The scale to be played is always found in direct line horizontally between the key-notes marked on the fingerboard, but the digitals may be touched at any point.

"The order of succession is always the same, and consequently the progression of fingering the scale is identical in every key.

"The first, second, fourth, and fifth tones of the scale are played by the white digitals, the third, sixth, and seventh by the coloured.

"The sharpened sixth and seventh of the modern minor scale are played by the round digitals. The round digital, two removes to the left as in the key of B flat, is related to

that in the key of C as 8 : 9, and supplies the sharpened sixth in the relative minor of C; so in all keys similarly related.

"Playing the scale in each key the following relations appear:—

"From white digital to white, say from the first to second and fourth to fifth of the scale, and from coloured to coloured, or from the sixth to the seventh of the scale, the relation is always 8 : 9.

From white to coloured, being from the second to the third, and from the fifth to the sixth of the scale, 9 : 10.

"From coloured to white, being from the third to the fourth, and from the seventh to the eighth of the scale, 15 : 16.

"From *white* to *white*, or *coloured* to *coloured*, is always the *major tone*, 8 : 9.

"From *white* to *coloured* is always the *minor tone*, 9 : 10.

"From *coloured* to *white*, the diatonic semitone, 15 : 16.

"The round digital is related to the coloured which succeeds it as 15 : 16, and to the white which precedes it as 25 : 24, being the imperfect chromatic semitone.

Looking *across* the fingerboard at the digitals *endwise*, from the end of each white digital to the end of each coloured immediately above it, in direct line, the relation is always 128 : 135, or the chromatic semitone; and from the end of each coloured digital to the white immediately above it, in direct line, the comma is found, 80 : 81.

"Between all enharmonic changes, such as between A flat $404\frac{44}{81}$ to G sharp 405, the interval of the schisma always occurs, 32,768 : 32,805, the difference being 37.

"These simple intervals and differences, 8 : 9, 9 : 10, 15 : 16, 24 : 25, 80 : 81, 128 : 135, and 32,768 : 32,805, comprise all the mathematical and musical relations of the scale. The larger intervals of the scale are composed of

so many of 8 : 9, 9 : 10, and 15 : 16, added together. The 'comma of Pythagoras,' being a comma and schisma added together, is found between every enharmonic change of key, as from C♭ to /B, or twelve removes of key.

"The digitals rise to higher levels at each end, differing by chroma and comma, or comma and chroma, alternately. This causes separate levels on the fingerboard at each change of colour. Though these are not essential, they will be found very useful in manipulation, and serve readily to distinguish the different keys.

The two long digitals in each key are touched with great convenience by the thumb. The lower end of each coloured digital always represents the seventh in its own key, and the borrowed, or chromatic sharp tone, in every other; thus the seventh in the key of G is the sharpened fourth, or F sharp, in the key of C; and so in relation to every other chromatic sharp tone.

The white digital is to every coloured digital as its chromatic flat tone; thus the fourth in the key of F is B flat, or the flat seventh in the key of C; so in relation to every other chromatic flat tone. In this way all chromatic sharp and flat tones are perfectly and conveniently supplied without encumbering the fingerboard with any extra digitals, such as the black digitals on the ordinary keyboard, the scale in each key borrowing from those related to it every possible chromatic tone in its own place, in perfect intonation. The tuning is remarkably easy, and as simple as it is perfect.

"While all the major keys upon the fingerboard, according to its range, have relative minors, the following, \B♭, /F, \C, \G, \D, A, E, B, F♯, C♯, G♯, and D♯, can all be played both as major and as perfect tonic minors.

"These secondary keys are more than appear at the first inspection of the fingerboard. A series of round

digitals placed upon the white, and a comma higher, additional to those placed upon the coloured digitals, would supply the scale in every form the most exacting musician could desire, but it is a question if such extreme extensions are either necessary or in true key relationship, and whether simplicity in the fingerboard is not more to be desired than any multiplication of keys which involve complexity and confusion."—*Stone.*

Helmholtz, in Appendix XVIII. to his work "On the Sensations of Tone," &c., refers to the organs of Thomson & Poole as under:—

"Since the publication of the first edition of this book, I have had an opportunity of seeing the *Enharmonic Organ*, constructed by General *Perronet Thomson*, which allows of performance in 21 different tonics harmonically connected. This instrument is much more complicated than my harmonium. It contains 40 pipes to the octave, and has three distinct manuals, with, on the whole, 65 digitals to the octave, as the same note has to be sometimes struck on two or all of the manuals. This instrument allows of the performance of much more extensive harmonies than my harmonium, without requiring any enharmonic interchange. It is even possible to execute tolerably quick passages and ornamentations upon it, notwithstanding its apparently involved fingering. The organ was erected in the Sunday School Chapel, 10 Jewin Street, Aldersgate, London, and was built by Messrs. Robson, 101 St. Martin's Lane, London. It contains only one stop of the usual *principal* work, has Venetian shutters to the swell, and is provided with a peculiar mechanism for correcting the influence of temperature on the intonation.

"Mr. H. W. Poole has lately transformed his organ so

as to get rid of stops for changing the intonation, and has constructed a peculiar arrangement of the digitals, which enables him to play in all keys with the same fingering. His scale contains not merely the just Fifths and Thirds in the series of major chords, but also the natural or sub-minor Sevenths for the tones of both series. There are 78 pipes to the octave, and *F*♭ has been identified with *E* &c., as upon my harmonium.

"Successions of chords on General Thomson's instrument are extraordinarily harmonious, and, perhaps, on account of their softer quality of tone, even more surprising in their agreeable character than on my harmonium. I had an opportunity, at the same time, of hearing a female singer, who had often sung to it, perform a piece to the accompaniment of the enharmonic organ, and her singing gave me a peculiarly satisfactory feeling of perfect certainty in intonation, which is usually absent when a pianoforte accompanies. There was also a violinist present who had not been much accustomed to play with the organ, and accompanied well-known airs by ear. He hit off the intonation exactly as long as the key remained unchanged, and it was only in some rapid modulations that he was not able to follow it perfectly."

APPENDIX C.

ON PITCH.

For the following extracts we are indebted to the paper "On the History of Musical Pitch," by Mr. A. J. Ellis, referred to in the former part of this work:—

"*Determining Pitch by Beats.*—Let the V of two notes be M and N, where M is greater than N, and let the ratio M and N be known, so that $n\mathrm{M} = m\mathrm{N}$. Let the sum of the beats in a second made by the interposed forks with the extreme notes and each other be b. Then $\mathrm{M} - \mathrm{N} = b$. These two equations give $(m - n)\ \mathrm{N} = nb$, and $(m - n)\ \mathrm{M} = mb$. If M is the Octave of N, then $m = 2$ and $n = 1$, and hence $\mathrm{N} = b$. Suppose that we do not know the ratio M : N exactly, but know that it is nearly that of $m : n$, and also whether M is too sharp or too flat, and have observed that the sum of the beats in a second of the forks interposed between M and N is c; then (1) if M is too sharp, we have $n\mathrm{M} - m\mathrm{N} = c$, and $\mathrm{M} - \mathrm{N} = b$, whence $(m - n)\ \mathrm{N} = nb - c$, $(m - n)\ \mathrm{M} = mb - c$; and (2) if M is too flat we have $m\mathrm{N} - n\mathrm{M} = c$ and $\mathrm{M} - \mathrm{N} = b$, whence $(m - n)\ \mathrm{M} = mb + c$, $(m - n)\ \mathrm{N} = nb + c$. The easiest and most important case is when M is nearly the Octave of N, and hence $m = 2$, $n = 1$. Then in the first case $\mathrm{M} = 2b - c$, $\mathrm{N} = b - c$, and in the second case $\mathrm{M} = 2b + c$, $\mathrm{N} = b + c$."—*Journal of the Society of Arts.*

"*Formula for finding pitch* from a heavy weighted suspended string. Let

"L = the vibrating length of the string from the suspending point to the movable bridge, expressed very accurately in English inches.

"l = the same in French millimètres.

"W = the stretching weight of the string, including the weight of non-vibrating part of the string, expressed in any unit.

"w = the weight of the vibrating length of the string in the same unit. These weights of the string are best obtained by stretching a similar string by the same weight and leaving it for some days till the stretching is complete, then cutting off a known length of it, weighing it, and dividing the whole weight of the string by the whole length to determine the weight of an inch, or a millimètre of it. The weights are then found by measurement.

"V = the number of double vibrations in a second. Then

$$2 \log V = 1{\cdot}98485 + \log W - (\log w + \log L)$$
$$= 3{\cdot}38968 + \log W - (\log w + \log l)."$$

—*Journal of the Society of Arts.*

On the Measurement of Pitch.

"*Measurers of Pitch.*—The following are the principal methods for determining the V of any note heard:—1. By a string. 2. By the syren. 3. By Professor McLeod's optical method. 4. By Professor Mayer's electrographic method. 5. By beats.

"*The String.*—A string, stretched by a constant weight, may be stopped at different places, and each sounding length will determine a different note, as on the violin. If the string were perfectly elastic and uniform, the V of these notes would be inversely proportional to the length of the string.

"Assuming this to be always the case, Mersenne (1648) took a string long enough to allow its vibrations to be seen and counted, and then shortened it till it was in unison with a given note, and, after multiplying the observed V

by the first length, divided it by the second (each expressed in the same unit), to find the V of the given note. He was, of course, very wrong, making an organ-pipe four French feet in length, speak, at one time, V 84, and at another, V 96, whereas, it probably spoke V 112. (See A 373·7, and A 376·6 in Table I.)

"J. H. Griesbach (1860) greatly improved on this method by tuning a string, one-fifth of an inch (more accurately 5½ mm.) thick, till one-quarter of its length was in unison with a given note, and then counting the vibrations of the full length of the string (which was kept in action by a continuous bow) by making the string, as it reached its upper position, mark a strip of paper passed over it, on which seconds were also marked as they elapsed. The instrument itself is in Room Q of the Scientific Collection at the South Kensington Museum, with a description from the *Journal of the Society of Arts* for 6th April 1860, p. 353. The extreme care with which Mr. Griesbach worked, and at the same time the untrustworthiness of the arrangement, which is crowded with sources of error, is shown by some of his results; thus, his V 416, V 521·6, and V 528, are shown by remeasurement of the forks to be V 422·5, V 524·8, and V 534·46.

"Euler and Bernouilli worked out the problem of the string mathematically, but the difficulties of determining the unison, measuring the lengths, finding the weight, and obtaining uniformity in the string, together with those arising from its thickness, are so great, that the method cannot be relied upon for any great accuracy. We are, however, indebted to it for some important measurements by Euler, Dr. Robert Smith, Marpurg, Fischer, and De Prony. (See Table I., under A 392·2, 414·4, (2) 424·2, 427·6, (1) 431·7, 437·3, 438·2, (1) 441·7, (1) 444·5.)

"Delezenne, of Lille, made the best use of the stretched string. Having proved that only the finest wire which would bear the strain would give satisfactory results, he stretched 700 millimètres of such a wire on a violoncello body, tuned it to Marloye's fork of V 128 (which was, probably, very accurate, as Marloye's V 256 was so), and then, by a movable bridge, cut off the length, which gave a unison with a given fork. Measuring this length in millimètres, he divided $128 \times 700 = 89{,}600$ by it, to find the V. For organ-pipes, he first tuned a fork with sliders in unison with the pipe, and then measured the fork so tuned by his sonometer. (See Table I., under A 450·5.) I am indebted to Delezenne for numerous important pitches, which he believed to be correct within three-tenths of a comma, or about V 37 in V 10,000, and they are, very probably, still more accurate. He estimates that those who use Euler's formula may be wrong by a comma, or V 1 in V 80, or V 125 in V 10,000, owing to the mere thickness of the string necessary to support the stretching weight.

"*The Syren* of Baron Caignard de la Tour consists of a perforated disc, which is driven round by a stream of air, and, allowing puffs to pass through the oblique holes, makes a musical sound, of which the V is the number of such puffs in a second counted by an appended mechanism. M. Cavaillé-Coll added a bellows, giving a constant pressure of wind, and, by its assistance, he tells me that Lissajous determined the pitch of the French Diapason Normal. M. Cavaillé-Coll also improved the counting apparatus, by which he has been able to obtain even more accurate results. The ordinary syren of commerce is very untrustworthy; for example, Mr. Hullah's forks, thus measured, and intended to make V 512, really made V 524·8 to V 525.

(See Table I., under A 441·3.) Even at the best it is a difficult instrument to manipulate. Probably all the determinations of pitch made for the French Commission in 1859 were made by Lissajous and Despretz with this instrument, as well as those cited by De la Fage as made by Lissajous. These and other pitches determined by the ingenious inventor himself are all cited in Table I.

"*The Optical Method*, invented by Professor Herbert McLeod and Lieutenant R. G. Clark, R.E., and described in the 'Proceedings of the Royal Society,' for January 1879 (vol. 28, p. 291), consists in viewing white lines, on a rotating cylinder, through the shadow of a constantly vibrating fork. The result is, apparently, a dark wave, which remains stationary when the V of the fork is the same as the number of white lines which pass before the eye in a second. For effecting this, and counting the lines that pass, there are elaborate contrivances. The machine is very difficult to manipulate, but, probably, extremely accurate in result. It will be seen that I am greatly indebted to it for several measures of vital importance to my investigations.

"*The Electrographic Method* was invented by Professor A. Mayer, of Stevens Institute, Hoboken, New Jersey, U.S., who is preparing for publishing it in all its details. In this method a camphor-smoked paper on a metallic rotating cylinder is inscribed with a wave-curve by an aluminium point fastened to one prong of a large fork, through which a powerful induction coil, actuated by a seconds' pendulum, throws a spark, which burns a single hole in the paper precisely every two seconds. By counting the sinuosities in the wave-curve between these holes the V is determined. The difference of pressure of the aluminium point makes no difference in the rate of vibration. The flattening

caused by the point is ascertained by beats. This instrument is, of course, expensive, and difficult to adjust, and is applicable only to large tuning-forks, the V of which it determines with great exactness. As will be seen, I have been greatly indebted to Professor Mayer for several measures of pitch taken by this instrument, but they could not be completed till 1st March, and hence must be communicated hereafter.

"*Musical Beats.*—When two musical notes of very nearly the same pitch are sounded together they produce beats, or loudnesses separated by silences, which, under ordinary circumstances, occur exactly as many times in a second as the V of one note exceeds the V of the other. The number of beats in a second can be counted easily when it lies between 2 and 5. Beyond 5 beats in a second there is considerable difficulty, arising from the rapidity of the loudnesses, and, after 6 beats in a second, the result cannot be depended on. Below 2 beats in a second there is also a difficulty, arising from the length of time occupied by each loudness. After 1 beat in a second the result can seldom be depended on. If, then, we know the exact interval between two notes, we can, by interposing forks and counting the beats, determine the exact V of each note. In particular, if the notes form an octave, the beats in a second between them is the V of the lower note.

"Sauveur, 1713, used beats of organ-pipes (see A 406·6 in Table I.), and his experiments were successfully repeated by M. Cavaillé-Coll (Association Scientifique de France, Bulletin Heb., No. 81, 16 Aug. 1868, p. 126), but they were difficult and uncertain; and the organ-pipe varies too much with temperature to make it useful for measuring the pitches of other notes. Sarty (see A 436 in Table I.) complicated the matter more by using a monochord in

addition, and his result is very uncertain. About 1865, Mr. Henry Willis, the well-known organ builder, also made a number of very careful experiments with organ-pipes, tuned by a slide on a slot, and actuated by bellows of constant pressure, of his own construction, with a view of determining difference of pitch by beats.

"*Tuning-fork Tonometer.*—If two tuning-forks, making an octave with each other very nearly, but not exactly, be held over a resonance jar, tuned to the higher by pouring in water, beats are heard, and may be counted for from 10 to 20 seconds, between the precise octave of the lower fork, and its approximate octave, while the low note itself is practically inaudible. If, then, a number of tuning-forks be interposed between the two, beating roughly four times in a second, two and two, and, after having rested sufficiently for their pitches to become permanent, are accurately counted, the V of the lower fork, and hence that of all the intermediate forks, can be determined. For verification, it is best to carry the series to at least a dozen forks beyond the octave. The forks should be good, beating at least 45 seconds audibly with each other, and furnished with wooden handles, but not screwed on to a resonance box. The difficulty is in counting with sufficient accuracy, for if the lower fork be about V 256, there will be 64 sets of beats to an octave, and an error of ·01 beat per second, would make the serious error of V 0·64 in determining the pitch of the lower note.

"The invention of this tonometer is due to Johann Heinrich Scheibler (born 11th November 1777, died 20th November 1837), a silk manufacturer of Crefeld, in Germany. His account of his method and mode of measurement, and the details of his tonometer of 52 forks, from A $219\frac{2}{3}$ to A $439\frac{1}{3}$, at 69° Fahr., is given in his pamphlet, 'The Physical and Musical Tonometer' (*Der Physikalische und*

Musikalische Tonmesser, Essen bei Bädeker, 1834, pp. 80, and plates). His method was much more laborious than that here suggested, but his counting seems to have been wonderfully perfect. These 52 forks have disappeared since Scheibler's death, and all efforts I have made to discover them, with the help of Herr Amels and Scheibler's existing descendants (to whom I am much indebted), have hitherto failed. But a tonometer of 56 forks, which belonged, at least, to Scheibler, if it was not made by him, still exists, only there are no records of its having been counted by Scheibler. It was inherited by Scheibler's daughter, Madame M. E. L. de Greiff (died 4th September 1854), and then by her son, Herr Aurel de Greiff, who gave it on a long loan to Herr Jean Amels, then of Crefeld, and now of 78 Newgate Street, London, silk merchant and musician, none of Scheibler's family caring for music. Herr Amels has kindly allowed me to have the use of this tonometer since 10th May 1879, to the present time, and I am able to show it you this evening.

"It was believed that this tonometer proceeded by 4 beats in a second, from 4 A 220 to 2 A 440. A very careful count showed me that only 32 out of the 55 sets of beats were 4 in a second, and that the others varied from 38 to 42 beats in 10 seconds. The best sum of all the beats that I could obtain was 219·27 in a second, which should, therefore, be the V in the lowest fork at 69° Fahr., the mean temperature used by Scheibler. It struck me then, as possible, that the extreme forks were really of the same pitch as those of the 52-fork tonometer, namely, V 219·6$\frac{2}{3}$ and 439·3·$\frac{1}{3}$ On that supposition I had made the trifling error of V 0·4 in counting, and I distributed this among 20 of the 23 sets of beats, which were not exactly 4 in a second. Then I reduced the whole to 59° Fahr., and obtained the following result:—

SCHEIBLER'S 56-FORK TONOMETER AT 59° FAHR.

No. of Fork.	Formerly presumed pitch.	Ellis's count.	Octaves of Ellis's.	No. of Fork.	Formerly presumed pitch.	Ellis's count.	Octaves of Ellis's.
1	440	439·54	879·08	29	328	327·62	655·24
2	436	435·74	871·48	30	324	323·61	647·22
3	432	431·84	863·68	31	320	319·54	639·08
4	428	427·96	855·92	32	316	315·54	631·08
5	424	423·96	847·92	33	312	311·54	623·08
6	420	419·96	839·92	34	308	307·54	615·08
7	416	415·74	831·48	35	304	303·61	607·22
8	412	411·74	823·48	36	300	299·39	598·78
9	408	407·74	815·48	37	296	295·57	591·14
10	404	403·77	807·54	38	292	291·70	583·40
11	400	399·76	799·52	39	288	287·70	575·40
12	396	395·79	791·58	40	284	283·70	567·40
13	392	391·67	783·34	41	280	279·69	559·38
14	388	387·57	775·14	42	276	275·69	551·38
15	384	383·57	767·14	43	272	271·69	543·38
16	380	379·60	759·20	44	268	267·77	535·54
17	376	375·60	751·20	45	264	263·82	527·64
18	372	371·68	743·76	46	260	259·81	519·62
19	368	367·56	735·12	47	256	255·64	511·28
20	364	363·63	727·26	48	252	251·67	503·34
21	360	359·63	719·26	49	248	247·67	495·34
22	356	355·63	711·26	50	244	243·67	487·34
23	352	351·63	703·26	51	240	239·66	479·32
24	348	347·63	695·26	52	236	235·69	471·38
25	344	343·62	687·24	53	232	231·69	463·38
26	340	339·62	679·24	54	228	227·77	455·54
27	336	335·62	671·24	55	224	223·77	447·54
28	332	331·62	663·24	56	220	219·77	439·54

"The difficulty was now to verify my count, which had been made with great care with the help of a ship chronometer, each set of beats having been counted repeatedly for 40 seconds. But then I could not feel sure of being right in my count within V 0·05 or V 0·025 at most, and this left the distributed error of V 0·02 imperceptible.

From this difficulty I was relieved by the kindness, first of Professor Herbert McLeod, and subsequently of Professor Alfred Mayer. Both counted for me by their instruments already described, five large French forks which I had had made, and Professor McLeod also lent me four of Koenig's forks, which he had carefully measured. On measuring these by Scheibler's forks, using the values given in the preceding table, I obtained results practically identical with those of Professor McLeod, as shown in the following Table, in which all the V are reduced to 59° Fahr. The final results by Professor Mayer have not reached me in time enough to insert in this place, but will be subsequently communicated to the Society.

Name of Fork.	Ellis.	McLeod.
1812, Conservatoire A	439·54	439·55
1820, Tuileries A	434·25	434·33
1818, Feydean A	423·09	423·02
1789, Versailles A	395·79	395·83
Nominally.		
Marloye Ut_3 256	255·96	255·98
Koenig Ut_3 256	256·30	256·31
,, Mi_3 320	320·30	320·37
,, Sol_3 384	384·43	384·44
,, Ut_4 512	512·55	512·55

"The extreme closeness of these results gave me perfect confidence in using this 56-fork tonometer of Scheibler for all the measurements made for this paper. My rule has been to determine the beats several times (generally 5, often 10) with each of two forks, and to take the mean of all the results. It is, therefore, probable that measures of forks which I could count for 10 seconds, are not so much as V 0·1 in error.

"Other tuning-fork tonometers have been made by Wölffel and Koenig, both of Paris, but I have had no

opportunity of examining and comparing them. Koenig is reported to have lately invented a new and exceedingly accurate counting instrument, but I have seen no description of it as yet. The great difficulty in verifying is one of the disadvantages of the tuning-fork tonometer. I have found it impossible to verify by imperfect Fifths, as their beats last too short a time to be counted with any approach to sufficient accuracy.

"By furnishing Messrs. Valantine and Carr, of 76 Milton Street, Sheffield, extensive tuning-fork makers to the music trade, with standards counted by means of this Scheibler's tonometer, as thus valued, I have enabled the public to obtain small forks, such as are usually employed for giving pitch, at moderate prices, and of great accuracy, that is, seldom or ever showing half a vibration in a second different from the number impressed on them. For those who wish to know the pitch of instruments or orchestras within the usual limits, I recommend pocket-boxes of 12 forks, either V 412 to V 456 for A or V 500 to V 544 for C. Such boxes, properly fitted, would cost, complete, about two guineas. Single forks above V 412 can be made for 3s., and below to V 256 for 4s. 6d. each. Larger forks are more expensive, 6-inch prongs costing 15s. It would be quite impossible to obtain such cheap forks elsewhere with anything like the same accuracy of pitch, and I consider it one of the principal results of my long and laborious countings that I am able to show investigators where they can obtain the tools they need. Messrs. Valantine and Carr also make complete octaves of 65 forks at similar reasonable prices, and then the operator can count for himself; they have already made two such sets.

"*Reed Tonometer.*—To remedy the deficient power of verification in the tuning-fork tonometer, and to accomplish many other desirable objects, Herr Georg Appunn, of

Hanau. invented his tonometers of 65, 33, and 57 reeds, of which copies may be seen in the South Kensington Museum, and of the two first at the Museum of King's College, London. Lord Rayleigh has also a copy of the first. I have carefully examined all of these copies. I was so pleased with the first, after long work upon it without discovering error, that I used it in my former paper for measuring pitch, notwithstanding that it differed considerably in its indications from other measurements. The cause of error could not be discovered by long work and counting with a single instrument. It was not till the Lords of the Committee of Council on Education permitted me to remove the 65 and 33-reed tonometers to King's College Museum, that I discovered the curious fact on which it depended. The decisive observations were of this kind. Suppose L, M, N are three consecutive reeds on one instrument, and P, Q, R three reeds of the same pitch on another. (In point of fact, the instruments, which had been made independently, were not exactly in unison, and this was the first shock that my belief in their accuracy sustained.) Then I beat L with M, and M with N, and adding the observed beats, I obtained what I term the 'internal beats' of L with N, that is, such as take place inside the box containing the reeds. Next, I beat L with Q, and Q with N, and adding them, I obtained the 'external beats' of L with N, that is, the beats made outside the box containing the reeds, in the free surrounding air. Now, the internal were always more than the external beats. My first experiment showed that the internal exceeded the external beats by about 1 per cent. Continuing the experiments for many weeks, till all the beats were thus counted, and varying the experiment occasionally by sounding L and B at different ends of the Museum, at least 50 feet apart, while I stood midway to count, I found, as a mean

of all my observations, that the number of internal beats must be reduced by $7\frac{1}{2}$ per 1000 to produce the external beats, and that, consequently, the values obtained for the reeds, by counting internal beats, must be reduced in the same proportion. The internal beats were counted on the King's College copy. Last autumn I varied this experiment by taking the pitch of each reed of all three tonometers in the South Kensington Museum, by beats with Scheibler's forks as counted above. The result was, that in order to reduce Appunn's numbers on the 65-reed tonometer (nominally V 256 to V 512) to Scheibler's, it was necessary, as a mean, to deduct 76 in 10,000, precisely confirming the former result (showing that the instrument went actually from V 254 to V 508). For the first octave of the 57-reed tonometer (nominally V 64 to V 128), the same reduction was obtained (showing that instrument went really from V $63\frac{1}{2}$ to V 127). For the 33-reed tonometer (nominally V 128 to V 256), the reduction was 83 in 10,000 (showing that the instrument went really from V 127 to V $253\frac{3}{4}$); but then, almost every reed of this tonometer (which had been sent from Germany afterwards, and without comparison) was flatter than its corresponding octave in the 65-reed tonometer, and the octave was not quite perfect.

"What the cause of this 'drawing' of the beats may be, has not yet been investigated. Its direction and amount was, I believe, entirely unknown previously, and hence Herr Appunn can hardly be blamed for overlooking it. Nor does it in any way detract from the use of the instrument. When the beats have been exactly counted, and hence the values of the reeds determined, we have only to reduce these values by 76 in 10,000, with the help of a little table (like that of Scheibler's forks given above), and the instrument is as useful as ever to determine pitch and make experiments. Thus Koenig's forks, when I take the

numbers from Appunn given in my former paper, and reduce them properly, are very near what we may suppose Koenig's to have really been, as calculated from his $Ut_3 = V$ 256·28, and supposed to be absolutely correct harmonics. Of course there were slight errors in my former measurements, which were early attempts, and fraught with difficulty, and, possibly, either slight errors in Koenig's own excellent workmanship, or slight subsequent changes due to rough usage.

No. of Harmonic.	Marked values in simple Vibrations.	Calculated values in double Vibrations.	Measured by Appunn's Tonometer.	Reduced by subtracting 76 per 10,000.	Difference from calculated values.	Harmonics of 64·105.
1	...	64·07	...	...	...	64·105
4	Ut_3 512	256·28	258·4	256·42	+ ·14	256·420
5	M_3 640	320·30	323·1	320·64	+ ·34	320·525
6	Sol_3 768	384·42	387·6	384·64	+ ·22	384·63
8	Ut_4 1024	512·56	516·7	512·75	+ ·19	512·84
10	M_4 1280	640·70	646·0	641·07	+ ·37	641·05
14	7. 1792	896·98	905·0	898·10	+ 1·12	897.47
16	Ut_8 2048	1025·12	1032·6	1024·7	− ·42	1024·68
18	Re_8 2304	1153·26	1163·3	1154·4	+ 1·14	1153·49
20	Mi_8 2560	1281·40	1292·0	1282·1	+ ·70	1282·1

"By examining the harmonics in the last column, deduced from a quarter of 256·42, it will be seen that they agree closely with the reduced numbers in the fifth column except in two cases (those in which the apparent error in the sixth column exceeds 1), and this seems to point to a small error in measurement in these two cases. All the four last pitches had to be counted by high partial tones of the reeds, and hence mistakes were very probable. But this crucial test shows that the errors made neither arose from grossly erroneous counting, nor from careless manufacture of the reed tonometer, but must have proceeded, as my other observations establish, from an acoustical pheno-

menon affecting all the nominal values of the reeds in the same ratio. And this is my answer to the first objection raised against my former paper. At the same time, I feel that I owe an apology to Herr Koenig, for my having been unfortunately misled by the unknown error of Appunn's instrument to attribute that error to him, and I make this apology most sincerely, for no one deserves more thanks from acousticians than Herr Koenig, both for the excellence of his workmanship, and the ingenuity of his contrivances.

"Besides this acoustical acceleration of the beats, there remain two other drawbacks to Appunn's tonometers: first, they do not retain their pitch with accuracy, and, secondly, their variation with temperature is unknown. Hence they are not, as I had hoped, instruments of scientific precision, though admirable for all purposes of lecture illustrations." —*Journal of the Society of Arts.*

On the Recording of Pitch.

"*Carriers of Pitch.*—As very few persons are able to reproduce a pitch at will after the lapse of a short time, it is necessary to have instruments by which a given pitch-note may be sounded at any moment. The oldest of these contrivances are the metal cylindrical open flue *organ-pipe*, and the stopped *pitch-pipe*. In later times, the *tuning-fork* and the *free reed* have been used. In orchestras, the *oboe*, a reed pipe, is generally sounded for the other instruments to tune to.

"*The Organ Pipe.*—The pitch rises with heat, and falls with cold, often making a semitone between its winter and summer pitches. When the V at any given temperature is known, the V at any other temperature may be found with sufficient exactness by increasing the first V by 4 per cent., dividing by 1000 to 2 places of decimals, multiplying the result by the number of degrees Fahrenheit by which the

observed differs from the required temperature, and adding or subtracting according as we reduce to a higher or a lower temperature. In this way all the organ pitches in Table I., which I have myself observed, have been reduced to the pitch they would have at 59° F. = 15° C. = 12° R. Thus, A 528, at 59° F., gives what at 73° F.? To 528 add 4 per cent., or 21·12, which gives 549·12, and this, divided by 1000 to 2 places of decimals, gives ·55, which, multiplied by 14 (the difference of 73° and 59°), gives 7·70, and, as the required temperature is greater, we have to add this 7·70 to 528, producing A 535·7 at 73° F. (See A 441·7 and A 443·1 in Table I.) As the wind used is often of a lower temperature to the air about the organ, and as the expansion of the air affects the temperature, the rule is not always perfectly accurate, but I have found it sufficiently so for the purposes of this paper. After touching an organ-pipe, or blowing it with the mouth, it should be left to cool before its pitch is taken. For the same temperature the pitch is mainly influenced by the length of the pipe, measured from the line where it is soldered on to the foot, up to the open end, and by the internal diameter. If these dimensions are taken in inches, the pitch or V of the pipe is very nearly 20,080, divided by the sum of three times the length added to five times the diameter, according to M. Cavaillé-Coll (adapted from *Comptes Rendus*, 1860, p. 176), and in the two-foot octave I have seldom found the result so much in error as a comma, or V 1 in V 80. If we actually find the V of a similar pipe, and multiply it by the sum of three times the length added to five times the diameter (expressed in inches), and use this product in place of 20,080, we may find the pitch of another pipe of the same kind, differing slightly in length and diameter, by dividing this product by the sum of three times the new length added to five times the new diameter, both taken in inches. I have

had to use this device frequently when the actual dimensions of pipes made for me differed from their intended dimensions, in order, from the pitch of the actual pipe, to deduce that of the intended pipe. (See Table I., A 373·7, 376·6, 396·4, 424·4, 434·7, 446·0, 445·8, 504·2, 505·8.)

"The strength of the wind used is important. The above rule supposes this pressure to be capable of supporting a column of water about $3\frac{1}{4}$ inches high. From experiments made by M. Cavaillé-Coll, as pressure varies from $2\frac{3}{4}$ to $3\frac{1}{4}$ inches, V increases by about 1 in 300, but, as pressure varies from $3\frac{1}{4}$ to 4 inches, V increases only by about 1 in 440; the whole increase of pressure from $2\frac{3}{4}$ to 4 inches increases V by about 1 in 180. Hence the pitch of A may vary by from V 1 to V $2\frac{1}{2}$ from this cause only. (See actual observations in Table I., under the pitches last cited.)

"The quantity of wind, regulated by the size of the wind-slit and the orifice at the foot, is another source of variation. The shape of the mouth, and especially the shading of the mouth or extremity, greatly influences pitch. Hence a solitary pipe removed from the organ where it was shaded by adjacent pipes is often sharper. Cleaning an organ sharpens it. Even removing a pipe and replacing it will often alter the pitch. An organ-pipe is slightly flattened by pressing in, and slightly sharpened by pressing out, the edges of its open end, as by the 'tuning cone;' but considerable changes require the pipe to be lengthened or shortened.

"It is clear, therefore, that, when the V of a pipe is not measured as it stands in the organ itself, the pitch given may be several vibrations in error. And even where we are fortunate enough to find an organ with pipes that have remained unaltered for 200 or 300 years, which is seldom

the case, we cannot be sure that it stands exactly at its original pitch. This must, of course, be borne in mind for all the cases of organ-pitch given in Table I. But the extreme amount of error will seldom be 1 per cent., which, for present purposes, is insignificant. In point of fact, the exact pitch of an organ cannot be ascertained, for it is so large that various parts of it are constantly at variable temperatures, and hence are constantly liable to be at different pitches, or out of tune with each other. Hence, in measuring the pitch of an organ, I always select the 2 A or 1 C of the open metal diapason, and, if possible, on the great organ, and consider that to be the pitch for which the organ was constructed.

"*The Pitch-pipe* (of which I am able, through the kindness of the Bellfoundry Colbacchini, at Padua, to show you two very curious Italian examples of 150 and 100 years old, described under A 425·2 in Table I. below) is subject to all the errors of an organ-pipe, and being blown by the warm breath at very different pressures cannot be depended on for accuracy. But its portability, and the easy production of one or two octaves of tone by sliding the piston in and out, formerly rendered it indispensable to singers who had no instrument to guide them. The same is true of pitch derived from flutes, clarionets, and oboes. (See remarks in Table I., under A 395·2, 410·0, 413·3, (2) A 418·0, 422·0, 425·2, 424·5.)

"*The Tuning-fork*, originally called the *Pitch-fork*, was invented by John Shore, Royal Trumpeter, in 1711, Sergeant-Trumpeter at the entry of George I. in 1714, and Lutist to the Chapel Royal in 1715. He died deranged in 1753. Hence the tuning-fork is probably not more than 150 years old. It was very rude at first, as in this example, which was dug up at Brixton in 1878 (see A 454·2, in Table I.), but has, in late years, become a beautiful philo-

sophical instrument, as in the larger forks before you. It is very permanent. I have reason to believe that Scheibler's forks have not varied by one vibration in ten seconds since his death in 1837. It varies very slightly for temperature, being (contrariwise to the organ-pipe) flattened by heat and sharpened by cold to the amount of about V 1 in V 21,000 for each degree Fahrenheit. When, therefore, careful experiments have to be made, a tuning-fork should never be touched by the hand at all (wood or paper being interposed), or carried in the pocket, or struck hard or often (every blow heats, and, therefore, flattens it very slightly); but, for ordinary purposes, this is immaterial. As forks are tuned by filing, which not only heats them, but unsettles their molecular arrangements—at least, in part—it is necessary to let them cool and rest for several days, sometimes for weeks, before their pitch can be depended on for scientific accuracy. They will often rise by several vibrations in ten seconds in the course of cooling. Hence copies are always apt to be too sharp, and should, if possible, be re-compared. This has often caused me much difficulty, and, in several cases, a doubt will necessarily remain on such copies which have been sent to me. The difficulty of tuning a fork in exact unison with another is also extremely great. Hence, in Table I., such pitches may be too sharp by half a vibration in a second, or even more. We seldom find a batch of tuning-forks at the same pitch. (See A 435·4 in Table I.) On the whole, however, no more accurate means of preserving pitch exist. Two great sources of permanent injury to a fork are wrenching or twisting the prongs (as by a fall, or screwing the forks in and out of resonance-boxes, when the prongs ought never to be touched; or fixing both prongs in a vice to file), and rust. To preserve from rust, never stroke the prongs with the fingers (as musicians have a habit of doing),

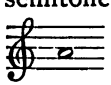
do not speak over the forks, keep them carefully from the damp (the large forks on resonance-boxes in chamois leather stalls, the smaller ones in cases, or folded in paper), and oil them occasionally with a film of limpid gun-lock oil (to be obtained from any gunsmith). If rust forms, prevent it spreading by applying oil, but be careful not to use sand-paper, as that will certainly injure the pitch still more. As most old forks are more or less rusty, it is important to have some notion of the amount of injury inflicted. Actual cases are investigated in Table I., under A 441·1, 441·8, and 443·2. But I found it advisable to try the following experiments:—Three ordinary forks, having been carefully measured, were immersed in water, one half way from the end of the prongs, another half way from the stem end, and a third totally. First experiment: they were left 48 hours in water, and then taken out without wiping, and allowed to dry during 24 hours, they were then wiped and tried. Second experiment: afterwards, they were repeatedly immersed for a day or two, and taken out, being left to dry by themselves; this produced a large quantity of rust, which was rubbed off with soft paper, and then the forks were well oiled. The following were the results:—

Forks.	Original Pitches.	Pitches after immersion.	Alteration in V.	Alteration per cent. of V.
First Experiment—				
Prongs immersed . .	518·77	518·79	·0	·0
Bend immersed . .	528·20	527·90	− ·3	− ·0566
Totally immersed . .	258·77	258·63	− ·14	− ·0541
Second Experiment—				
Prongs immersed . .	518·77	518·44	− ·33	− ·06
Bend immersed . .	528·20	526·30	− 1·90	− ·36
Totally immersed . .	258·77	257·72	− 1·05	− ·41

"These experiments show that a slight amount of rust is imperceptible, and that with a very large amount, such as could not occur without the greatest carelessness, as in the old fork, described under A 454·2 in Table I., the error is never likely to exceed 4 in 1000. For measuring pitch this would be fatal, but for merely conveying the history of a pitch it is perfectly unimportant. Observe that rust towards the extremity of the prongs is of slight importance, and, in case of complete rusting, almost the whole effect is due to rust at the bend. In all cases, the effect is to flatten the fork.

"*The Reed.*—Harmonium reeds, placed in little tubes and blown by the mouth, may be classed with the pitch-pipes, convenient, but untrustworthy. The reed itself is apt to vary, and the pitch also depends greatly on the force of the wind. (See A 442·5 and A 488·0 in Table I.)" —*Journal of the Society of Arts.*

On the Variations of Pitch.

The first column of this table gives, in semitones and tenths of semitones, the gradual rise in the pitch of the note *a*,  and the second column gives the vibration numbers:—

Outline History of Musical Pitch.

S.	A.		
0·0	370	Ideal lowest, or zero-point.	Church Pitch lowest.
0·2	374	Hospice Comtesse, 1700.	
0·3	377	Schlick, low, 1511; Bedos, 1766.	
1·0	392	Euler's Clavichord, 1739.	Church Pitch low.
1·1	395	R. Smith, 1759; Roman pitch pipes, 1720.	
1·2	396	De Caus, 1615; Versailles Chapelle, 1789.	
1·4	403	Mersenne Spinet, 1648.	Chamber Pitch low.
1·6	407	Sauveur, 1713.	
1·7	408	Mattheson, Hamburg, 1762.	
1·7	409	Pascal Taskin, court tuner, 1783.	
2·0	415	Dresden chained fork, 1722.	European Mean Pitch for two centuries.
2·2	420	Freiberg, 1714; Seville, 1785.	
2·3	422	Mozart, 1780.	
2·3	423	Handel, 1751.	
2·4	424	Praetorius's suitable pitch, 1619; original Philharmonic, 1813.	
2·5	428	R. Harris, 1696; Opéra Comique, 1823.	
2·7	433	Sir George Smart's fork, 1820–26.	Compromise Pitch.
2·8	435	French Diapason Normal, 1859.	
3·0	440	Scheibler's Stuttgart Standard, 1834.	Modern Orchestral Pitch, and *Ancient Medium Church Pitch.
3·1	442	*Bernhardt Schmidt, low, 1690.	
3·2	445	Madrid, 1858; San Carlo, Naples, 1857.	
3·2	446	Broadwood's Medium, 1849; French Opera, 1856; Griesbach's A. 1860.	
3·4	449	=C 534; Griesbach's C 528, 1860.	
3·5	451	Lille Opera, 1848; British and Belgian Army, 1879.	
3·5	453	Mean Philharmonic, 1846–54.	
3·6	455	Highest Philharmonic, 1874; Broadwood, Erard, and (English) Steinway, 1879.	
3·6	456	Vienna, high, 1859.	
3·7	457	(American) Steinway, 1879.	
3·8	458	Great Franciscan Organ, Vienna, 1640.	Church Pitch high.
4·0	466		
4·3	474	Tomkins, 1668; B. Schmidt, high, 1683.	
4·5	481	St. Catherine's, Hamburg, 1543.	
4·8	489	St. James's, Hamburg, 1688.	
5·0	494	St. James's, Hamburg, 1879.	Church Pitch highest.
5·1	496	Rendsburg, 1668.	
5·3	504	Schlick, high, 1511; Mersenne, ton de chapelle, 1636.	
5·4	506	Halberstadt Cathedral, 1361.	
6·0	523		
7·0	554		Chamber Pitch hig hest.
7·3	563	Mersenne, ton de chambre, 1636.	
7·4	567	Praetorius, North German, very old.	

APPENDIX D.

EQUAL TEMPERAMENT.

The following extract from Mr. A. J. Ellis's paper, quoted from in the body of the work, will be found of interest:—

Art. 5. *Equal Semitones as a Measure of Relative Pitch.*—If we supposed that, between each pair of adjacent notes, forming an equal semitone as a piano is now intended to be tuned, 99 other notes were interposed, making exactly an equal interval with each other, we should divide the octave into 1200 equal hundredths of an equal semitone, or *cents*, as they may be briefly called. We generally estimate intervals in music by the number of semitones they contain; thus, the minor Third has 3, the major Third 4, the Fourth 5, the Fifth 7 semitones, and so on. In the same way, very small intervals, less than a semitone, may be estimated in cents. Thus, S 3·56 means an interval of 3 semitones and 56 cents. In this way, in Table I. the interval formed by the initial value of A in each entry with A 370 is given.[1] The interval between

[1] *To Calculate the Cents in any Interval.*—(1) For intervals less than an equal semitone—that is, when the larger V is not more than 6 per cent. larger than the smaller V: Divide 100 times the difference of the V by 6 per cent. (less 1 per 2000) of the smaller V to the nearest whole number. Thus, to find the interval between A 422·5 and A 440; 100 times the difference is 1,750, and 6 per cent. of 422·5 is 25·3, and this, less ·2 (or 1 per 2000 in 422·5), is 25·1; then, dividing 1750 by 25·1, we obtain 70 cents.

(2.) If the interval is more than an equal semitone, we can continually form equal tones and semitones, above the lowest, by adding 12½ per cent. for a tone, and 6 per cent. (less 1 per 2000) for a semitone, till we obtain a V which is less than an equal semitone from the larger number. Then, we find the cents in this smaller interval by the last rule, and add 100 for each equal semitone added on to the lower V.

any two such values of A is the difference of the corresponding S. Thus, the interval between A 455·3, S 3·59, and A 422·5, S 2·30, is S 1·29, which is the interval between Handel's and Erard's concert pitch.

At p. 785 of his translation of Helmholtz, Mr. Ellis gives the following rule for tuning sensibly in equal temperament:—

Equal Intonation is seen to be so bad in every respect, except the Fourth and Fifth (which produce the same

Thus, for A 422·5 and A 455·3, we form an equal semitone above ·422·5 by adding 25·1 (or 6 per cent., giving 25·3; less 1 per 2000, that is, ·2, giving 447·6). Next, we find the cents in the interval V 447·6 to V 455·3 to be 29, as in the last case, and we have 129 cents for the whole interval.

(3.) For intervals less than a just major Third—that is, when 8 times the larger V is not greater than 10 times the smaller V—multiply 3477 by the difference of the V's, and divide by their sum. If the result lies between 150 and 300, subtract 1 from the quotient; the result is exact. Thus, for the last example, 3477, multiplied by the difference 32·8, gives 114,045·6; and this, divided by the sum 877·8, gives 129 cents, as before. It is evident that this may be applied to any interval by continually reducing it by a just major Third till it is less than a major Third. This is effected by continually subtracting 10 times the smaller from 8 times the larger V, and adding 386⅓ cents to the result, for every such reduction. When the interval exceeds an Octave, divide the larger V continually by 2 till it is less than double the smaller; then proceed as before, and add 1200 to the result for each division by 2.

(4.) For any interval, by logarithms (by far the most convenien method for those who can use them), multiply the difference of the logarithms of the two V's by 4000 (which will be enough for intervals under a semitone); correct by subtracting 1 in 300 and 1 in 1000 from the former product. The result will be correct to one-tenth of a cent. Thus, log. 455·3 = 2·65830, log. 422·5 = 2·62583; difference = ·03247, which, multiplied by 4000, gives 129·88. Subtract 1 in 300, or ·43, and 1 in 10,000, or ·01 (sum ·44), and the result is 129·44 cents, or 129 to the nearest cent. When many cases have to be calculated, it is best to form a little table of the multiples of 39·86314, and, by its means, multiply the difference of the logarithms by that number. This was the method pursued for Table I.

results as in the unequal chords of practical intonation), that nothing but the fewness of the notes which it requires (as a cycle) to modulate infinitely, could have brought it into notice. The great harshness of its major Thirds and minor Sixths, and of its minor Thirds and major Sixths, its strikingly inharmonic differential tones, and their very distinct beats (all of which stand out in strong relief on the ordinary harmonium), render it quite unfit for the performance of any music with sustained tones and delicate harmonies except with qualities of tone for which the partial tones above the fourth are scarcely audible. It is totally unfit for part singing or for stringed quartets, or 'double-stop passages' on a violin, or for the compound stops of organs. This refers only to its effect in harmonies. Its effect in melody is altogether different, because in that it can produce no beats. Those whose musical ear has grown to the melody of equal temperament necessarily find mean tone temperament, or just intonation, with flatter major Thirds, Sixths, and Sevenths, dull, heavy, and disagreeable, and probably in view of the melody they like, overlook the harmony, and do not remark its bad effects. A concertina-tuner, who tuned justly intoned chords for me, told me he did not like them; he missed the beat. This reminds one of the man who could not sleep at nights in the country, it was so dreadfully quiet. But a little habit produces a very different state of feeling. At the same time it is well to remember that our rules of harmony grew up under the mean-tone temperament, and that although the Chinese prince Tsai-yu, in A.D. 1573, that is, two centuries before equal temperament was common in Europe, gave the exact lengths and bores of 12 pipes in correctly equal temperament (given on Chinese authority by Amiot on his p. 105), yet the Chinese harmony never extended beyond the occasional use of an Octave, for which, of

course, equal temperament is perfectly suitable. As mentioned on p. 656, strictly equal temperament is a thing unknown in practice, owing to the extreme difficulty of tuning the Fifths by estimation of ear. 'Many years ago,' says Prof. de Morgan ('On the Beats of Imperfect Consonances,' Trans. of the Cambridge Phil. Soc. 9, Nov. 1857, vol. 10, p. 142), 'I had two *dulcimers*, as I suppose they must be called, of a couple of Octaves each. The notes were given by single strings, and the sound was produced by a hammer held in the hand; they stood exceedingly well in tune, and the sound was as pure as that of a tuning-fork. When I tuned one to equal temperament, as I thought, and then the other, I never found agreement, though each was satisfactory by itself. I soon left off, setting down the discordance to my own inexperience. But an old professional tuner, to whom I mentioned the subject, assured me that he did not believe either that any tuner *gained* equal temperament, or that any one tuner agreed with himself or with any other. He summed up by saying that "equal temperament was equal nonsense."' For all ordinary pitches of c' a very decent (although unequal) *imitation* of equal temperament may be effected by making the Fifths $c'\ g'$, $c'\sharp\ g'\sharp$, $d'\ a'$, $d'\sharp\ a'\sharp$, $e'\ b'$ (lying within the Octave $c'\ c''$) beat *once* in *one* second, and the Fifths $f'\sharp\ c''\sharp$, $g'\ d''$, $g'\sharp\ d''\sharp$, $a'\ e''$, $b'\flat\ f''$, $b'\ f''\sharp$ (in which the higher note is in the Octave above) beat *three* times in *two* seconds; in this case $f'\ c''$ will beat about *once* in between *one* and *two* seconds. The order of tuning is $c'\ g'$, $g'\ d''$, $d'\ g'\ d''$, $d'\ a'$, $a'\ e''$, $e'\ a'\ e''$, $e'\ b'$, $b'\ f''\sharp$, $f''\sharp\ b'\ f''\sharp$, $f''\sharp\ c''\sharp$, $c''\sharp\ f''\sharp\ c''\sharp$, $c''\sharp\ g''\sharp$, $g''\sharp\ d''\sharp$, $d''\sharp\ g''\sharp\ d''\sharp$, $d''\sharp\ a''\sharp$, $a''\sharp\ f''$, that is $b'\flat\ f''$. The beats can be counted by a half-second's pendulum of an ordinary clock, or by a weighted string having a length from centre of suspension to centre of bob, of 9·7848 inches, or nearly enough $9\frac{7}{8}$ inches. The following table will shew the

extreme closeness of the approximation thus obtained. It must be remembered that an interval of ·01 equal semitones is barely perceptible, and that to distinguish an interval of ·0087 equal semitones by means of delicate apparatus is considered to be a great feat. The vibrational numbers have been calculated from the formulæ.

$$2g' = 3c' - 1,\quad 4d'' = 3g' - 1{\cdot}5,\quad 2a' = 3d'' - 1,\quad 4e' - 3a' - 1{\cdot}5,$$
$$2b' = 3e' - 1,\quad 4f' = 3b' - 1{\cdot}5,\quad 4c'\sharp = 3f'\sharp - 1{\cdot}5,\quad 2g'\sharp - 3c'\sharp - 1,$$
$$4d''\sharp = 3g''\sharp - 1{\cdot}5,\quad 2a' = 3d''\flat - 1,\quad 4f'' = 3a''\sharp - 1{\cdot}5,$$

taking c' first = 256, and next = 264, and the resulting numbers are contrasted with the corresponding exact numbers for equal temperament in adjoining columns, and a final column gives the error from the true equal tones expressed in equal semitones for each case.

Note.	Equal.	Imitation.	Too flat by equal Semitones.	Note.	Equal.	Imitation.	Too flat by equal Semitones.
c'	256	256	·000	c'	264	264	·000
$c'\sharp$	271.23	271·09	·009	$c'\sharp$	279·70	279·64	·004
d'	287·35	287·25	·006	d'	296·33	296·25	·005
$d'\sharp$	304.44	304·23	·011	$d'\sharp$	313·95	313·84	·007
e'	322·61	322·41	·007	e'	332·62	332·53	·005
f'	341·72	341·51	·001	f'	352·40	352·32	·004
$f'\sharp$	362·04	361·96	·003	$f'\sharp$	373·35	373·35	·000
g'	383·57	383·50	·003	g'	395·55	395·50	·002
$g'\sharp$	406·37	406·14	010	$g'\sharp$	419·07	418·96	·005
a'	430·54	430.38	·006	a'	443·99	443·88	·004
$a'\sharp$	456·15	455·84	·011	$a'\sharp$	470·39	470·26	·005
b'	483·26	483·11	·001	b'	498·37	498·30	·002

APPENDIX E.

THE VOICE.

Mr. A. J. Ellis gives, in his paper "On the History of Musical Pitch," the following results of his investigations into the compass of the human voice. It is hardly possible to praise too highly the industry and enthusiasm which have led Mr. Ellis to crowd into one paper information of such immense value:—

"*Compass of the Human Voice.*—From the remarks just made, it will be readily seen that the importance of musical pitch consists in the nice adjustment of the work required from the voice to the work it is capable of performing. Hence, the first requisite to understanding the variations of pitch, or the value of any pitch proposed, is to know, in numbers of vibrations, the average limits of each kind of voice for which composers write, and on which instruments, by accompanying, impose an interpretation. It is clearly essential that no instrument should make a composer expect an impossible performance. Now, I was unable to find any satisfactory solution of this problem. The ranges of the several voices are given by Mr. Alberto Randegger, in his Primer on 'Singing' (Novello, 1879), expressed in notes, which, as he kindly informed me, referred to Broadwood's medium pitch A 446·2, so that they can be immediately translated, assuming equal temperament, into numbers of vibrations. But, proceeding only by equal semitones, the intervals were too large, and although, doubtless, due to great experience, they did not seem to be founded on a sufficiently extensive observation of chorus singers; hence, I felt compelled to undertake the

investigation myself. I am indebted to the great kindness and liberality of the choir conductors, Messrs. Henry Leslie, W. G. McNaught, J. Proudman, Ebenezer Prout, L. C. and G. I. Venables, and 542 members of the choirs they conduct, for having been able to try it upon a sufficiently large number of voices, to furnish a trustworthy mean. The following Table gives the numbers of each kind of voice examined, rejecting duplicates :—

Choirs in Order of Examination.	S.	A.	T.	B.	Totals.
1. Bow and Bromley Institute Choir; conductor, Mr. W. G. McNaught	22	20	23	29	94
2. Mr. Proudman's Voice-training Class, at the Tonic Sol-fa College Winter Classes	8	1	10	2	21
3. South London Choral Association, Advanced Choir; conductor, Mr. L. C. Venables	40	25	26	33	124
4. Tonic Sol-fa Choral Association, Select Choir; conductor, Mr. Joseph Proudman	20	11	10	12	53
5. South London Choral Association, Intermediate Choir; conductor, Mr. G. I. Venables	30	13	11	23	77
6. Borough of Hackney Choral Association; conductor, Mr. Ebenezer Prout	38	25	19	24	106
7. Mr. Henry Leslie's Choir; conductor, Mr. Henry Leslie	18	13	18	18	67
Total independent voices	176	108	117	141	542

"These numbers by no means represent the full numbers of the members of these choirs, but only the few who happened to be present on the practising nights that I attended, during very unfavourable weather. Thus, I attended Mr. Henry Leslie's choir during the dense fog of 27th January 1880, when the room itself was full of fog, and the attendance was, consequently, extremely limited.

"The method adopted was as follows :—I procured four

forks, tuned to V 507, 522·5, 528, and 540·7, of which the three first represent the *just* C, corresponding to Handel's A 422·5, French Normal A 435·4, and Scheibler's A 440, and the last is Mr. Hipkins' fork, representing the highest Philharmonic pitch in 1874, giving EA 454·7, but JA 450·6. The first and last forks are a diatonic semitone apart; the two middle ones are a little less than half that from each extreme. I had printed four descending scales marked *do, si, la, sol, fa, mi, re, do,* si_1, &c., in words or letters, and four ascending scales marked *do, re, mi, fa, sol, la, si,* do^1, in the same way. Then I asked the conductor to pitch the voices of the choir to *do* from the first fork, and the choir to sing the scale down twice, the first time marking, by scratching out, the lowest note to which each voice (after noting its own name) could sing *easily*, and the *extreme* lowest note it could sing at all, and at the second singing to revise its marks. Next, the voices took an ascending scale to the same pitch, and marked the highest *easy* note (male voices avoiding *falsetto*), and highest *extreme* note (admitting *falsetto*). The three other pitches were treated in the same way. As unaccompanied singers (especially the Tonic Sol-faists) naturally sing in just intonation, it was easy to calculate almost exactly the number of vibrations in each of the limiting notes. Then taking from the paper of each voice its easy and extreme limits, certainly found within a quarter of a tone, I was able, by adding the vibrations, and dividing by the number of voices, to get the average compass of each kind of voice, expressed in vibrations, and to compare them with Mr. Randegger's, expressed in the same way. The following Tables express the results obtained:—

"The first Table gives the more important *easy*, lower and higher limiting notes of the compass of each kind of voice. The column headed *mean* gives, in name and

number of vibrations, the mean note reached by the voices in the first column. The column headed *actual* gives the highest and lowest note actually sung, of all the notes from

TABLE I.—MEAN AND ACTUAL COMPASS OF THE HUMAN VOICE.

Voices observed.	Easy Lower Limit.	
	Mean.	Actual.
149 Sopranos	4 F 180·2	4 PB 253·4 to 4 PC 135·2
91 Altos	4 E *flat* 161·3	4 HA 211·3 to 4 SC 132
107 Tenors	8 G 98·2	4 FE 163·3 to 8 PD 76
125 Basses	8 E 81·2	8 HA 105·6 to 8 SC 66

Voices observed.	Easy Higher Limit.	
	Mean.	Actual.
145 Sopranos	1 B 993·2	½ SF 1408 to 1 SG 704
83 Altos	1 G *sharp* 835·7	½ PD 1216·4 to 1 PE 675·8
114 Tenors	1 C 520·8	1 PD 608·2 to 2 HE 316·9
120 Basses	2 F *sharp* 375.2	1 PC 540·7 to 2 FD 293·9

which the mean was calculated. In this column the letters H, F, S, P, refer to the pitches derived from Handel's, the French, Scheibler's, and the Philharmonic forks, all calculated in just intonation.

TABLE II.—MEAN AND ACTUAL COMPASS OF THE HUMAN VOICE.

Voices observed.	Extreme Lower Limit.	
	Mean.	Actual.
173 Sopranos . . .	4 E *flat* 161·9	4 PG 202·8 to 4 FC 130
108 Altos	4 D 147·1	4 SG 198 to 8 SB 123·8
114 Tenors	8 E 84·7	8 PB 126·7 to 8 SC 66
140 Basses	8 C *sharp* 71·6	8 PF 90·1 to 16 PA 56·3

Voices observed.	Extreme Higher Limit.	
	Mean.	Actual.
173 Sopranos . . .	½ C *sharp* 1124·4	½ HA 1690 to 1 PG 811
105 Altos	1 B *flat* 951·6	½ SG 1584 to 1 PF 721
112 Tenors	1 D 616·9	1 PG 811 to 2 SG 396
139 Basses	2 B *flat* 482·9	½ PC 1081·4 to 2 SE 330

"The second Table gives the extreme limits in the same way, but these are not of so much importance, and in particular, the extreme higher limits of the bass and tenor, which include falsetto, are rather curiosities than otherwise.

"The third Table gives Randegger's regular and exceptional limits of the same voices, but, for comparison, the soprano and mezzo-soprano are placed together, and the tenor and baritone, instead of, as is more usual and natural, the mezzo-soprano with the alto, and the baritone with the bass.

TABLE III.—RANDEGGER'S STATEMENT OF LIMITING TONES EXPRESSED IN NUMBER OF VIBRATIONS.

REGULAR.				
Voice.	Lower Limit.		Upper Limit.	
Soprano	4 B *flat*	236·4	½ C	1061·2
Mezzo Soprano	4 G	198·8	1 B *flat*	945·4
Alto	4 E	167·2	1 F	708·4
Tenor	4 C	132·6	2 B *flat*	472·7
Baritone	8 A *flat*	105·0	2 F	354·2
Bass	8 F	88·6	2 E *flat*	315·5
EXCEPTIONAL.				
Voice.	Lower Limit.		Upper Limit.	
Soprano	4 B *flat*	236·4	½ F	1416·8
Mezzo soprano	4 G	198·8	½ C	1061·2
Alto	4 E	167·2	1 G	795·0
Tenor	4 C	132·6	1 C *sharp*	562·2
Baritone	4 F	88·6	2 G	397·5
Bass	8 D	74·5	2 F	354·2

"On the chart I exhibit, the whole results are written out in musical notation, in Broadwood's medium pitch, tailed white notes representing the means, and black notes with the tails turned different ways, the highest and lowest. In doing so, the notes were taken as named, except those marked H, which were taken half a tone flatter, thus HA is represented by A *flat*. On a piano, in concert pitch, no note will be then more than a quarter of a tone in error, and the table can easily be played. In the present paper, musical notation had to be avoided, for typographical reasons.

"A strict comparison with Randegger will show great differences, for which probably good reasons can be given; and it should be borne in mind that Randegger's 'exceptional' does not exactly correspond to my 'extreme.' The limits of this paper do not allow of a full discussion of these results, but reference will have to be made to them under A 503·7 and A 567·3, in Table I. below, for which the observations were mainly undertaken. In the meantime, observe that music for choruses should *not* be written for the average or mean limits, as probably one half the chorus could not reach those limits easily, and those that could would be distressed by them if they occurred frequently. But it is always safe to write from the actual highest form of the upper limit. Thus, it would not be safe to write for soprano choristers from the mean 4 F 180·2 to the mean 1 B 993·2, but it would be quite safe to write from the higher actual form of the lower limit, 4 PB 253·4, to the lower actual form of the upper limit, 1 SG 704. But even then, long frequent sustained and forte passages involving these notes would distress. Taken in the last-named form, the limits will be found to agree better with Randegger's."
—*Journal of the Society of Arts.*

APPENDIX F.

The following notes of two lectures on "Musical Acoustics," delivered at the Royal Academy of Music by Mr. Sedley Taylor, will be useful as aids to memory before an examination :—

1. Sound can pass through solid, liquid, and gaseous bodies, but not through a vacuum.

2. Waves have a progressive motion, which is not that of the materials of which they, at any given instant, consist.

3. The particles, or smallest conceivable portions, of a medium which is being traversed by a wave, perform successive vibratory movements.

4. During the time occupied by any individual particle in performing one complete vibration, the system of waves advances through one whole wave-length.

5. Sound-waves are due to longitudinal vibrations of the particles of the conveying medium.

6. Each sound-wave consists of a pulse of condensation, and a pulse of rarefaction.

7. The mechanical cause of the transmission of sound-waves is the elasticity of the medium they traverse.

8. A musical sound is due to a continuous series of rapid equal vibrations.

9. Musical sounds may differ from each other in loudness, pitch, and quality.

10. Loudness depends on extent of vibration.

11. Pitch depends on rate of vibration.

12. The instrument called the Syren enables us to de-

termine the number of vibrations, per second, which correspond to a single note of assigned pitch.

13. This number is called the vibration-number of the note in question.

14. The interval between any two notes is measured by the ratio of their respective vibration-numbers.

15. When one sounding instrument causes another to give out sound, the effect is called resonance.

16. Resonance is due to the accumulated impulses of a large number of sound-waves.

17. An air cavity of given volume and form resonates most powerfully to a note of one definite degree of pitch.

18. Resonance gives us the means of singling out a note of assigned pitch from among an assemblage of notes of various degrees of pitch sounding at the same time.

19. Each sound produced by any of the musical instruments ordinarily in use is not a single musical note, but consists of a number of such notes differing from each other very considerably, both in pitch and loudness.

20. These notes are called the partial-tones of the compound sound. They form a fixed series, and their vibration-numbers, reckoned from the lowest partial-tone upwards, are proportional to the numbers, 1, 2, 3, 4, 5, &c.

21. The quality of a musical sound depends on the number, orders, and relative intensities of the partial-tones into which it is resolved by the ear.

22. Sounds of different quality may, to a certain extent, be artificially imitated by combining simple tones independently produced.

23. Two sets of equal waves traversing a medium at the same time and in the same direction, may, under certain circumstances, entirely obliterate each other.

24. An instance of this phenomenon occurs when two sounds produce silence.

25. When two simple tones, very nearly, but not quite, in unison, are sounded together, slow alternations both of loudness and of pitch succeed each other at equal intervals of time.

26. These alternations are called beats.

27. The number of beats per second is equal to the difference betweeen the vibration-numbers of the two beating tones.

28. When beats become too rapid to be separately perceived, they produce the effect we call dissonance.

29. The dissonance arising from the beats of two simple tones is harshest when the interval between the tones is about half a note, considerably less harsh when the interval amounts to a whole note, and for intervals greater than a note and a half, imperceptible.

30. The last mentioned interval may conveniently be called the "beating-distance" for two simple tones.

31. When two composite sounds are heard at once, dissonance arises between every pair of non-coincident partial-tones which are within beating-distance of each other.

32. The whole roughness of the combination is equal to the sum of the separate dissonances thus arising.

33. Two composite sounds are said to form an absolute concord if none of their partial-tones are so situated as to produce dissonance with each other. In the opposite case, the combination is classed according to the number, intensity, and situation of the beating partial-tones, as a more or less imperfect concord, or as a discord.—(HELMHOLTZ'S *Theory of Harmony.*)

34. The system of "equal temperament," on which instruments with fixed sounds are now tuned, tends to efface the sharpness of definition by which, with perfect tuning, the smoother concords of the scale are separated from the neighbouring discords.

Sound is not something solid or tangible
[illegible] the air.
Sound is produced by the passing of air
[illegible] portion of air from
one point [illegible]
[illegible] may pass along
[illegible] motion of the
[illegible] by it.

EXAMINATION QUESTIONS.

CHAPTER I.

SENSATION AND EXTERNAL CAUSE OF SOUND.

Define sound.

What is the difference between the cause of sound and the sensation of sound?

What is the actual cause of all sound?

Prove that a medium is necessary for the transmission of sound.

What is the difference between sound and noise?

What are the limits beyond which sounds are not audible to an average ear?

What are the limits of sounds available for music?

CHAPTER II.

TRANSMISSION OF SOUND.

How are vibrations transmitted from one place to another?

Describe an experiment for illustrating the elasticity of air.

State three important principles respecting the transmission of sound.

How is the velocity of sound determined?

What is the velocity of sound at 30° Fah? At 40°? At 50°?

CHAPTER III.

NATURE OF WAVE MOTION IN GENERAL.

Describe by diagrams the formation of waves of water.

What is the leading principle to remember in connection with the progress of waves?

Give a familiar proof that waves are formed by limited movements among single particles.

What are the characteristics of waves?

What is the difference between length and width of wave? Between length and form?

Draw waves of the same length but different width; the same width but different length; the same length and width but different form.

With what musical characteristic does each of these features correspond?

What is the cause of these differences in wave-form, length and width?

Draw diagrams illustrating the formation of wave-forms by a pendulum.

State how the motions of single drops affect length, width and form?

CHAPTER IV.

APPLICATION OF THE WAVE THEORY TO SOUND.

What is the action of a tuning-fork on a particle of air?

Draw a particle of air in (*a*) condensation and (*b*) rarefaction, and show how a wave-form is produced by the alternation of these two conditions.

What governs the length of the wave? What the width?

What is "Mariotte's Law"? How does it apply to condensed or rarefied air?

CHAPTER V.

ELEMENTS OF A MUSICAL SOUND.

Prove that loudness depends upon the width of the air-wave.

Show how to demonstrate that the pitch of a sound depends upon the rate of vibration.

Describe a machine for demonstrating this fact.

How many revolutions per second are requisite to produce C = 256 from a disc of 16 holes? of 12 holes? of 8 holes?

Give the vibration-number of each of the following, if the tonic = 128: fifth; major third; fourth; major sixth; major second; major seventh.

Write in order the fractions showing the proportion of each tone of a major scale to the tonic.

What is the principle of the syren? Describe two or three varieties of the instrument.

A syren has six rows of holes as follows: 8, 10, 12, 14, 16, 18. If the disc revolves 12 times per second, and the tone produced by each row at that pace (96, 120, 144, &c.) be taken as a tonic, calculate the vibration numbers of each note in these six scales.

How would you find the vibration number of any note struck on the pianoforte?

What do you mean by "absolute" and "relative" pitch?

If the absolute pitch of the first note of the tune known as the "Old 100" be 440 vibrations per second, calculate the vibration numbers of the whole tune, and write them under their respective notes.

Describe four different modes of measuring pitch.

State what you know of the variations in standard pitch at different periods.

CHAPTER VI.

RESONANCE.

What is resonance?

Give familiar examples.

If a tuning-fork be struck and held near another of the same pitch, what is the result?

What is the mechanical cause of resonance?

What must be the exact length of a column of air in a closed vessel to reinforce a particular sound?

Describe the effect of resonance in organ pipes.

What is the length of a column of air which will reinforce a tenor C tuning-fork in a closed pipe? What in an open pipe?

What proof of resonance is given by the pianoforte?

What use has Helmholtz made of the phenomena of resonance?

CHAPTER VII.

THE ANALYSIS OF COMPOUND SOUNDS.

What is the difference between a simple and a compound sound?

Write down musically the first six compound tones of

What name has been given to the constituent parts of compound sounds?

What is the law as to their vibration numbers?
Calculate the vibration numbers of the first six upper partials of a tone whose vibration number is 96.
What other name has been given to these upper partials?
Describe an experiment to prove the compound nature of pianoforte tones.
What are the intervals between a tone and its first, second, third, and fourth upper partials respectively?
Write down C on the second ledger line below the bass stave and add to it as many of its upper partials as you can.
What artificial aid did Helmholtz use to detect upper partial tones?
How is quality affected by upper partials?
How many degrees of intensity are possible for tones having three upper partials—that is, three partials beside the prime tone?
What is the characteristic of simple tones?
What is meant by pendular vibrations?
What is Fourier's theorem with regard to compound vibration forms?
What is meant by a difference of phase?
Explain Ohm's law with regard to the analysis of compound sounds.
When the ear cannot dissect a compound tone, by what means does it detect the presence of upper partials?

CHAPTER VIII.

HELMHOLTZ'S THEORY OF MUSICAL QUALITY.

Name some musical tones in which there are no upper partials.
By what name do you distinguish these?
What is the peculiarity of tones compounded of the prime and the first five upper partials?
Account for the crisp and cutting quality of tones having many high upper partials.
Can simple tones vary in quality? Give a reason for your answer.
Name some tones characterised by harmonic upper partials.
What regulates the quality of tone produced by a struck string?
Which is the softer, a tone produced by plucking a string with the finger, or one by striking it with a sharp instrument? Why?
Which of the two qualities named in the last question has the greater number of high upper partials, and why?
What is the result if any upper partial has a node at the point struck?
Explain the usual position of the blow in pianoforte strings, and show its advantages.
What peculiarity attends the upper partials of violin tones?

Why does bowing near the bridge make a louder, and near the finger-board a softer, quality of tone?
Draw a rough diagram showing the form of wave of a violin tone.
What is the actuating cause in producing the sound of organ flue pipes?
What is the effect of increased pressure of wind?
What effect has a wide pipe on the upper partials, and what a narrow one?
Explain the reason why an unbroken series of upper partials is given by a stopped pipe.
What is the character of reed tones?
Explain why upper partials prevail in reed tones.
What effect has the pipe on reed tones?
What is Fourier's theorem? Draw a diagram illustrating it.
What is the effect of contrary and equal waves on each other.
Give diagrams showing the effects of various waves upon each other.

CHAPTER IX.

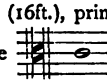
THE MOTIONS OF SOUNDING STRINGS.

What is the simplest form of string vibration?
Describe a node and a ventral segment.
Apply to string-vibration the principle referred to at the end of Chapter VIII., and state the effect of crest on crest, trough on trough, and crest on trough, of a segment of a vibrating string, giving diagrams on the model of those on p. 187.
How does the length of a string (other things being equal) affect its vibration?
What qualities of a string affect its pitch?
How many upper partials are there in the tone of a string vibrating in six segments?
What is the principle connecting segments with upper partials?
Describe an experiment proving that strings vibrate in segments.
Describe the effects of heat upon strings.

CHAPTER X.

THE MOTION OF SOUNDING AIR-COLUMNS.

What are the peculiarities of vibration in stopped pipes and in open pipes? Draw diagrams on the model of those on p. 208, &c., to illustrate your answer.

Describe Tyndall's experiments with organ pipes.
What is the law governing vibrations of air columns?
Show the connection between the segments of vibrating air-columns and the formation of upper partials.
Give the rule for finding the pitch of a pipe from its length.
The length of a stopped pipe being 2 feet, what is the vibration number of its tone?
Make the calculation for an open pipe of the same length.
Give the rule for finding the length of a pipe from its pitch.
Calculate the length of a stopped pipe if its prime vibrates 256 times per second.
An open pipe gives a note vibrating 1024 times per second; how long is the pipe?
How long must a stopped pipe be to give the same note, vibrating 1024 times per second?
What class of pipes form the main body of an organ?
What is the length of the lowest pipe of the principal, twelfth, and fifteenth respectively?
If the open diapason, bourdon (16ft.), principal, twelfth, and fifteenth are drawn, and the note  is pressed down, write musically the notes which will be heard.
How are the tones of reed pipes varied?
What are the three leading varieties of orchestral wind instruments?
Give a short description of the acoustic qualities of the oboe, flute, clarionet, and bassoon.
Give the open notes of the French horn, trumpet, and other similar instruments.
What is the principle governing the production of the series of notes in instruments of the horn class?

CHAPTER XI.

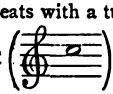
THE HUMAN VOICE.

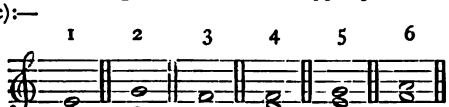
To what class of instruments does the voice belong?
What is the actual voice-producing organ?
Describe separately the parts producing the voice.
Name the cartilages of the larynx.
Draw a diagram showing the tension and relaxation of the vocal cords (see pp. 245–6).
Say how the vocal organs are situated with respect to the organs which assist them in their work.

What is the average compass of each of the four usual voices?
Is the voice a simple tone?
Explain your answer. Describe Helmholtz's experiments in relation to this point.
State what you know of the peculiarities of human vowel-tones.

CHAPTER XII.

BEATS.

How are beats produced?
Give a familiar experiment showing that one vibration will nullify another so as to produce no sound.
What is the general principle of "interference"?
Describe in detail the mode in which beats are produced, say by two tones vibrating respectively 120 and 123 tones per second.
Draw a figure (after that on p. 266) showing how 4 and 4½ vibrations per second will produce beats.
What is the rule for calculating beats?
If each of the two wires giving "tenor C" on a pianoforte vibrates 256 times per second, and one be tuned with the hammer until it vibrates 263 times per second, how many beats per second will the two sounds make?
If, without altering the sharper wire, the other is sharpened till it vibrates 260 per second, how many beats per second will then result?
Write a short essay of ten or twelve lines describing the phenomenon of beats.
How would you produce beats with a tuning-fork?
If one prong of a treble C () tuning-fork makes 8 beats per second with the other, give the vibration number of each prong, that of "tenor C" being 256, and each being equally out of tune.
What is the limit of beating distance?
What becomes of beats which cannot be counted? and what result have they on the quality of the tones producing them?
What is meant by the beats of upper-partials?
Show musically the beats between the upper-partials of these pairs of notes, calculating as far as the sixth upper-partial (seventh harmonic):—



To what practical uses are beats put?
Of what use is the syren in calculating beats?
Describe Tyndall's mode of rendering beats visible.

CHAPTER XIII.

HELMHOLTZ'S THEORY OF CONSONANCE AND DISSONANCE.

Define the words "consonance" and "dissonance."
What have beats to do with "perfect" and "imperfect" intervals?
What do you mean by one interval being "rougher" than another?
Calculate the vibration numbers caused (1) by the primes, and (2) by the upper partials of the pairs of notes given in the questions on Chapter XII.
What causes "consonance" and "dissonance?"
At what rate per second do beats cease to be recognisable?
What intervals does Helmholtz class as consonant?
Are all consonant intervals harmonious in a like degree? Give reasons for your answer.

CHAPTER XIV.

COMBINATIONAL TONES.

What are combinational tones?
What is the rule for calculating their vibration numbers?
What are (1) differential tones; and (2) summational tones?
Write out the chief intervals of the octave and the combinational tones they make with each other.
What instruments are best suited for experiments with combinational tones?

CHAPTER XV.

CONSONANT CHORDS.

By what is the consonance of chords affected?
What intervals are improved by having one of their tones transposed an octave? Which are made worse?
What are the most perfect positions of (1) major, and (2) minor triads?
What are the most perfect positions of (1) major and (2) minor chords of four notes?
Give notable illustrations of the application of these truths in compositions by the great masters.

CHAPTER XVI.

SCALES AND TEMPERAMENTS.

Are our modern major and minor scales "founded on nature" or not? Give reasons for your answer.

Calculate the vibration numbers of the major scale on "tenor C."

Are the intervals from C to D and from D to E in the scale of C equal? If there is any difference, state it.

State the "greater" tones of the major scale, and the "lesser" tones.

How does Helmholtz account for the selection of the notes of the present scale?

What is Airy's explanation?

What nations do not take our major scale as the basis of their music?

What are the Greek modes? Name them, and state wherein they differ from each other.

What is the Greek greater scale? Give a diagram of it, and select from it the scale of each of the modes.

What do you mean by a tempered scale?

Why is it necessary to temper scales?

Which is the higher—B♯ or C?

Describe Just Intonation, Meantone Temperament, and Equal Temperament.

What devices have organ-builders adopted to obviate the difficulties of temperament?

What system was in vogue until about half a century ago?

Which system is usually adopted now?

Is it possible to divide an octave into "twelve equal semitones"? Give reasons for your answer.

Can octaves be formed by just fifths, or by just thirds, or by both combined?

C being 264, find the vibration number of A below.

A being 444, find C above it.

Give the rule for tuning in "equal temperament."

CHAPTER XVII.

SYSTEMS OF PITCH NOTATION.

What are the essential features of a good system of musical notation?

Name the chief systems in use.

What are the great advantages of the "staff notation" over all others?

Name two important features possessed by the staff notation which are absent from all other systems.

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